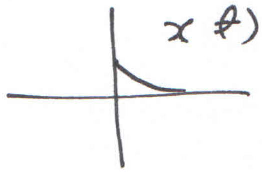
EXAMPLE 9.1

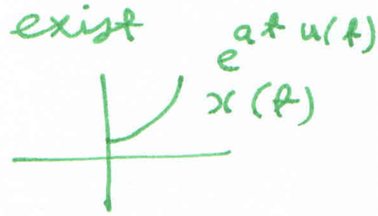
$$x(t) = e^{-at}u(t)$$



IF $a < 0$

$$x(t) = e^{at}u(t)$$

F.T does not exist



$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$X(s) = \int_{-\infty}^{\infty} e^{-at}u(t)\, e^{-st}\, dt$$

$$X(s) = \int_{0}^{\infty} e^{-at}e^{-st}\, dt$$

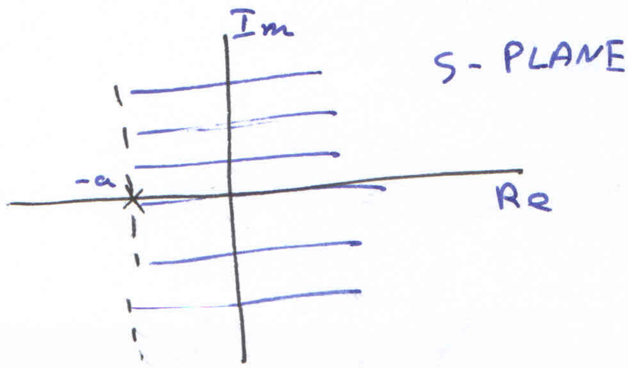
$$X(s) = \int_{0}^{\infty} e^{-(s+a)t}\, dt$$

$$= -\frac{1}{s+a}\left(e^{-(s+a)t} \Big|_{0}^{\infty} \right)$$

$$= -\frac{1}{s+a}\left(e^{-\infty} - e^{0} \right)$$

$$= -\frac{1}{s+a}(0 - 1)$$

$$= \frac{1}{s+a}$$



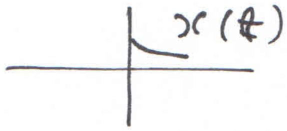
$$\text{Real}\{s\} > -a$$

$$\frac{1}{s+a}$$

$s + a = 0$

$s = -a$

RIGHT SIDED SIGNAL



$\therefore$ USING PROPERTY 4

~~Real{~~

$\text{Real}\{s\} > -a$

EXAMPLE 9.2

$x(t) = -e^{-at} u(-t)$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} \, dt$$

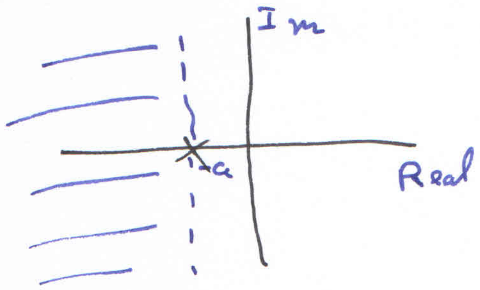
$$X(s) = -\int_{-\infty}^{\infty} e^{-at} u(-t) e^{-st} \, dt$$

$$= -\int_{-\infty}^{0} e^{-(s+a)t} \, dt$$

$$X(s) = \frac{1}{s+a}$$

$s + a = 0$

$s = a$



LEFT SIDED SIGNAL $(u(-t))$

$\therefore$ USING PROPERTY 5

$\text{Real}\{s\} < -a$

EXAMPLE 9.3

$$x(t) = 3e^{-2t}u(t) - 2e^{-t}u(t)$$

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$$

$$= \int_{-\infty}^{\infty} \left[3e^{-2t}u(t) - 2e^{-t}u(t)\right]e^{-st}\,dt$$

$$= 3\int_{-\infty}^{\infty} e^{-2t}e^{-st}u(t)\,dt + (-2)\int_{-\infty}^{\infty} e^{-t}e^{-st}u(t)\,dt$$

FROM EXAMPLE 9.1 (BOTH RIGHT SIDED)

$$X(s) = \frac{3}{s+2} - \frac{2}{s+1}$$

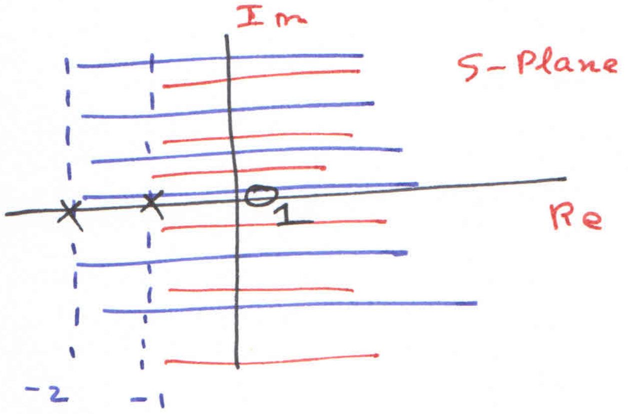
$s+2=0$

$s=-2$

$s+1=0$

$s=-1$

BOTH RIGHT SIDED



$e^{-t}u(t) \overset{L}{\longleftrightarrow} \frac{1}{s+1}$ Real $\{s\} > -1$

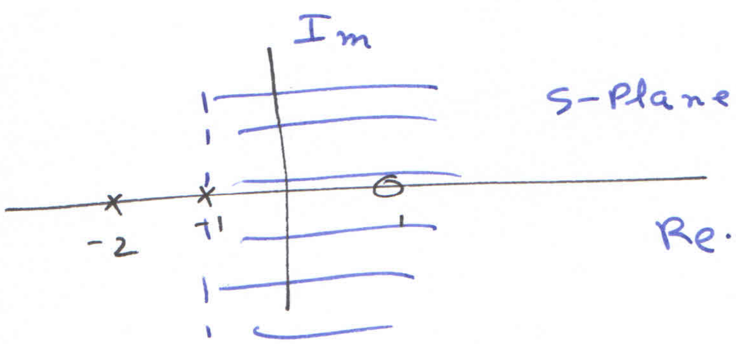
$e^{-2t}u(t) \overset{L}{\longleftrightarrow} \frac{1}{s+2}$ Real $\{s\} > -2$

$\therefore 3e^{-2t}u(t) - 2e^{-t}u(t) \overset{L}{\longleftrightarrow} \frac{3}{s+2} - \frac{2}{s+1}$

ZERO AT $s-1=0 \Rightarrow s=1$

$\Rightarrow \dfrac{s-1}{s^2+3s+2}$

USING PROPERTY 8



EXAMPLE 9.4

$$x(t) = e^{-2t} u(t) + e^{-t} \cos 3t \; u(t)$$

$$x(t) = \left[e^{-2t} + \frac{1}{2} e^{-(1-3j)t} + \frac{1}{2} e^{-(1+3j)t} \right] u(t)$$

AS $\cos(\theta) = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} \, dt$$

$$= \int_{-\infty}^{\infty} e^{-2t} u(t) e^{-st} \, dt + \frac{1}{2} \int_{-\infty}^{\infty} e^{-(1-3j)t} u(t) e^{-st} \, dt$$

$$+ \frac{1}{2} \int_{-\infty}^{\infty} e^{-(1+3j)t} u(t) e^{-st} \, dt$$

SIMILAR TO EXAMPLE 9.1 (ALL RIGHT SIDED)

$$e^{-2t} u(t) \xleftrightarrow{L} \frac{1}{s+2} \qquad \text{Real}\{s\} > -2$$

$$e^{-(1-3j)t} u(t) \xleftrightarrow{L} \frac{1}{s+(1-3j)} \qquad \text{Real}\{s\} > -1$$

$\text{Real}\{s+(1-3j)\} = 0 \Rightarrow s+1=0 \quad s=-1$

$$e^{-(1+3j)t} u(t) \xleftrightarrow{L} \frac{1}{s+(1+3j)} \qquad \text{Real}\{s\} > -1$$

FROM PROPERTY 8

FOR ALL LAPLACE TRANSFORMS TO CONVERGE ~~Real~~ Real{s} > -1.

$$\therefore \quad \frac{1}{s+2} + \frac{1}{2}\left(\frac{1}{s+(1-3j)}\right) + \frac{1}{2}\left(\frac{1}{s+(1+3j)}\right), \quad \text{Real}\{s\} > -1$$

COMBINING THEM ALL TOGETHER

$$e^{-2t}u(t) + e^{-t}\cos 3t\, u(t) \xleftrightarrow{L} \frac{2s^2+5s+12}{(s^2+2s+10)(s+2)}, \quad \text{Real}\{s\} > -1$$

$$2s^2+5s+12 = 0$$

$$s = -1.25 + 2.1065j$$

$$s = -1.25 - 2.1065j$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2-4(2)(12)}}{4}$$

$$x = \frac{-5 \pm \sqrt{-46}}{4}$$

USING MATLAB $s = [2 \quad 5 \quad 12]$

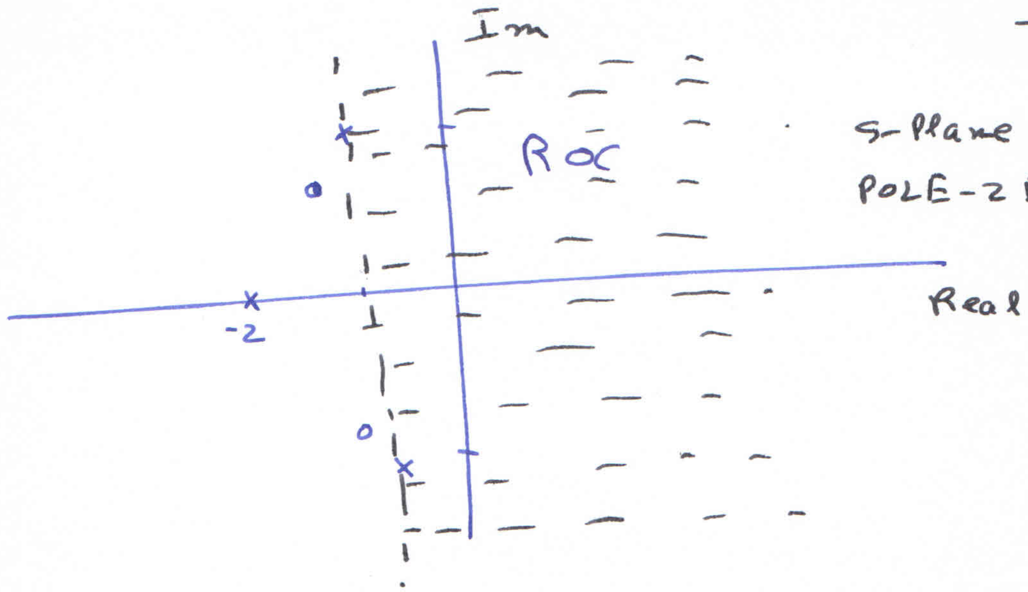
roots(s) => $s = -1.25 + 2.1065i$

$s = -1.25 - 2.1065i$

$$(s^2+2s+10)(s+2) = 0$$

~~s~~ = \a

$s = -2, \quad s = -1+3j, \quad s = -1-3j$



s-plane
POLE-ZERO PLOT

EXAMPLE 9.5

$$x(t) = \delta(t) - \frac{4}{3} e^{-t} u(t) + \frac{1}{3} e^{2t} u(t)$$

SIMILAR TO EXAMPLE 9.1

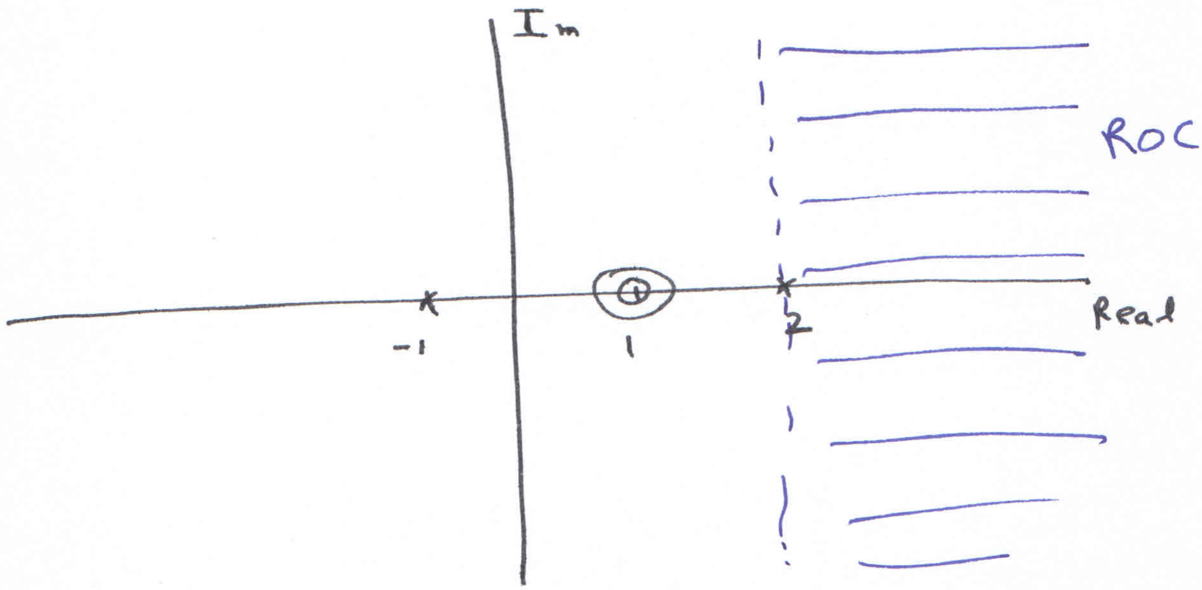
$$L\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t) e^{-st} dt = 1$$

ROC IS ENTIRE S-PLANE.

$$X(s) = 1 - \frac{4}{3} \frac{1}{s+1} + \frac{1}{3} \frac{1}{s-2}, \quad \text{Real}\{s\} > 2.$$

OR

$$X(s) = \frac{(s-1)^2}{(s+1)(s-2)} \quad \text{Real}\{s\} > 2$$



EXAMPLE 9.6

$$x(t) = \begin{cases} e^{-at}, & 0<t<T \\ 0, & \text{o.w} \end{cases}$$

SIGNAL FINITE
ROC ENTIRE
S-PLANE

$$X(s) = \int_0^T e^{-at} e^{-st} \, dt$$

$$= \frac{1}{s+a} \left(1 - e^{-(s+a)T} \right)$$

$s+a=0$
$s=-a$

$s+a=0$
$s=-a$

L' HÔPITAL'S RULE

IF EITHER

$$\lim_{x \to c} f(x) = \lim_{x \to c} g(x) = 0$$

or $$\lim_{x \to c} |f(x)| = \lim_{x \to c} |g(x)| = \infty$$

then $$\lim_{x \to c} \frac{f(x)}{g(x)}$$

$$\Rightarrow \lim_{x \to c} \frac{f'(x)}{g'(x)} = L$$

$$\lim_{s \to -a} X(s) = \lim_{s \to -a} \frac{1 - e^{-(s+a)T}}{s+a}$$

$$= \lim_{s \to -a} \left[\frac{\frac{d}{ds}\left(1 - e^{-(s+a)T}\right)}{\frac{d}{ds}(s+a)} \right]$$

$$= \lim_{s \to -a} \frac{0 - (-T)\, e^{-aT} e^{-sT}}{1}$$

$$= \lim_{s \to a} T e^{-aT} e^{-sT}$$

$$X(-a) = T\left(e^{-aT} e^{-(-a)T}\right)$$

$$= T\left(e^{-aT+aT}\right)$$

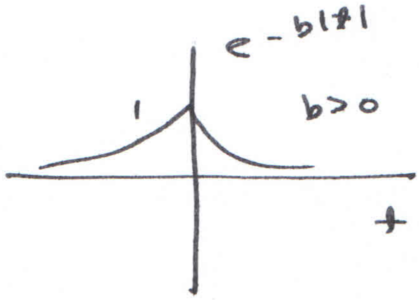
$$= Te^{0}$$

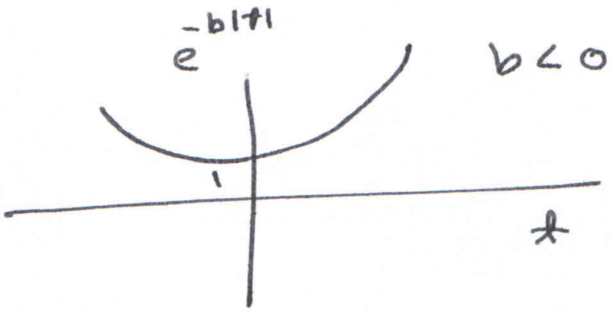
$$= T$$

EXAMPLE 9.7

$$x(t) = e^{-b|t|}$$

$$x(t) = e^{-bt}u(t) + e^{bt}u(-t)$$





NO ROC

FROM EXAMPLE 9.1

$$e^{-bt}u(t) \overset{L}{\longleftrightarrow} \frac{1}{s+b} \qquad \text{Real}\{s\} > -b$$

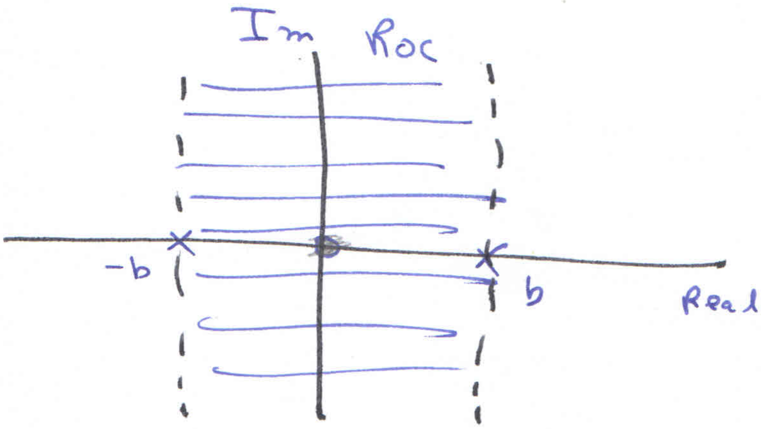
FROM EXAMRLE 9.2

$$e^{bt}u(-t) \overset{L}{\longleftrightarrow} -\frac{1}{s-b} \qquad \text{Real}\{s\} < b$$

FOR ~~R<0~~ $b > 0$

$$e^{-b|t|} \xleftrightarrow{\mathcal{L}} \frac{1}{s+b} - \frac{1}{s-b} = \frac{-2b}{s^2-b^2}$$

$$-b < \text{Real}\{s\} < b$$



Read 1st Paragraph Page 669

Read EXAMPLE 9.8

EXAMPLE 9.9

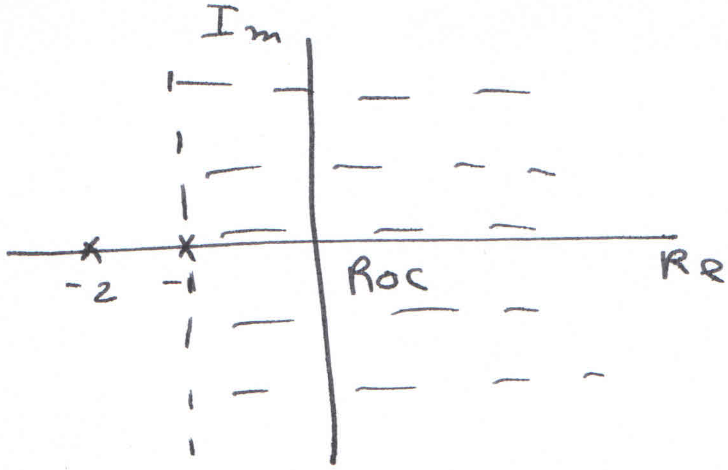
Page 671

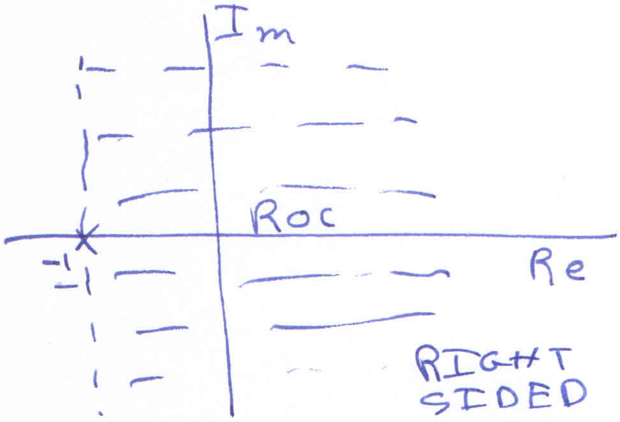
$$X(s) = \frac{1}{(s+1)(s+2)} \qquad \text{Real}\{s\} > -1$$

PARTIAL FRACTION EXPANSION

$$X(s) = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

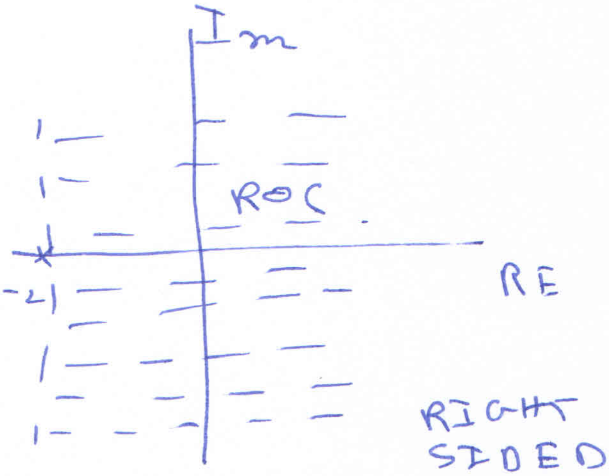
$$X(s) = \frac{1}{s+1} - \frac{1}{s+2}$$





$$\frac{1}{s+1} \overset{L}{\longleftrightarrow} e^{-t}u(t)$$

$$\text{Real}\{s\} > -1$$



$$\frac{1}{s+2} \overset{L}{\longleftrightarrow} e^{-2t}u(t)$$

$$\text{Real}\{s\} > -2$$

$$\therefore \quad \frac{1}{(s+1)(s+2)} \overset{L}{\longleftrightarrow} \left[e^{-t} - e^{-2t}\right]u(t)$$

$$\text{Real}\{s\} > -1$$

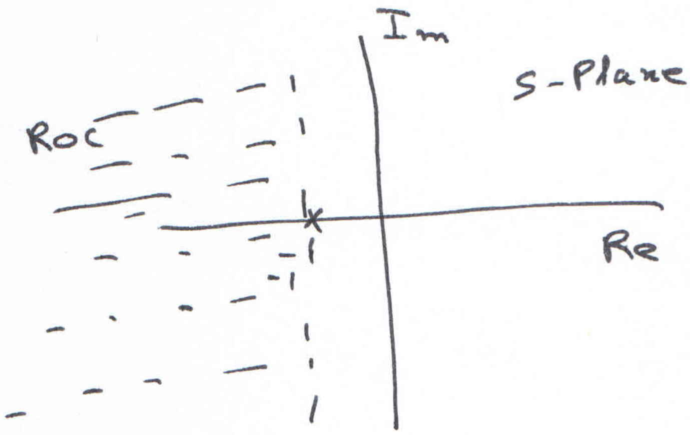
EXAMPLE 9.10

$$X(s) = \frac{1}{(s+1)(s+2)}, \quad \text{Real}\{s\} < -1$$

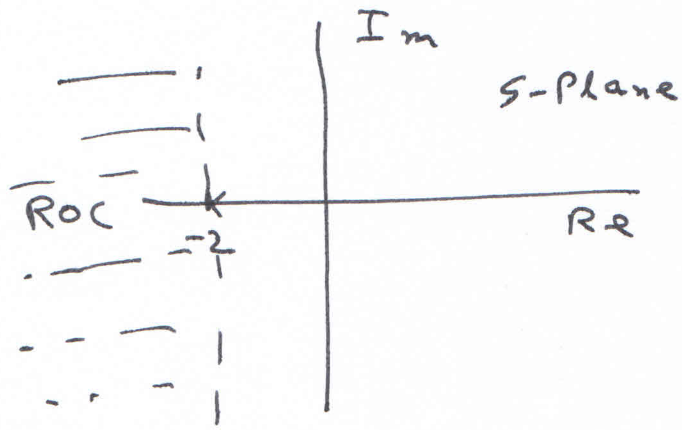
$$X(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$s+1=0 \qquad s+2=0$

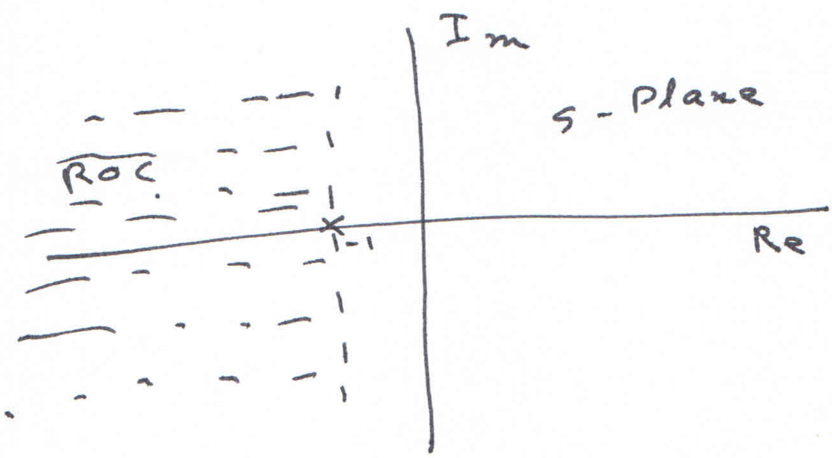
$s=-1 \qquad s=-2$



LEFT SIDED



LEFT SIDED



LEFT SIDED

$$-e^{-t}u(-t) \xleftrightarrow{L} \frac{1}{s+1} \qquad \text{Real}\{s\} < -1$$

$$-e^{-2t}u(-t) \xleftrightarrow{L} \frac{1}{s+2} \qquad \text{Real}\{s\} < -2$$

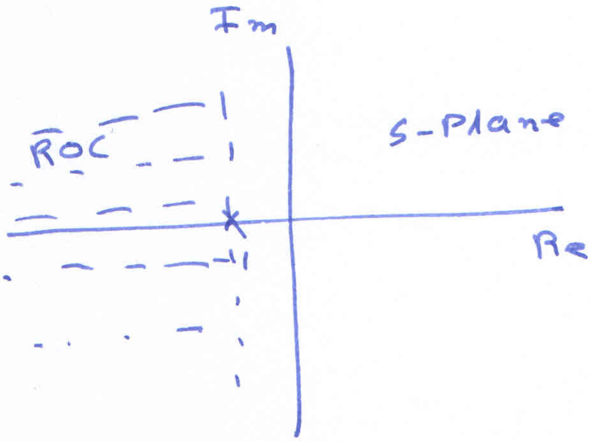
$$x(t) = \left[-e^{-t} + e^{-2t}\right] u(-t) \xleftrightarrow{L} \frac{1}{(s+1)(s+2)}$$

$$\text{Real}\{s\} < -2$$

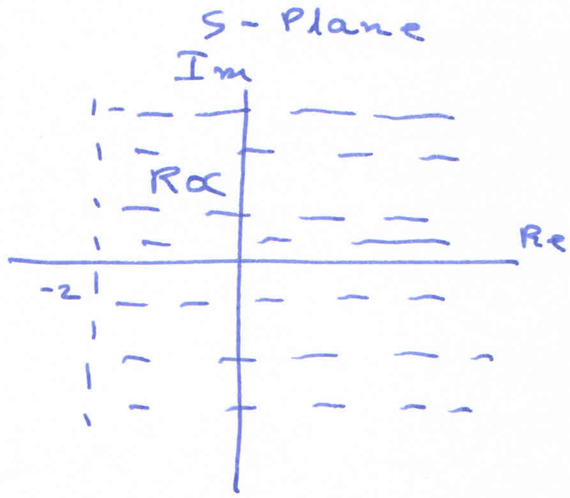
EXAMPLE 9.11
page 673

$$X(s) = \frac{1}{(s+1)(s+2)} \qquad -2 < \text{Real}\{s\} < -1$$

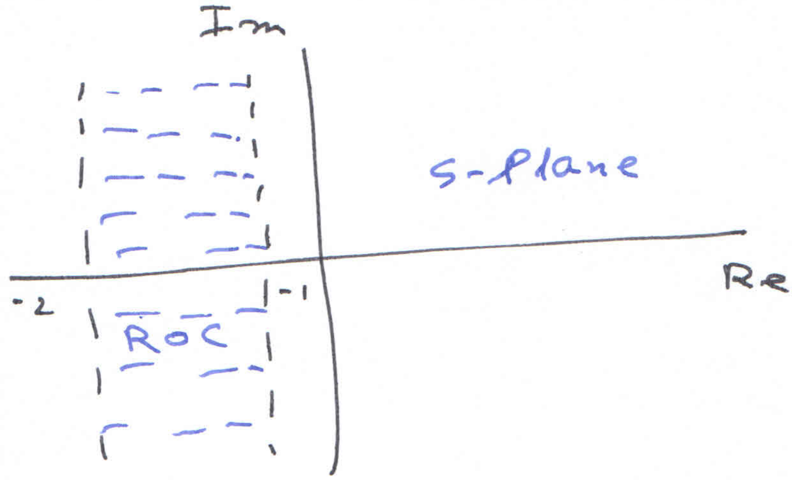
$$X(s) = \frac{1}{s+1} - \frac{1}{s+2}$$



LEFT SIDED



RIGHT SIDED



$$x(t) = \underbrace{-e^{-t}u(-t)}_{\text{LEFT SIDED}} - \underbrace{e^{-2t}u(t)}_{\text{RIGHT SIDED}} \overset{L}{\longleftrightarrow} \frac{1}{(s+1)(s+2)},$$

$$-2 < Real\{s\} < -1$$

EXAMPLE 9.13

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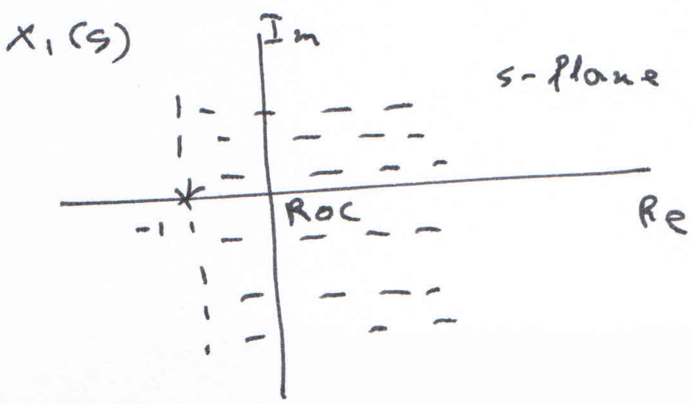
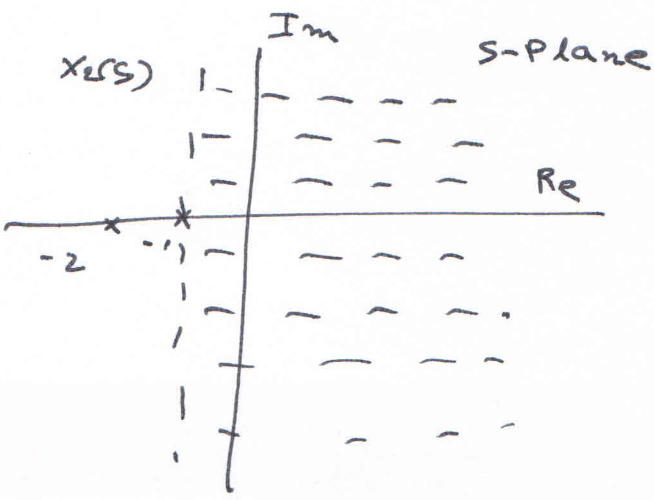
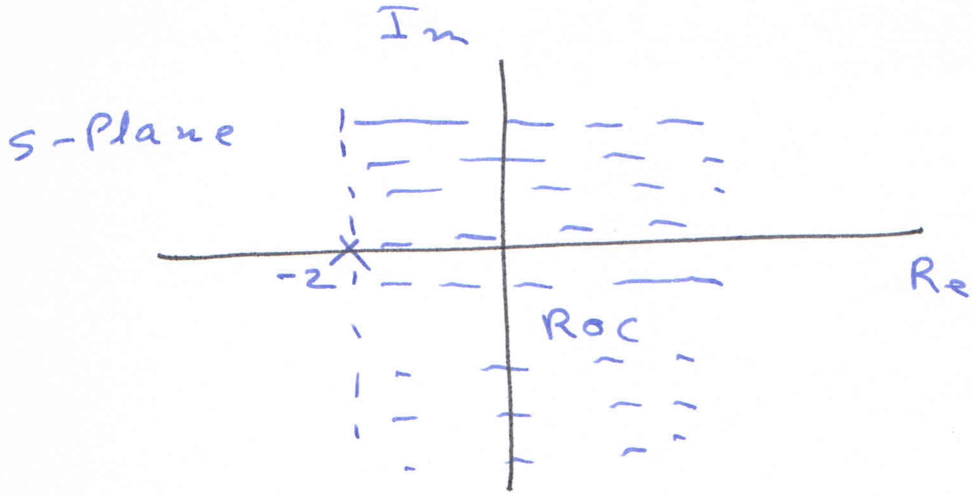
$$x(t) = x_1(t) - x_2(t)$$

$$X_1(s) = \frac{1}{s+1} \qquad Re\{s\} > -1$$

$$X_2(s) \quad \frac{1}{(s+1)(s+2)} \qquad Re\{s\} > -1$$

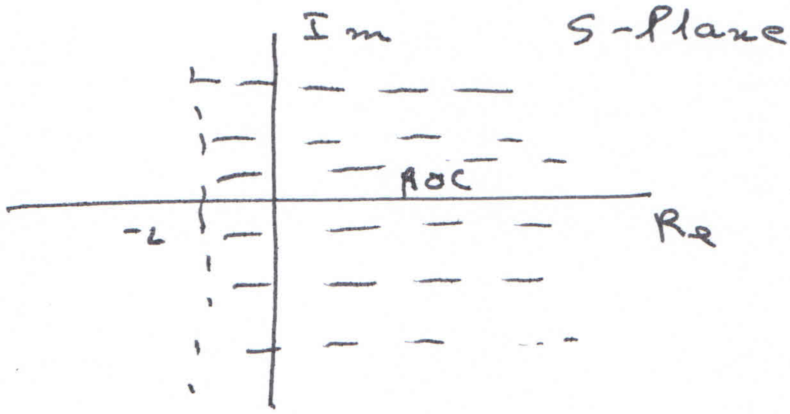






$$X(s) = \frac{1}{s+1} - \frac{1}{(s+1)(s+2)}$$

$$= \frac{s+1}{(s+1)(s+2)} = \frac{1}{s+2}$$



EXAMPLE 9.14

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$$x(t) = t e^{-at} u(t)$$

$$x(t) \xleftrightarrow{L} X(s) , \quad ROC = R$$

$$-t\,x(t) \xleftrightarrow{L} \frac{dX(s)}{ds} , \quad ROC = R$$

$$e^{-at}u(t) \xleftrightarrow{L} \frac{1}{s+a}, \quad \text{Real}\{s\} > -a$$

$$t\,e^{-at}u(t) \xleftrightarrow{L} -\frac{d}{ds}\left[\frac{1}{s+a}\right]$$

$$= \frac{1}{(s+a)^2}, \quad \text{Real}\{s\} > -a$$

REPEATING

$$\frac{t^2}{2}e^{-at}u(t) \xleftrightarrow{L} \frac{1}{(s+a)^3}, \quad \text{Real}\{s\} > -a$$

AND GENERALLY

$$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t) \xleftrightarrow{L} \frac{1}{(s+a)^n}, \quad \text{Re}\{s\} > -a$$

EXAMPLE 9.15

$$X(s) = \frac{2s^2+5s+5}{(s+1)^2(s+2)}, \quad \text{Real}\{s\} > -1$$

$$X(s) = \frac{2}{(s+1)^2} - \frac{1}{s+1} + \frac{3}{s+2}, \quad \text{Real}\{s\} > -1$$

ROC RIGHT OF $s=-1$ and $s=-2$

$\therefore$ ALL RIGHT SIDED SIGNALS

$$x(t) = \left[2te^{-t} - e^{-t} + 3e^{-2t}\right] u(t)$$