

Tutorial_3

Find Big-O, Ω and n_0 for

① $5n^6 + 4n^3 - 2n^2 + 1 = O(n^6)$

$$C = 5 + 4 + 2 + 1 = 12$$

$$n_0 = 1$$

② $\log(n^7 + 2n^3 - 10)$

$$n^7 + 2n^3 - 10 \leq 13n^7 \quad \text{for all } n \geq 1$$

$$\begin{aligned} \log(n^7 + 2n^3 - 10) &\leq \log(13n^7) \\ &= \log 13 + 7 \log n \\ &\leq 8 \log n \end{aligned}$$

True if $n \geq 13$

Thus

$$\log(n^7 + 2n^3 - 10) = O(\log n)$$

$$C = 8$$

$$n_0 = \max(1, 13) = 13$$

③ for Ω of $\log(n^7 + 2n^3 - 10)$
we have

$$\log(n^7 + 2n^3 - 10)$$

$$n^7 + 2n^3 - 10 \geq \frac{1}{2}n^7 \quad \text{for } n_0 \geq \frac{2+10}{1-1/2} = 24$$

$$\begin{aligned} \log(n^7 + 2n^3 - 10) &\geq \log\left(\frac{1}{2}n^7\right) \\ &= \log \frac{1}{2} + 7 \log n \\ &= -1 + 7 \log n \\ &\geq 6 \log n \end{aligned}$$

True if $n \geq 2$

$$\Rightarrow \log(n^7 + 2n^3 - 10) = \Omega(\log n)$$

$$C = 6 \quad n_0 = \max(24, 2) = 24$$