- **<u>Fourier Series Representation (in CT)</u>:-**

Similar to what we have done in convolution, we are going to represent signals as a set of harmonically related sinusoids (Harmonically related means multiple of; i.e. if the fundamental frequency is ω_o then the 3^{rd} Harmonic is $3\omega_o$). Since these sinusoids are periodic, it's logical to assume that we could only use them to represent periodic signals.

From Euler formula, we know that we could write sinusoids as a complex exponential. The response of the system to a complex exponential is ($e^{st} \rightarrow \boxed{\text{system}} \rightarrow H(s)\, e^{st}$), where H(s) at a certain s is the eigenvalue associated with the eigenfunction e^{st}.

Therefore, for an **LTI** system we could represent the periodic signal as $\boxed{x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{jk\omega_o t}}$

The FS coefficients are defined as $\boxed{a_k = \frac{1}{T_o} \int_{T_o} x(t)\, e^{-jk\omega_o t} dt}$, Hence $\boxed{a_0 = \frac{1}{T_o} \int_{T_o} x(t)\, dt}$ *(DC)*

Representing the signal as a complex exponential gives us several mathematical advantages; which are shown in the following table [Please review the proofs from the textbook or from *Dr A. I. Sulyman* notes].

- For FS to converge, x(t) must have:-

i)Finite number of Maxima&Minima in one period ii)Finite number of discontinuities in one period iii)Absolutely Integrable

- **<u>Important mathematical eq:-</u>**

<u>Euler's eq:-</u>

$\boxed{e^{jx} = \cos(x) + j\sin(x)}$

$\boxed{\cos(x) = Re\{e^{jx}\} = [e^{jx} + e^{-jx}]/2}$

$\boxed{\sin(x) = Img\{e^{jx}\} = [e^{jx} - e^{-jx}]/2j}$

$\boxed{e^{jk\pi} = \cos(k\pi) = (-1)^k}$

$\cos(-x) = \cos(x) \;\; || \;\; \sin(-x) = -\sin(x)$

$\cos(x+y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\sin(x+y) = \sin(x)\cos(y) + \sin(y)\cos(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1$

$\cos^2(x) + \sin^2(x) = 1$

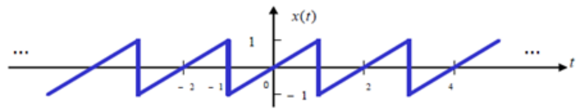
TABLE 3.1: PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$x(t)$, $y(t)$ } Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k, b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t}x(t) = e^{jM(2\pi/T)t}x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t-\tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$a_k = a^*_{-k}$; $\mathcal{Re}\{a_k\} = \mathcal{Re}\{a_{-k}\}$; $\mathcal{Im}\{a_k\} = -\mathcal{Im}\{a_{-k}\}$; $\lvert a_k\rvert = \lvert a_{-k}\rvert$; $\measuredangle a_k = -\measuredangle a_{-k}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$x_e(t) = \mathcal{E}v\{x(t)\}$ [$x(t)$ real]; $x_o(t) = \mathcal{O}d\{x(t)\}$ [$x(t)$ real]	$\mathcal{Re}\{a_k\}$; $j\mathcal{Im}\{a_k\}$

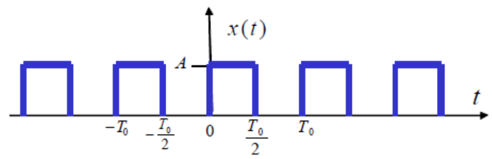
Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

Q1) Compute the FS representation for the signals shown, and determine the DC part and the 1st harmonic



Q2) (i) Compute the complex Fourier series (FS) representation for the periodic signal $x(t)$ shown.
(ii) Determine the DC part and the first harmonics of the signal $x(t)$.
(iii) Write an expression for the FS coefficients of the signal $x(t - T_o/4)$
(iv) Write an expression for the FS coefficients of the signal $dx(t)/dt$



Q3) Determine and sketch the FS coeffecients for $x(t) = \sin(2\pi t + \pi/4)$