Problem:

The normalized lowpass filter with a cutoff frequency of 1 rad/sec is given as:

$$H_P(s) = \frac{1}{s+1}.$$

a. Use the given $H_p(s)$ and the BLT to design a corresponding digital IIR lowpass filter with a cutoff frequency of 15 Hz and a sampling rate of 90 Hz.

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Solution:

First, we obtain the digital frequency as

$$\omega_d = 2\pi f = 2\pi(15) = 30\pi \,\text{rad/sec}, \text{ and } T = 1/f_s = 1/90\,\text{sec}.$$

$$\omega_a = \frac{2}{T}\tan\left(\frac{\omega_d T}{2}\right) = \frac{2}{1/90}\tan\left(\frac{30\pi/90}{2}\right)$$

$$\omega_a = 180 \times \tan(\pi/6) = 180 \times \tan(30^\circ) = 103.92 \text{ rad/sec.}$$

Then perform the prototype transformation (lowpass to lowpass) as follows:

$$H(s) = H_P(s)_{s=\frac{s}{\omega_a}} = \frac{1}{\frac{s}{\omega_a}+1} = \frac{\omega_a}{s+\omega_a},$$

$$H(s) = \frac{103.92}{s+103.92}$$

Apply the BLT, which yields

$$H(z) = \left.\frac{103.92}{s+103.92}\right|_{s=\frac{2}{T}\frac{z-1}{z+1}}.$$

$$H(z) = \frac{103.92}{180\times\frac{z-1}{z+1}+103.92} = \frac{103.92/180}{\frac{z-1}{z+1}+103.92/180} = \frac{0.5773}{\frac{z-1}{z+1}+0.5773}.$$

$$H(z) = \frac{0.5773(z+1)}{\left(\frac{z-1}{z+1} + 0.5773\right)(z+1)} = \frac{0.5773z + 0.5773}{(z-1) + 0.5773(z+1)}$$

$$= \frac{0.5773z + 0.5773}{1.5773z - 0.4227}.$$

$$H(z) = \frac{(0.5773z + 0.5773)/(1.5773z)}{(1.5773z - 0.4227)/(1.5773z)} = \frac{0.3660 + 0.3660z^{-1}}{1 - 0.2679z^{-1}}.$$

Problem:

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Solution:

$$\omega_d = 2\pi f = 2\pi(3400) = 6800\pi \text{ rad/sec, and } T = 1/f_s = 1/8000 \text{ sec.}$$

$$\omega_a = \frac{2}{T}\tan\left(\frac{\omega_d T}{2}\right) = 16000 \times \tan\left(\frac{6800\pi/8000}{2}\right) = 6.6645 \times 10^4 \text{ rad/sec.}$$

Since the order of 2 is given in the specification, we directly pick the second-order lowpass prototype from Table 8.3:

$$H_P(s) = \frac{1}{s^2 + 1.4142s + 1}.$$

$$H(s) = H_P(s)\big|_{\frac{s}{\omega_a}} = \frac{4.4416 \times 10^9}{s^2 + 9.4249 \times 10^4 s + 4.4416 \times 10^9}.$$

Carrying out the BLT yields

$$H(z) = \left.\frac{4.4416 \times 10^9}{s^2 + 9.4249 \times 10^4 s + 4.4416 \times 10^9}\right|_{s=16000(z-1)/(z+1)}.$$

$$H(z) = \frac{4.4416 \times 10^9}{\left(16000\frac{z-1}{z+1}\right)^2 + 9.4249 \times 10^4\left(16000\frac{z-1}{z+1}\right) + 4.4416 \times 10^9}.$$

$$H(z) = \frac{17.35}{\left(\frac{z-1}{z+1}\right)^2 + 5.8906\left(\frac{z-1}{z+1}\right) + 17.35}.$$

$$H(z) = \frac{17.35(z+1)^2}{(z-1)^2 + 5.8906(z-1)(z+1) + 17.35(z+1)^2}.$$

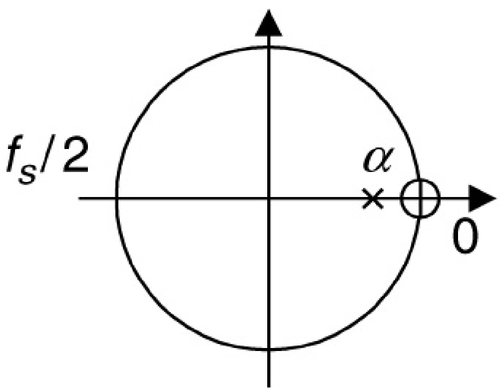
$$H(z) = \frac{17.35\left(z^2+2z+1\right)}{(z^2-2z+1) + 5.8906(z^2-1) + 17.35(z^2+2z+1)}$$

$$= \frac{17.35z^2 + 34.7z + 17.35}{24.2406z^2 + 32.7z + 12.4594}.$$

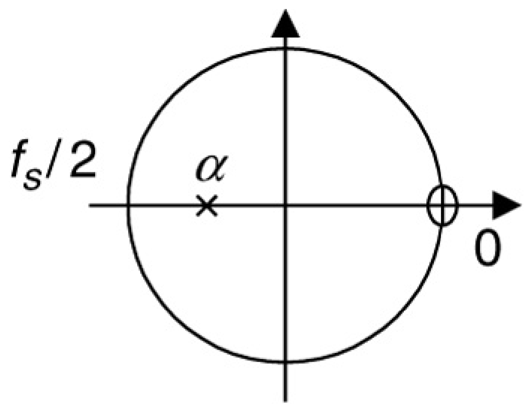
$$H(z) = \frac{0.7157 + 1.4314z^{-1} + 0.7151z^{-2}}{1 + 1.3490z^{-1} + 0.5140z^{-2}}.$$

Pole Zero Placement Method

First-Order HPF Design



When $f_c < f_s/4$, $\alpha \approx 1 - 2 \times (f_c/f_s) \times \pi$



When $f_c > f_s/4$, $\alpha \approx -(1 - \pi + 2 \times (f_c/f_s) \times \pi)$

$$H(z) = \frac{K(z-1)}{(z-\alpha)} \qquad K = \frac{(1+\alpha)}{2}$$

Problem:

A first-order highpass filter is required to satisfy the following specifications:

- Sampling rate $= 8{,}000\,\text{Hz}$
- 3 dB cutoff frequency: $f_c = 3{,}800\,\text{Hz}$
- Zero gain at 0 Hz.

a. Find the transfer function using the pole-zero placement method.

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A first-order highpass filter is required to satisfy the following specifications:

- Sampling rate = 8,000 Hz
- 3 dB cutoff frequency: $f_c = 3{,}800$ Hz
- Zero gain at 0 Hz.

a. Find the transfer function using the pole-zero placement method.

Solution:

Since the cutoff frequency of 3,800 Hz is much larger than $f_s/4 = 2{,}000$ Hz, we determine the pole as

$$\alpha \approx -(1 - \pi + 2 \times (3800/8000) \times \pi) = -0.8429,$$

$$K = \frac{(1 - 0.8429)}{2} = 0.07854$$

$$H(z) = \frac{0.07854(z - 1)}{(z + 0.8429)} = \frac{0.07854 - 0.07854z^{-1}}{1 + 0.8429z^{-1}}.$$