

• Properties of Systems

Theoretical summary:-

- 1- **MemoryLess System:-** memoryless if present output depends only on present input;

Ex. A) $y(t) = a x(t)$ B) $y[n] = (2x[n] - x^2[n])^2$

Otherwise it's a system with memory;

Ex. A) $y[n] = x[n-1]$ B) $y(t) = \int_{-\infty}^t x(T) dT$ C) $y[n] = y[n-1] + x[n]$

- 2- **Causal System:-** causal if the output at any time depend on the input at present and/or past times.

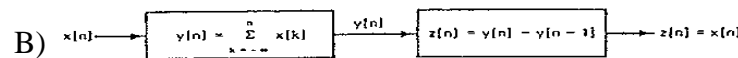
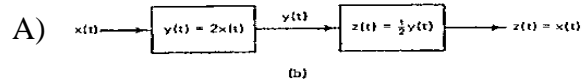
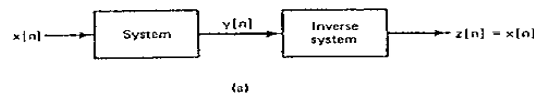
(\therefore all memoryless systems are causal) Ex; see memoryless examples

Non causal; output depends on future input

Ex. A) $y(t) = x(t+1)$ B) $y[n] = x[-n]$

- 3- **Invertibility:-** a system $y[t]$ is invertible if an inverse system $w[t]$ exists, such that when $w[t]$ is cascaded with $y[t]$; the output is the original input $x[t]$.

Ex.



Non Invertible system Ex. $Y(t) = x^2(t)$ (we cannot determine the sign of the input)

- 4- **Time Invariance:-** invariant if a shift in the input results in in an identical shift in the output

Ex. A) $y(t) = \sin[x(t)]$

Time variant Ex. A) $y[n] = n x[n]$

- 5- **Linearity:-** a system is linear if satisfies;

a) additivity :- $x_3(t) = x_1(t) + x_2(t) \quad \rightarrow \quad y_3(t) = y_1(t) + y_2(t)$

b) homogeneity:- $x_4(t) = \alpha x_1(t) \quad \rightarrow \quad y_4(t) = \alpha y_1(t)$

or check both together $x_3(t) = \alpha x_1(t) + b x_2(t) \quad \rightarrow \quad y_3(t) = \alpha y_1(t) + b y_2(t)$

- 6- **Stability:-** if bounded inputs results in bounded outputs (BIBO);

Ex. $Y[n] = \frac{1}{2M+1} \sum_{k=-M}^M x[n-k]$ (Note; always assume $x[n]$ a bounded signal)

non stable system Ex. $Y(t) = t x(t)$.

Q1) State YES/NO to indicate whether or not, the systems in (a)-(c) in the table below possess / do not possess the properties shown in the table. [Note: $x(t)$, $x(n)$ are system inputs while $y(t)$, $y(n)$ are the corresponding outputs].

	Memoryless	Causal	Stable	Time Invariant	Linear	Invertible
a) $y[n]=\cos(x[n])$						
b) $y(t)=x(t/3)$						
c) $y(t)=\int_{-\infty}^{2t} x(T)dT$						