- ## LTI systems & Convolution

Theoretical summary

If a system is linear and time-invariant (LTI), if the input is the unit impulse, the output is called the impulse response h[n].

$$\delta[n] \rightarrow \boxed{\text{system}} \rightarrow h[n]$$

$$\delta[n-k] \rightarrow \boxed{\text{system}} \rightarrow h[n-k]$$

$$x[k]\delta[n-k] \rightarrow \boxed{\text{system}} \rightarrow x[k]h[n-k]$$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \rightarrow \boxed{\text{system}} \rightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

∴ Response of an LTI system to an arbitrary input is;

$$x[n] \rightarrow \boxed{\text{LTI}} \rightarrow y[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \equiv (x * h)[n]$$

This operation is called **convolution**.

It is customary (but confusing) to abbreviate this notation:

$$\boxed{y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = (x * h)[n] = x[n] * h[n]}$$

Note:- $x[1] * h[1] \neq (x * h)[1]$; Convolution is an **operation on signals** not samples [1].

➔ Similarly in CT we find → $\boxed{y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau}$

- **Properties of convolution:-**

1- Commutative:- $x(t) * h(t) = h(t) * x(t)$

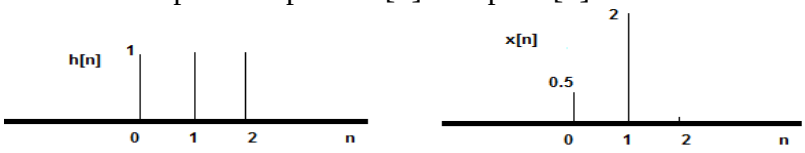
2- Distributive:- $x(t) * [h_1(t) + h_2(t)] = x(t)*h_1(t) + x(t)*h_2(t)$

3- Associative :- $x(t) * (h_1(t) * h_2(t)) = (x(t) * h_1(t)) * h_2(t)$

Note:- convolution with unit Impulse; $\boxed{x(t) * \delta(t-to) = x(t-to)}$

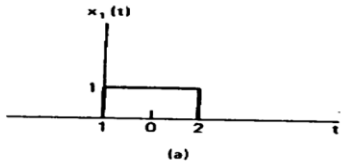
[1] Prof. Dennis Freeman, 6.002, OCW, MIT.

Q1) Consider an LTI system with impulse response h[n] & input x[n] as illustrated. Find the convolution sum.

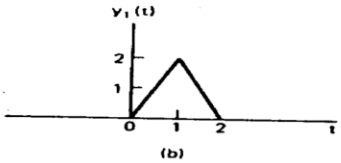


Q2) For a LTI system given that $h(t) = e^{-2t}\,u(t)$ and $x(t) = e^{2t}\,u(-t)$, determine the output $y(t)$.

Q3) Consider an LTI system whose response to the input $x_1(t)$ is the signal $y_1(t)$. Determine and sketch the response of system to $x_2(t)$ $x_3(t)$



(a)



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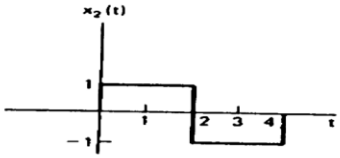

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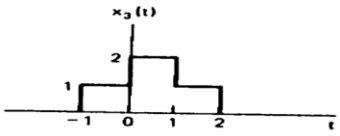

(d)