# Type I Error, Type II Error and Power Function

1. Let $X$ have a binomial distribution with the number of trials $n = 10$ and with $p$ either 1/4 or 1/2. The simple hypothesis $H_0: p = \frac{1}{2}$ is rejected, and the alternative simple hypothesis $H_1: p = \frac{1}{4}$ is accepted, if the observed value of $X_1$, a random sample of size 1, is less than or equal to 3. Find the significance level and the power of the test.
2. Let $X_1, X_2 \dots., X_{25}$ be a random sample from $N(\mu, 16)$. If the test: Reject $H_0: \mu = 1$ and accept $H_1: \mu = 3$ when $\bar{x} \geq 2$. Find the Type I error and the power function of the test.
3. Let $X_1, X_2$ be a random sample of size $n = 2$ from the distribution having pdf form$f(x;\theta) = \frac{1}{\theta} e^{-x/\theta}, x > 0, \theta > 0$. We reject $H_0: \theta = 2 \; vs \; H_1: \theta = 1$ if the observed values of $X_1, X_2$, say $x_1, x_2$ are such that
$$\frac{f(x_1, 2) f(x_2, 2)}{f(x_1, 1) f(x_2, 1)} \leq \frac{1}{2}$$
Here $\Omega = \{\theta: \theta = 1,2\}$. Find the significance level of the test.
4. Let $X$ have a Poisson distribution with mean $\theta$. Consider the simple hypothesis $H_0: \theta = \frac{1}{2}$ and the alternative composite hypothesis $H_1: \theta < \frac{1}{2}$. Thus $\Omega = \left\{\theta: 0 < \theta \leq \frac{1}{2}\right\}$. Let $X_1, \dots., X_{12}$ denote a random sample of size 12 from this distribution. We reject $H_0$ if and only if observed value of $Y = X_1 + \cdots. + X_{12} \leq 2$. If $\gamma(\theta)$ is the power function of the test, find the power $\gamma\left(\frac{1}{2}\right), \gamma\left(\frac{1}{3}\right), \gamma\left(\frac{1}{4}\right), \gamma\left(\frac{1}{6}\right)$ and $\gamma\left(\frac{1}{12}\right)$. What is the significance level of the test?
5. Let $X_1, X_2 \dots., X_8$ be a random sample of size $n = 8$ from a Poisson distribution with mean $\mu$. Reject the simple null hypothesis $H_0: \mu = 0.5$ and accept $H_1: \mu > 0.5$ if the observed sum $\sum_{i=1}^{8} x_i \geq 8$.
   (a) Compute the significance level α of the test.
   (b) Find the power function $\gamma(\mu)$ of the test as a sum of Poisson probabilities.
   (c) Using the Poisson Table, determine $\gamma(0.75), \gamma(1)$ and $\gamma(1.25)$.
6. Let X be a Bernoulli random variable with probability of success $p$. suppose we want to test at size $\alpha$, $H_0: p = 0.7 \; vs \; H_1: p = 0.5$. Reject $H_0$ if $\sum_{i=1}^{20} x_i \leq c$. Find $c$ if the size of test is equal to 0.048. Then, find the power function of the test.
7. Let $X_1, \dots, X_n$ is normally distributed with mean $\mu$ and variance 100. Reject $H_0: \mu = 75 \; vs \; H_1: \mu = 77$ if $\sum_{i=1}^{n} x_i > nc$. Determine $n$ and $c$ so that the power function $\gamma(\mu)$ of the test has the values $\gamma(75) = 0.159$ and $\gamma(77) = 0.841$.
8. Let $X$ have a pdf of the form $f(x, \theta) = \theta x^{\theta - 1}, 0 < x < 1$, zero elsewhere, where $\theta \in \{\theta: \theta = 1, 2\}$. To test the simple hypothesis $H_0: \theta = 1$ against the alternative simple hypothesis $H_1: \theta = 2$, use a random sample $X_1, X_2$ of size $n = 2$ and define the critical region to be $C = \left\{(x_1, x_2): \frac{3}{4} \leq x_1 x_2\right\}$. Find the power function of the test.

9. Let $Y_1 < Y_2 < Y_3 < Y_4$ be the order statistics of a random sample of size $n = 4$ from a distribution with pdf $f(x;\, \theta) = {}^1\!/_\theta\,, 0 < x < \theta$, zero elsewhere, where $0 < \theta$. The hypothesis $H_0\!: \theta = 1$ is rejected and $H_1\!: \theta > 1$ is accepted if the observed $Y_4 \geq c$.
   (a) Find the constant $c$ so that the significance level is $\alpha = 0.05$.
   (b) Determine the power function of the test.
10. Let $X$ be a single observation from the density $f(x, \theta) = (2\theta x + 1 - \theta), 0 < x < 1$, where $-1 \leq \theta \leq 1$. To test $H_0\!: \theta = 0$ vs $H_1\!: \theta > 0$, the following procedure was used: Reject $H_0$ if $X$ exceeds $\frac{1}{2}$. Find the power and the size of the test
11. Let $X_1, \dots, X_n$ denote a random sample from $f(x;\, \theta) = \left({}^1\!/_\theta\right), 0 < x < \theta$, and let $Y_1, \dots, Y_n$ be the corresponding ordered sample. To test $H_0\!: \theta = \theta_0$ versus $H_1\!: \theta \neq \theta_0$, the following test was used: Accept $H_0$ if $\theta_0\left(\sqrt[n]{\alpha}\right) \leq Y_n \leq \theta_0$; otherwise reject. Find the power function of this test.
12. Let $X_1, \dots, X_n$ be a random sample of size $n$ from $f(x;\, \theta) = \theta^2 x e^{-\theta x}\ x > 0$. In testing $H_0\!: \theta = 1$ versus $H_1\!: \theta > 1$ for $n = 1$ (a sample of size 1) the following test was used: Reject $H_0$ if and only if $X_1 \leq 1$. Find the power function and size of the test.