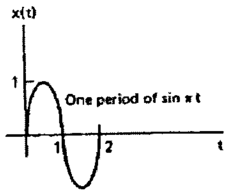
- **Classification of LTI systems** (using the impulse response):-

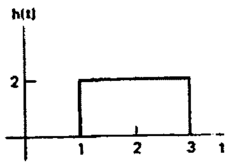
1- Memoryless:- if $h(t) = k\,\delta(t)$ || $h[n] = k\,\delta[n]$
2- Casual :- if $h(t)=0$ for $t < 0$ || $h[n] = 0$ for $n < 0$
3- Stable :- if $\int |h(t)|dt < \infty$ | $\sum_{n=-\infty}^{\infty} |h[n]| < 0$

Q1) state weather the following is Memoryless, causal, and stable? (Ex.2.29)
a) $h(t) = e^{-4t}\,u(t-2)$

Q2) Asses the stability of the following LTI systems with
a) $h[n] = u[n]$ b) $h(t) = \sum_{k=0}^{\infty} 0.5^k\,\delta(t-k)$

Q3) Find y(t) if x(t) and h(t) is given as :- (Ex.2.22c)





Q4) Find y[n] if $x[n] = (1/2)^{n-2}\,u[n-2]$ and $h[n] = u[n-+2]$ (Ex.2.3)

Q1 a) • $h(t) \neq k\,\delta(t)$ ∴ Not Memoryless
• $h(t) = 0$ for $t<0$ ∴ causal
• $\int_2^{\infty} e^{-4t}\,dt = \left.\frac{e^{-4t}}{-4}\right|_2^{\infty} = \frac{-1}{4}\left[0 - e^{-8}\right] = \frac{e^{-8}}{4} < \infty$ ∴ Stable

Q2
a) $\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} 1 = \infty$ ∴ not stable

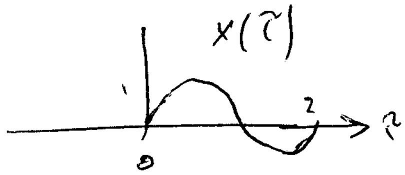
b) $\int_{-\infty}^{\infty} |h(t)|\,dt = \int \sum_{0}^{\infty} 0.5^k\,\delta(t-k)\,dt$

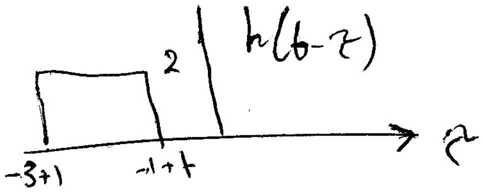
$= \sum_{0}^{\infty} 0.5^k \int \delta(t-k)\,dt$

$= \sum_{0}^{\infty} 0.5^k \cdot 1$

$= \frac{1}{1-0.5} < \infty$ ∴ Stable

Q3]





$y(t) = \int x(\tau)\, h(t-\tau)$

- for $-1+t < 0 \Rightarrow \underline{t < 1} \longrightarrow \underline{y(t) = 0}$
- for $-1+t > 0$ and $-3+t < 0 \Rightarrow \underline{1 < t < 3}$

$$\Rightarrow y(t) = \int_0^{-1+t} 2\sin \pi t = \frac{-2\cos \pi t}{\pi}\Bigg|_0^{-1+t} = \frac{-2}{\pi}\left[\cos[\pi(-1+t)] - 1\right]$$

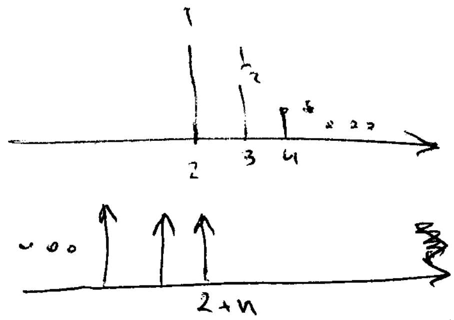
- for $-1+t > 2$ and $-3+t < 2 \Rightarrow \underline{3 < t < 5}$

$$\Rightarrow y(t) = \int_{-3+t}^{2} 2\sin(\pi t) = \frac{-2}{\pi}\cos \pi t\Bigg| = \frac{-2}{\pi}\left[\cos(2\pi) - \cos(\pi(-3+t))\right] = \frac{-2}{\pi}\left[1 - \cos(\pi(-3+t))\right]$$

- for $-3+t > 2 \Rightarrow \underline{t > 5} \longrightarrow \underline{y(t) = 0}$

$$\therefore y(t) = \begin{cases} 0 & t < 1 \\ \frac{-2}{\pi}\left[\cos(\pi(-1+t)) - 1\right] & 1 < t < 3 \\ \frac{2}{\pi}\left[1 - \cos(\pi(-3+t))\right] & 3 < t < 5 \\ 0 & t > 5 \end{cases}$$

Q4] $y[n] = \sum x[k]\, h[n-k]$



- for $2+n < 2 \rightarrow n < 0 \Rightarrow y[n] = 0$
- for $2+n > 2 \rightarrow n > 0$

$$\Rightarrow y[n] = \sum_{k=2}^{2+n} \frac{1}{2}^{k-2} = \left(\frac{1}{2}\right)^{-2} \sum_{k=2}^{2+n} \frac{1}{2}^{k}$$

$$= \left(\frac{1}{2}\right)^{-2} \cdot \frac{\left(\frac{1}{2}\right)^2 - \frac{1}{2}^{n+3}}{1 - 1/2}$$

$$= 2\left[1 - \frac{1}{2}^{n+1}\right]$$

$$\boxed{\text{Note:-} \quad \sum_{n=a}^{b} \alpha^n = \frac{\alpha^a - \alpha^{b+1}}{1 - \alpha} \quad \text{for } \alpha \neq 1}$$