

Tutorial 5 Z Transform

Exercise 1

Find the z-transform for each of the following sequences:

a. $x(n) = 4u(n)$

b. $x(n) = (-0.7)^n u(n)$

c. $x(n) = 4e^{-2n} u(n)$

d. $x(n) = 4(0.8)^n \cos(0.1\pi n) u(n)$

e. $x(n) = 4e^{-3n} \sin(0.1\pi n) u(n)$

Exercise 2

Using the properties of the z-transform, find the z-transform for each of the following

a. $x(n) = u(n) + (0.5)^n u(n)$

b. $x(n) = e^{-3(n-4)} \cos(0.1\pi(n-4)) u(n-4)$, where $u(n-4) = 1$ for $n \geq 4$ while $u(n-4) = 0$ for $n < 4$

Exercise 3

Given two sequences

$$x_1(n) = 5\delta(n) - 2\delta(n-2) \text{ and } x_2(n) = 3\delta(n-3)$$

- determine the z-transform of the convolution of the two sequences using the convolution property of z-transform.
- determine the convolution by the inverse z-transform.

Exercise 4

Using Z transform Table and the z-transform properties, find the inverse z-transform for each of the following functions:

a. $X(z) = 4 - \frac{10z}{z-1} - \frac{z}{z+0.5}$

b. $X(z) = \frac{-5z}{(z-1)} + \frac{10z}{(z-1)^2} + \frac{2z}{(z-0.8)^2}$

c. $X(z) = \frac{z}{z^2 + 1.2z + 1}$

d. $X(z) = \frac{4z^{-4}}{z-1} + \frac{z^{-1}}{(z-1)^2} + z^{-8} + \frac{z^{-5}}{z-0.5}$

Exercise 5

Using the partial fraction expansion method, find the inverse of the following z-transforms:

a. $X(z) = \frac{1}{z^2 - 0.3z - 0.24}$

b. $X(z) = \frac{z}{(z - 0.2)(z + 0.4)}$

c. $X(z) = \frac{z}{(z + 0.2)(z^2 - z + 0.5)}$

d. $X(z) = \frac{z(z + 0.5)}{(z - 0.1)^2(z - 0.6)}$

Exercise 6

A system is described by the difference equation

$$y(n) + 0.5y(n - 1) = 2(0.8)^n u(n)$$

Determine the solution when the initial condition is $y(-1) = 2$.

Exercise 7

Given the following difference equation with the input-output relationship of a certain initially relaxed system (all initial conditions are zero),

$$y(n) - 0.7y(n - 1) + 0.1y(n - 2) = x(n) + x(n - 1)$$

- find the impulse response sequence $y(n)$ due to the impulse sequence $\delta(n)$;
- find the output response of the system when the unit step function $u(n)$ is applied.