Tut #7

Q1

a) $h(t) = \frac{ds(t)}{dt}$

$= 0 + (-0.5)\, e^{-0.5t}\, u(t) + e^{-0.5t}\, \delta(t)$

$= -0.5\, e^{-0.5t} + \delta(t)$

b) $h[n] = s[n] - s[n-1]$

$= 0.5^{n}\, u[n] - 0.5^{n-1}\, u[n-1]$

Q2 a) $g(t) = u(t) * h(t)$

$= h(t) * u(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau$

$= \int_{-\infty}^{t} h(\tau)\, d\tau = \int_{0}^{t} e^{3\tau}\, d\tau = \frac{e^{3t} - e^{0}}{3}$

b) $h[n] = \delta(n) - 0.5\,\delta[n-1]$

$\therefore s[n] = u[n] * h[n]$

$= u[n] * [\delta(n) - 0.5\,\delta[n-1]] = u[n] - 0.5\, u[n-1]$

Q3 $y[n] = 2x[n] + \frac{1}{3}\, y[n-1]$; $\because$ Causal $\therefore y[-1] = 0$

let $x[n] = \delta(n)$

$n=0$ $y[0] = 2\,\delta(0) + \frac{1}{3}\, y[-1] = 2 + 0 = 2$

$y[1] = 2\,\delta(1) + \frac{1}{3}\, y[0] = 0 + \frac{1}{3}\cdot 2 = 2\cdot\frac{1}{3}$

$y[2] = 2\,\delta(2) + \frac{1}{3}\, y[1] = 0 + \frac{1}{3}\cdot 2\cdot\frac{1}{3} = 2\cdot\left(\frac{1}{3}\right)^2$

$\vdots$

$y[n] = 2\left(\frac{1}{3}\right)^n u(n) \Rightarrow h[n] = 2\left(\frac{1}{3}\right)^n u[n]$ (IIR $N > 1$)

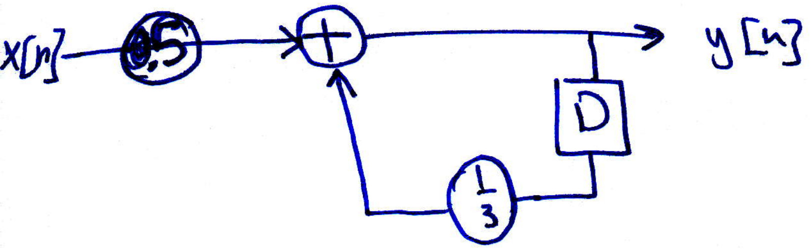
if $x(n) = 2\delta(n-1)$

$\Rightarrow y[n] = x(n) * h(n) = 2\, h(n-1) = 4\left(\frac{1}{3}\right)^{n-1} u(n-1)$

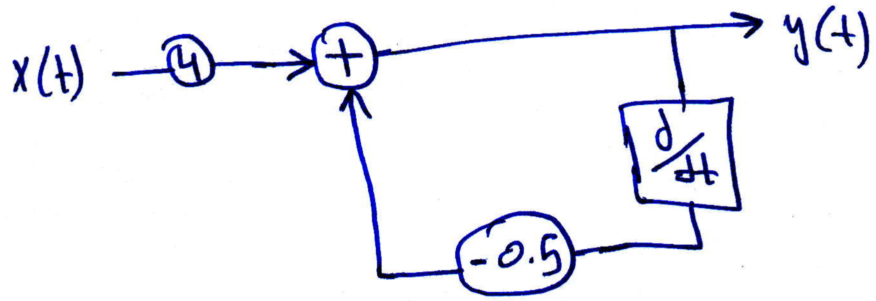
Tut # 7

Q4

a) $y[n] = \frac{1}{3} y[n-1] + 0.5\, x[n]$



b) $y(t) = -0.5 \frac{dy(t)}{dt} + 4x(t)$



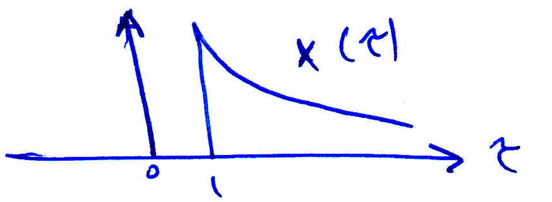
Q1] Calculate the autocorrelation function, and the total energy of the signal

a) $x(t) = e^{-t}u(t-1)$
b) $x(t) = u(t) - u(t-1)$
c) $x[n] = (1/3)^n u(n-1)$

Q2] Calculate the cross-correlation of the following signals

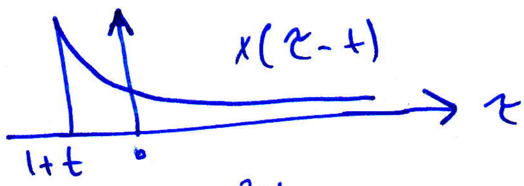
$x(t) = u(t) - u(t-1)$ & $y(t) = u(t - 3/2) - u(t - 5/2)$

Q1] a) $R_{xx}(t) = \int x(\tau - t)\, x(\tau)\, d\tau$



• for $1+t < 1 \Rightarrow t < 0$

$$R_{xx}(t) = \int_1^\infty e^{-\tau} e^{-(\tau-t)}\, d\tau = e^{t}\int_1^\infty e^{-2\tau} = e^{t}\cdot \frac{e^{-2\tau}}{-2}\Bigg|_1^\infty = \frac{e^{-2+t}}{2}$$



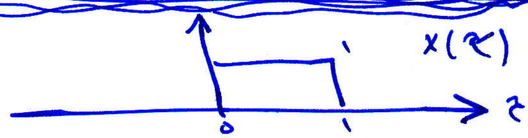
• for $1+t > 1 \Rightarrow t > 0$

$$R_{xx}(t) = \int_{1+t}^\infty e^{-\tau} e^{-(\tau-t)} = e^{t}\cdot \frac{e^{-2\tau}}{-2}\Bigg|_{1+t}^\infty = \frac{e^{-2-t}}{2}$$

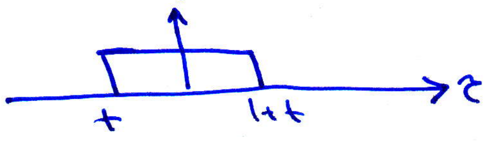
$$\therefore E = R_{xx}(0) = \frac{e^{-2}}{2}$$

$$R_{xx}(t) = \begin{cases} \frac{e^{-2+t}}{2} & t < 0 \\ \frac{e^{-2-t}}{2} & t > 0 \end{cases}$$

b) $R_{xx} = \int x(\tau-t)\, x(\tau)\, d\tau$



• for $1+t < 0 \rightarrow t < -1 \Rightarrow \boxed{R_{xx} = 0}$



• for $t < 0$ & $1+t > 0 \rightarrow -1 < t < 0$

$\Rightarrow R_{xx} = \int_0^{1+t} d\tau = 1+t$

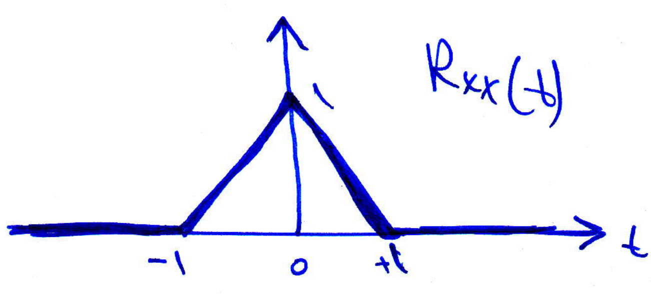
• for $t < 1$ & $1+t > 1 \rightarrow 0 < t < 1$

$\Rightarrow R_{xx} = \int_t^1 d\tau = 1-t$

$E = R_{xx}(0) = 1$

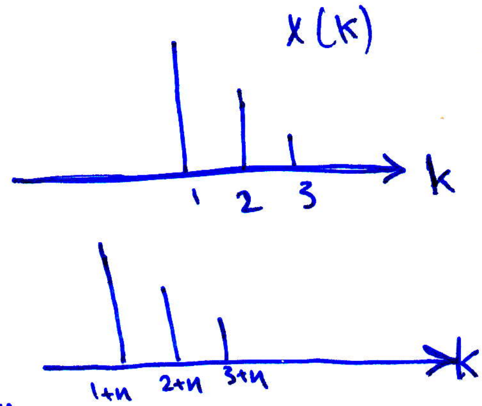
• for $t > 1 \Rightarrow \boxed{R_{xx} = 0}$

$$\therefore R_{xx}(t) = \begin{cases} 0 & t < -1 \\ 1+t & -1 < t < 0 \\ 1-t & 0 < t < 1 \\ 0 & t > 1 \end{cases}$$



Tut #8

c) $R_{xx}(n) = \sum_{k} x(k-n)\, x(k)$



• for $1+n < 1 \rightarrow n < 0$

$R_{xx} = \sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^{k} \left(\frac{1}{3}\right)^{k-n}$

$= \left(\frac{1}{3}\right)^{-n} \sum_{1}^{\infty} \left(\frac{1}{3^2}\right)^{k}$

$= \left(\frac{1}{3}\right)^{-n} \cdot \frac{(1/9)^{1} - (1/9)^{\infty}}{1 - 1/9} = \left(\frac{1}{3}\right)^{-n} \cdot \frac{9}{8} \cdot \frac{1}{9} = \frac{1}{8}\left(\frac{1}{3}\right)^{-n}$

• for $1+n > 1 \longrightarrow n > 0$

$R_{xx} = \sum_{k=1+n}^{\infty} (1/3)^{k} \left(\frac{1}{3}\right)^{k-n}$

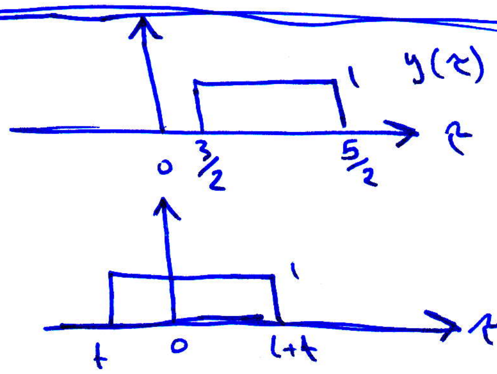
$= \left(\frac{1}{3}\right)^{-n} \sum_{1+n}^{\infty} \left(\frac{1}{9}\right)^{k}$

$= \left(\frac{1}{3}\right)^{-n} \cdot \frac{(1/9)^{1+n} - (1/9)^{\infty}}{1 - 1/9} = \left(\frac{1}{3}\right)^{-n} \cdot \frac{9}{8} \cdot \frac{1}{9} \cdot \left(\frac{1}{3}\right)^{2n} = \frac{1}{8}\left(\frac{1}{3}\right)^{n}$

$$\boxed{R_{xx}(n) = \begin{cases} \frac{1}{8}\left(\frac{1}{3}\right)^{-n} & n < 0 \\ \frac{1}{8}\left(\frac{1}{3}\right)^{n} & n > 0 \end{cases}}$$

$\Rightarrow \boxed{E = R_{xx}(0) = \frac{1}{8}}$

Q2) $R_{xy}(t) = \int x(\tau - t)\, y(\tau)\, d\tau$



• for $1+t < \frac{3}{2} \rightarrow t < \frac{1}{2} \Rightarrow R_{xy} = 0$

• for $t < \frac{3}{2}$ & $1+t > \frac{3}{2} \rightarrow \frac{1}{2} < t < \frac{3}{2}$

$R_{xy}(t) = \int_{3/2}^{1+t} d\tau = 1 + t - \frac{3}{2} = t - \frac{1}{2}$

• for $t < \frac{5}{2}$ & $1+t > \frac{5}{2} \rightarrow \frac{3}{2} < t < \frac{5}{2}$

$R_{xy}(t) = \int_{t}^{5/2} d\tau = \frac{5}{2} - t$

• for $t > \frac{5}{2} \Rightarrow R_{xy} = 0$

$$\therefore R_{xy}(t) = \begin{cases} 0 & t < 1/2 \\ t - 1/2 & 1/2 < t < 3/2 \\ 5/2 - t & 3/2 < t < 5/2 \\ 0 & t > 5/2 \end{cases}$$