- ## Correlation

Is a measure of similarity between two functions or signals.

→ Auto-Correlation:- $\text{Rxx(t)} = \int_{-\infty}^{\infty} \mathfrak{x}(\tau)\, x(\tau + t)\, d\tau \;=\; \int_{-\infty}^{\infty} \mathfrak{x}(\tau - t)\, x(\tau)\, d\tau$

$\text{Rxx(n)} = \sum_k \mathfrak{x}(k) x(k+n) \;=\; \sum_k \mathfrak{x}(k-n) x(k)$

→ Cross-Correlation:- $\text{Rxy(t)} = \int_{-\infty}^{\infty} \mathfrak{x}(\tau)\, y(\tau + t)\, d\tau \;=\; \int_{-\infty}^{\infty} \mathfrak{x}(\tau - t)\, y(\tau)\, d\tau$

$\text{Rxy(n)} = \sum_k \mathfrak{x}(k) y(k+n) = \sum_k \mathfrak{x}(k-n) y(k)$

where $\mathfrak{x}$(t) denotes the complex conjugate of x(t).

You could notice that the difference between the convolution & correlation is the complex conjugate and the time reversal. Moreover, note that correlation isn't commutative. Therefore, for real signals, which we focus on, we could write:- $\boxed{\text{Rxy(t)} = \text{x(-t)} * \text{y(t)}}$

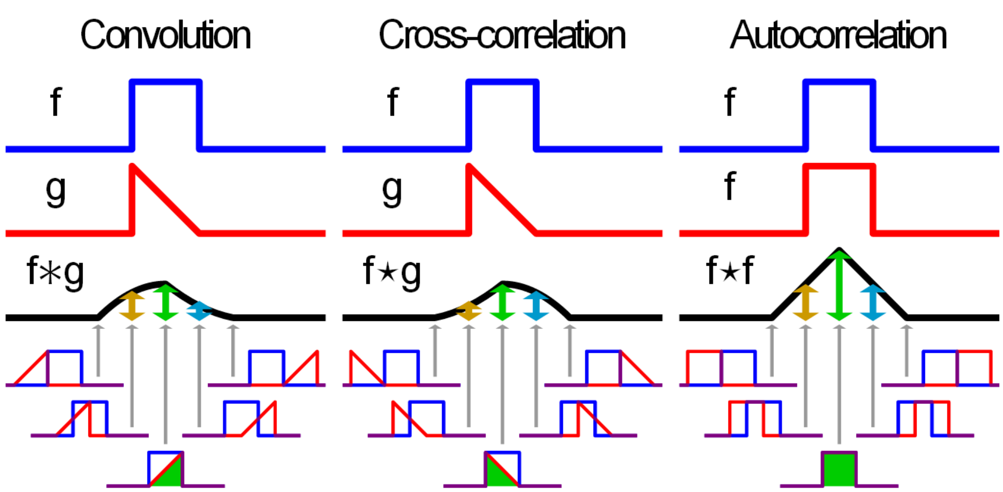
Properties of correlation

i) Rxx(t) = Rxx(-t) [even function]
ii) Rxx(0) = Signal Energy
iii) Rxy(t) = Ryx(-t) [Reflection]
iv) Rxx(0) ≥ Rxx(t) for all t

For DT, all the above plus |Rxy(n)| ≤ sqrt[Rxx(0) Ryy(0)]

* The proof of those properties can be found in the textbook or in *Dr A. I. Sulyman* notes.

Here's a nice visual representation showing the difference for a real signal



(done by wikipedia.org/wiki/User:Cmglee)

Q1] Calculate the autocorrelation function, and the total energy of the signal

a) $x(t) = e^{-t} u(t-1)$
b) $x(t) = u(t) - u(t-1)$
c) $x[n] = (1/3)^n u(n-1)$

Q2] Calculate the cross-correlation of the following signals

$x(t) = u(t) - u(t-1)$ & $y(t) = u(t - 3/2) - u(t - 5/2)$