جامعة
الملك سعود
King Saud University

College of Computer and Information Sciences
Department of Computer Science

CSC 220: Computer Organization

Unit1
Number Systems

Decimal review

- Decimal numbers consist of digits from 0 to 9, each with a weight.

1	6	2	.	3	7	5	digits
100	10	1		1/10	1/100	1/1000	weights

- Notice that the weights are all powers of the base, which is 10.

1	6	2	.	3	7	5	digits
10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}	weights

- To find the decimal value of a number, you can multiply each digit by its weight and sum the products:

$$(1 \times 10^2) + (6 \times 10^1) + (2 \times 10^0) + (3 \times 10^{-1}) + (7 \times 10^{-2}) + (5 \times 10^{-3}) = 162.375$$

Common Number Systems

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, … 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, … 7	No	No
Hexadecimal	16	0, 1, … 9, A, B, … F	No	No

Quantities/Counting (1 of 3)

Decimal	Binary	Octal	Hexa-decimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7

Quantities/Counting (2 of 3)

Decimal	Binary	Octal	Hexa-decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Quantities/Counting (3 of 3)

Decimal	Binary	Octal	Hexa-decimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17

Etc.

Binary numbers

- Binary, or base 2, numbers consist of only the digits 0 and 1. The weights are now powers of 2.
- For example, consider the binary number 1101.01:

1	1	0	1	.	0	1	binary digits, or bits
2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	weights in decimal

- The decimal value of 1101.01 is computed just like before:

$$
\begin{array}{ccccccccccccc}
(1 \times 2^3) & + & (1 \times 2^2) & + & (0 \times 2^1) & + & (1 \times 2^0) & + & (0 \times 2^{-1}) & + & (1 \times 2^{-2}) & = & \\
8 & + & 4 & + & 0 & + & 1 & + & 0 & + & 0.25 & = & 13.25
\end{array}
$$

Some powers of 2

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

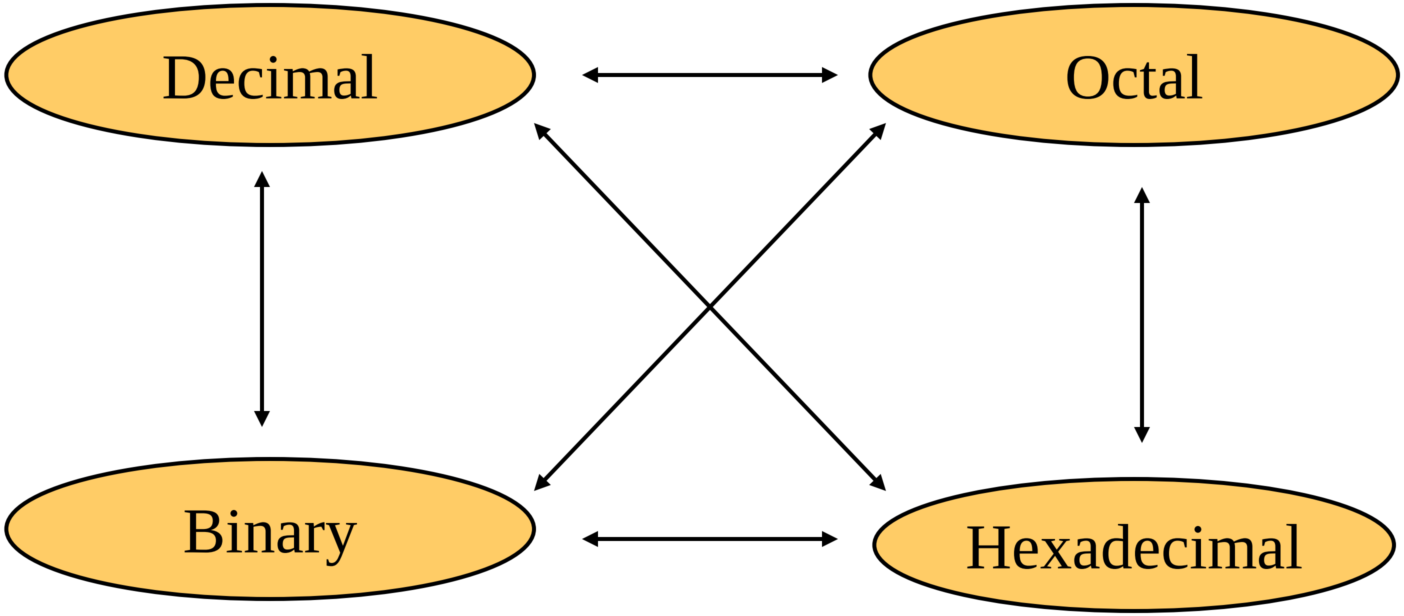
$2^8 = 256$

$2^9 = 512$

$2^{10} = 1024$

Conversion Among Bases

- The possibilities:

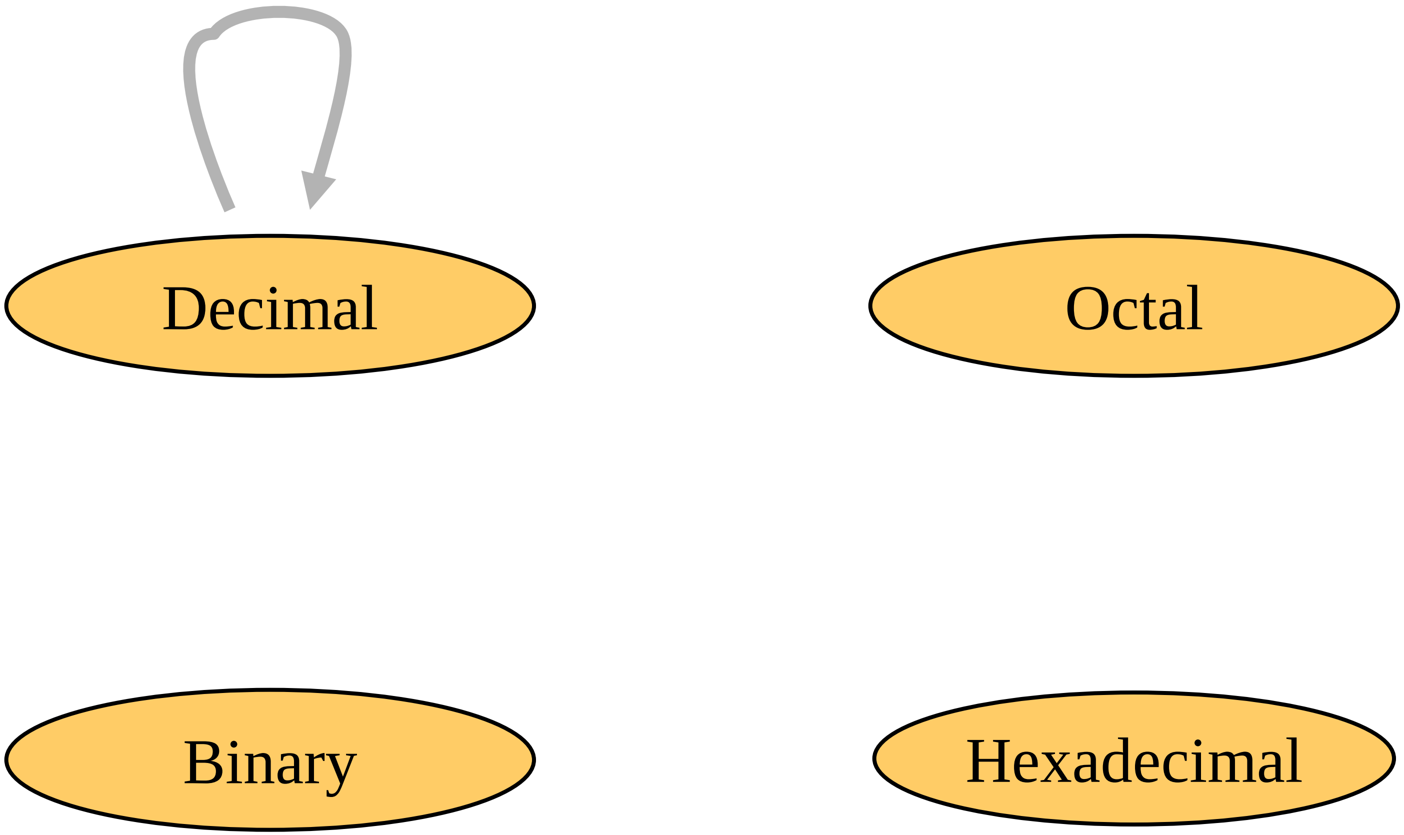


Quick Example

$$25_{10} = 11001_2 = 31_8 = 19_{16}$$

Base

Decimal to Decimal (just for fun)

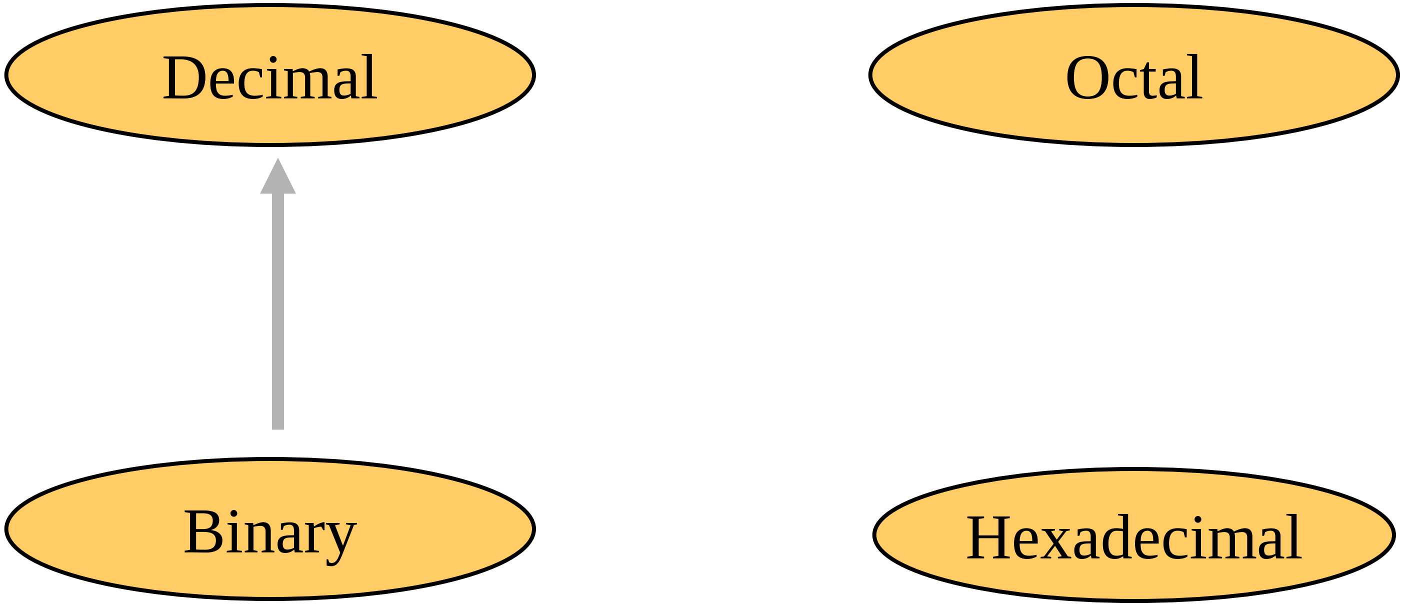


Weight

$$
\begin{array}{lllr}
125_{10} \Rightarrow & 5 \times 10^{0} & = & 5 \\
 & 2 \times 10^{1} & = & 20 \\
 & 1 \times 10^{2} & = & 100 \\
\cline{4-4}
 & & & 125
\end{array}
$$

Base

Binary to Decimal



Binary to Decimal

- Technique
  - Multiply each bit by 2^n, where n is the “weight” of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Example

Bit “0”

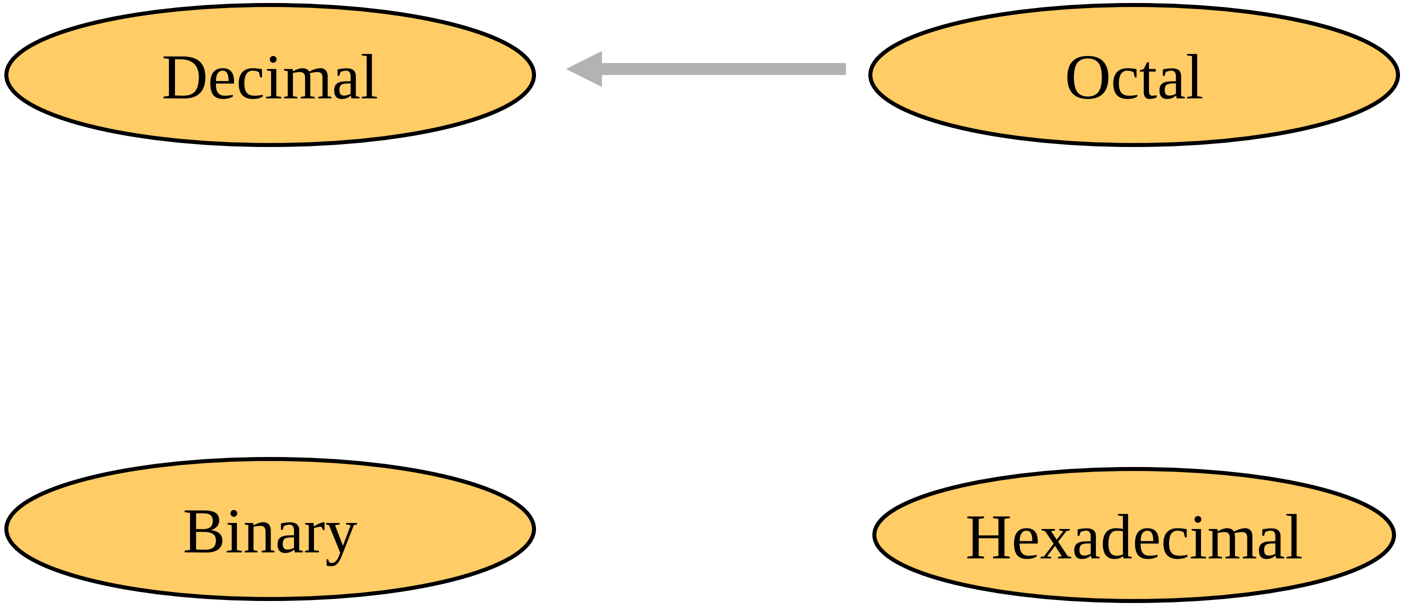
$$
\begin{array}{llr}
101011_2 \Rightarrow & 1 \times 2^0 = & 1 \\
 & 1 \times 2^1 = & 2 \\
 & 0 \times 2^2 = & 0 \\
 & 1 \times 2^3 = & 8 \\
 & 0 \times 2^4 = & 0 \\
 & 1 \times 2^5 = & 32 \\
\hline
 & & 43_{10}
\end{array}
$$

Converting Binary to Decimal

What is the decimal equivalent of the binary number 1101110?

1 x 2^6 = 1 x 64 = 64
+ 1 x 2^5 = 1 x 32 = 32
+ 0 x 2^4 = 0 x 16 = 0
+ 1 x 2^3 = 1 x 8 = 8
+ 1 x 2^2 = 1 x 4 = 4
+ 1 x 2^1 = 1 x 2 = 2
+ 0 x 2^o = 0 x 1 = 0
= 110 in base 10

Octal to Decimal



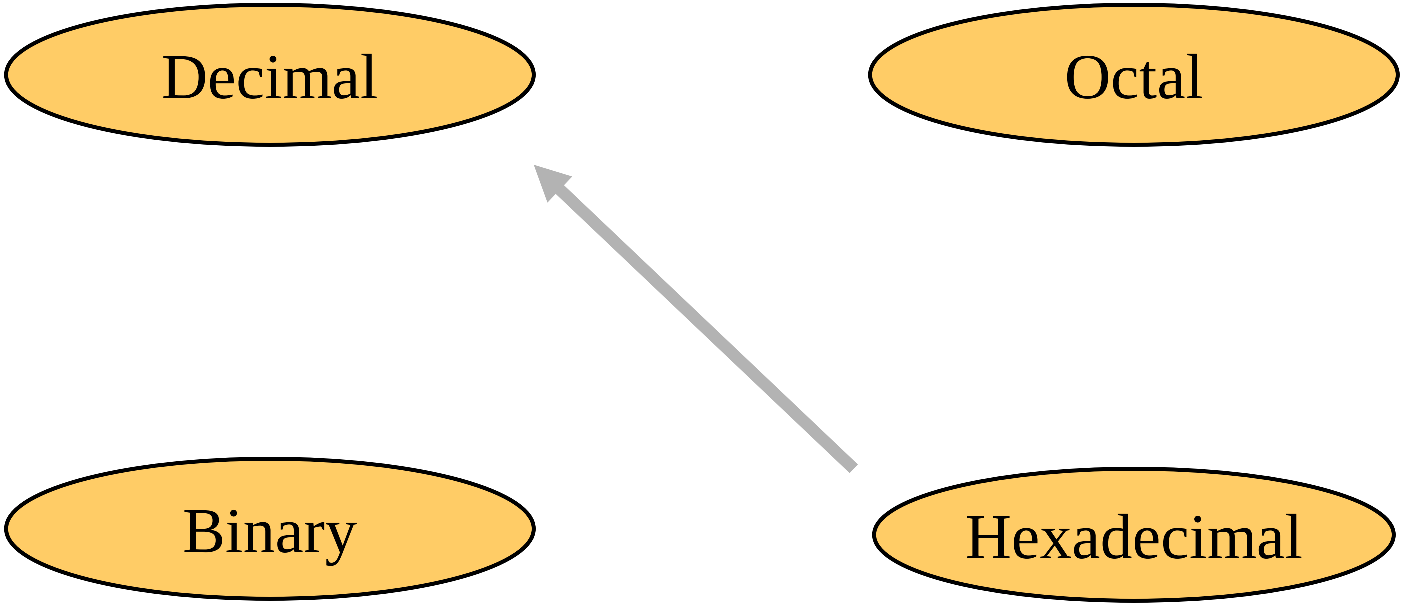
Octal to Decimal

- Technique
  - Multiply each bit by 8^n, where n is the “weight” of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Example

$$
\begin{array}{lrr}
724_8 \Rightarrow & 4 \times 8^0 = & 4 \\
 & 2 \times 8^1 = & 16 \\
 & 7 \times 8^2 = & 448 \\
\hline
 & & 468_{10}
\end{array}
$$

Hexadecimal to Decimal



Hexadecimal to Decimal

- Technique
  - Multiply each bit by 16^n, where n is the “weight” of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Base 16 is useful too

- The hexadecimal system uses 16 digits:

  0 1 2 3 4 5 6 7 8 9 A B C D E F

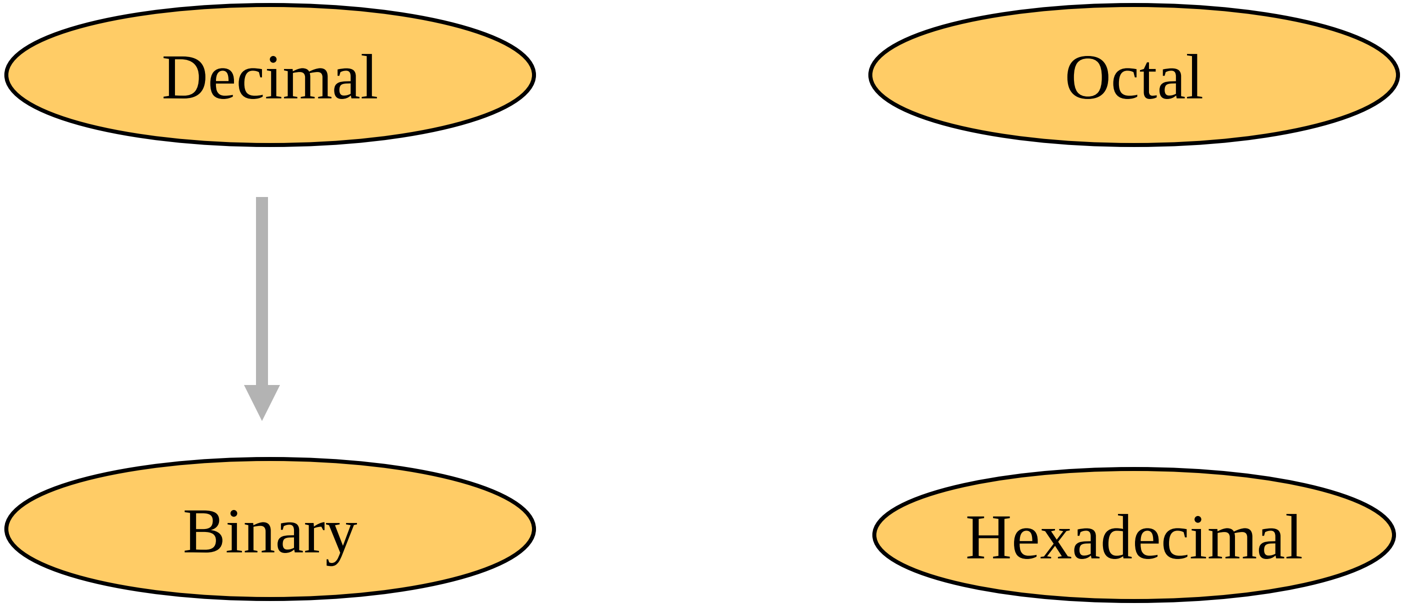
- Hexadecimal is useful as a shorthand for binary numbers.
  - Since 16 = 2^4, one hex digit is equivalent to four bits (including leading 0s).
  - It's often easier to work with numbers like "B4" instead of "10110100".
- Hex shows up in many different contexts.
  - IP addresses, such as "80.AE.05.27".
  - RGB color triplets, like "C0C0FF".
- You can convert between base 10 and base 16 using the same method as for converting from decimal to binary.

Decimal	Binary	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Example

$$
\begin{array}{llr}
\mathtt{ABC}_{16} \;\mathtt{=>} & \mathtt{C} \; \mathtt{x} \; 16^{0} = 12 \; \mathtt{x} \; 1 = & 12 \\
 & \mathtt{B} \; \mathtt{x} \; 16^{1} = 11 \; \mathtt{x} \; 16 = & 176 \\
 & \mathtt{A} \; \mathtt{x} \; 16^{2} = 10 \; \mathtt{x} \; 256 = & 2560 \\ \cline{3-3}
 & & 2748_{10}
\end{array}
$$

Decimal to Binary



Decimal

Octal

Binary

Hexadecimal

Decimal to Binary

- Technique
  - Divide by two, keep track of the remainder
  - First remainder is bit 0 (LSB, least-significant bit)
  - Second remainder is bit 1
  - Etc.

Converting decimal to binary

- To convert a decimal integer into binary, keep dividing by two until the quotient is 0. Then collect the remainders in reverse order.
- To convert a decimal fraction into binary, keep multiplying the fractional part by two until it becomes 0. Collect the integers in forward order.
- An example will make it all clear. Let's convert 162.375 to binary.

$162 / 2 = 81$ rem 0

$81 / 2 = 40$ rem 1

$40 / 2 = 20$ rem 0

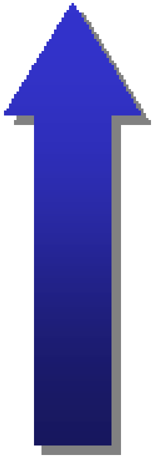
$20 / 2 = 10$ rem 0

$10 / 2 = 5$ rem 0

$5 / 2 = 2$ rem 1

$2 / 2 = 1$ rem 0

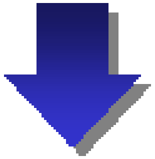
$1 / 2 = 0$ rem 1



$0.375 \times 2 = 0.750$

$0.750 \times 2 = 1.500$

$0.500 \times 2 = 1.000$



- So $162.375_{10} = 10100010.011_2$

Why does this work?

- This same idea works for converting from decimal to any other base.
- Think about "converting" 162 from decimal to decimal:

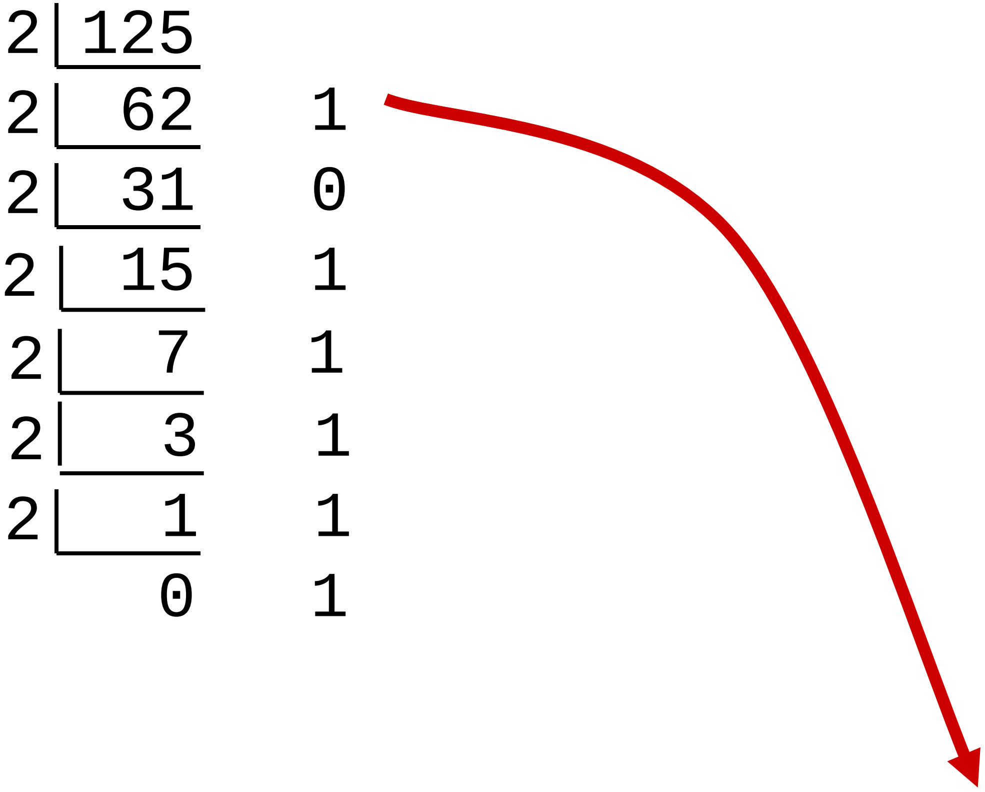
$$\begin{array}{rcrl} 162 / 10 & = & 16 & \text{rem } 2 \\ 16 / 10 & = & 1 & \text{rem } 6 \\ 1 / 10 & = & 0 & \text{rem } 1 \end{array}$$

- After each division, the remainder contains the rightmost digit of the dividend, while the quotient holds the remaining digits.
- Similarly when converting fractions, each multiplication strips off the leftmost digit as the integer result, leaving the remaining digits in the fractional part.

$$\begin{array}{rcl} 0.375 \times 10 & = & 3.750 \\ 0.750 \times 10 & = & 7.500 \\ 0.500 \times 10 & = & 5.000 \end{array}$$

Example

$125_{10} = ?_2$



2 | 125

2 | 62   1

2 | 31   0

2 | 15   1

2 | 7   1

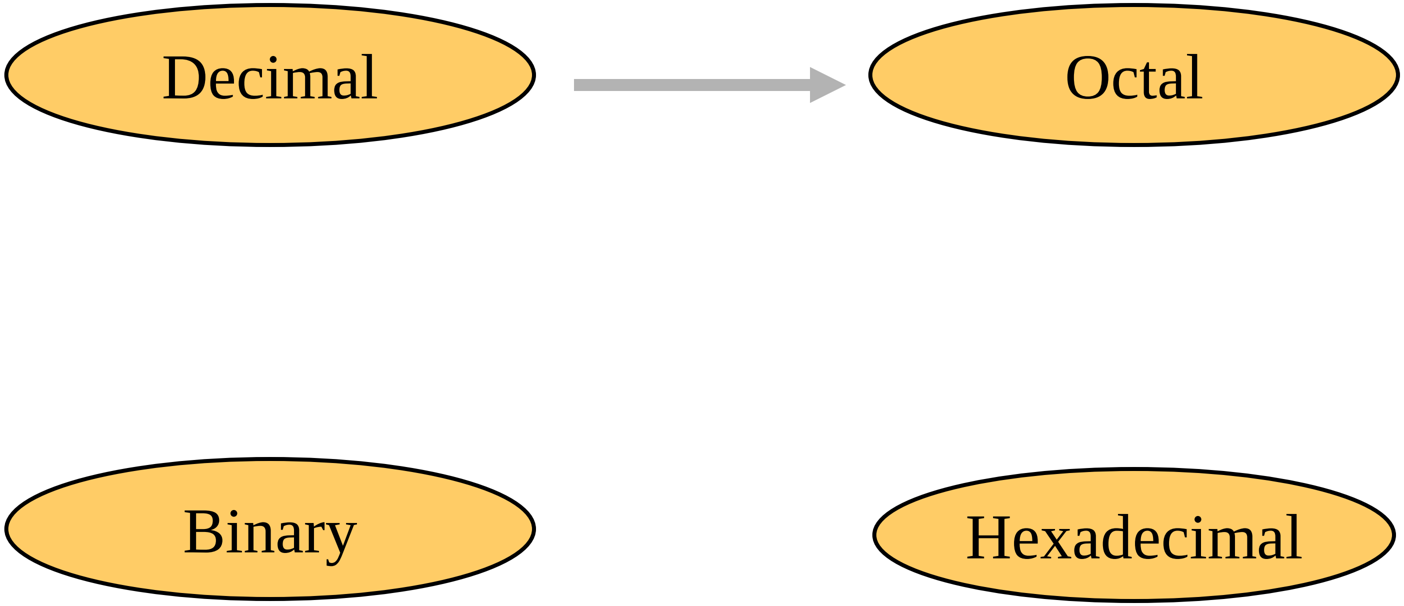
2 | 3   1

2 | 1   1

0   1

$125_{10} = 1111101_2$

Decimal to Octal

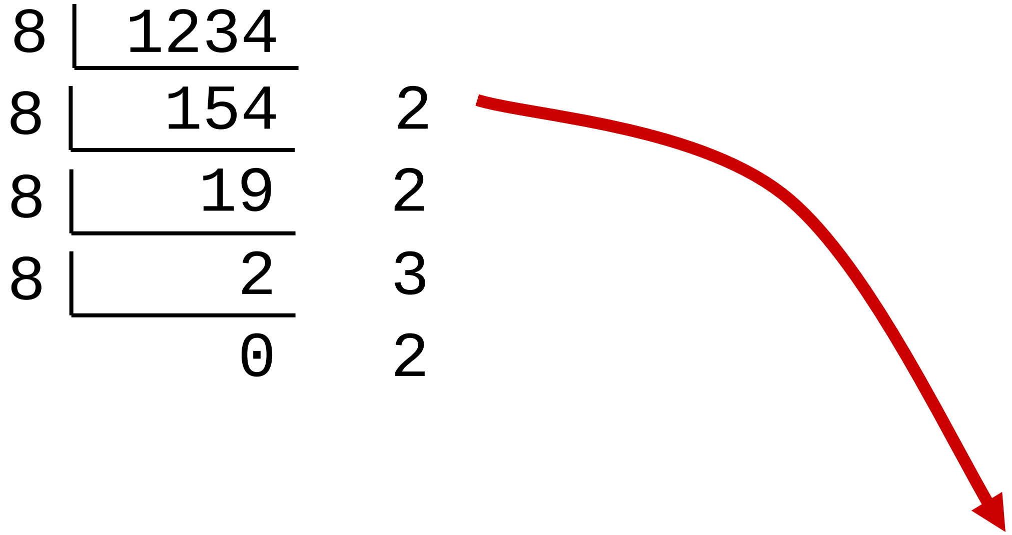


Decimal to Octal

- Technique
  - Divide by 8
  - Keep track of the remainder

Example

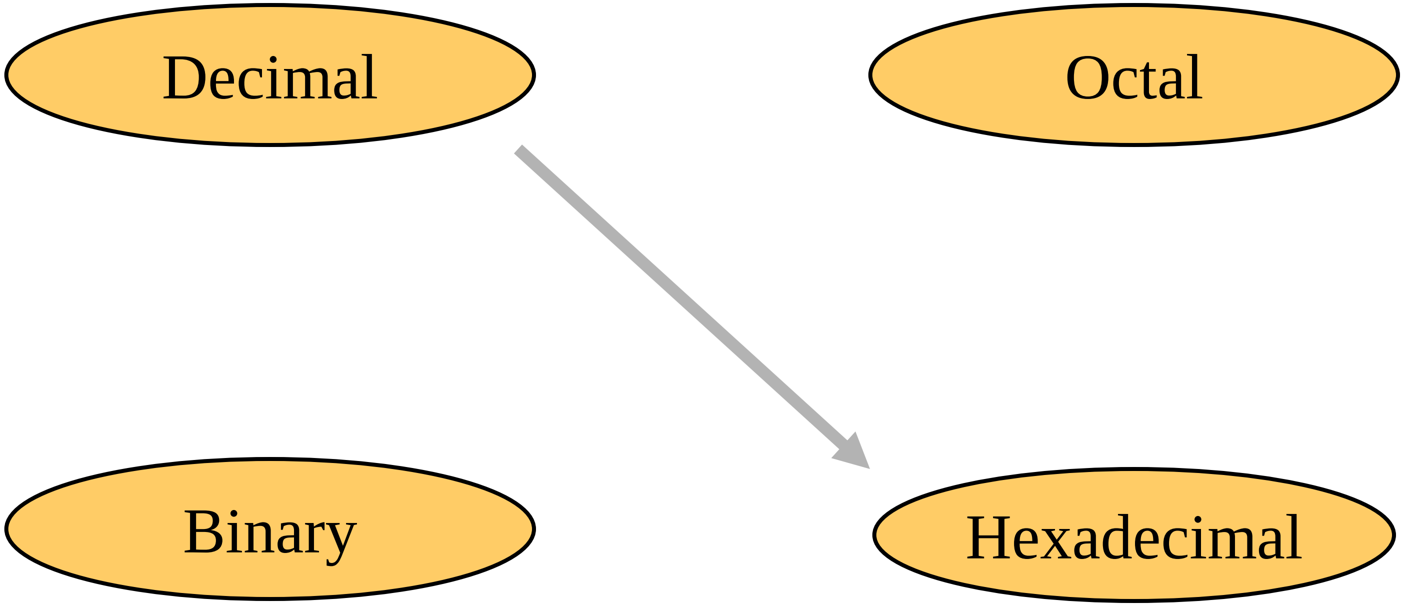
$1234_{10} = ?_8$



```
8 | 1234
8 |  154   2
8 |   19   2
8 |    2   3
       0   2
```

$1234_{10} = 2322_8$

Decimal to Hexadecimal

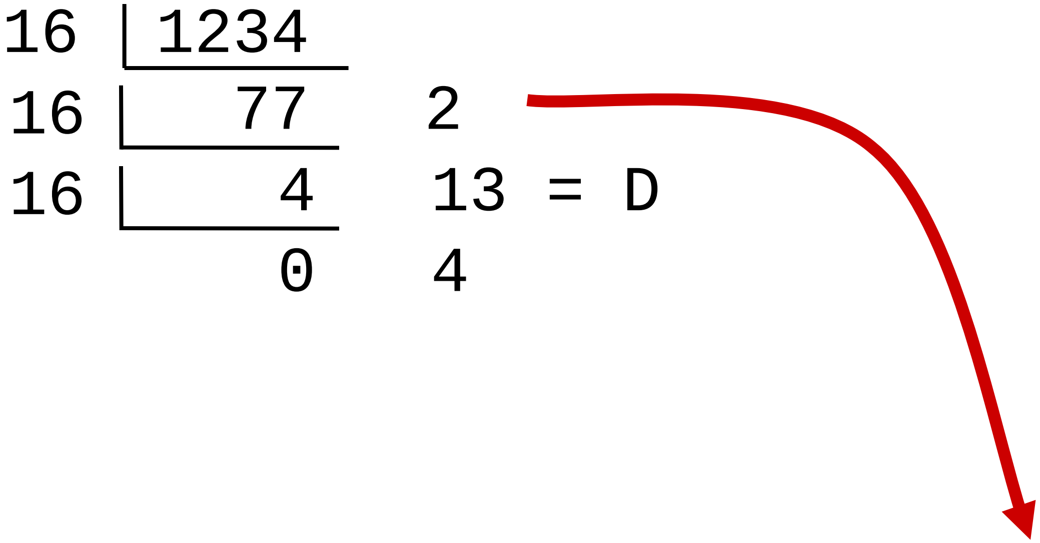


Decimal to Hexadecimal

- Technique
  - Divide by 16
  - Keep track of the remainder

Example

$1234_{10} = ?_{16}$



16 | 1234

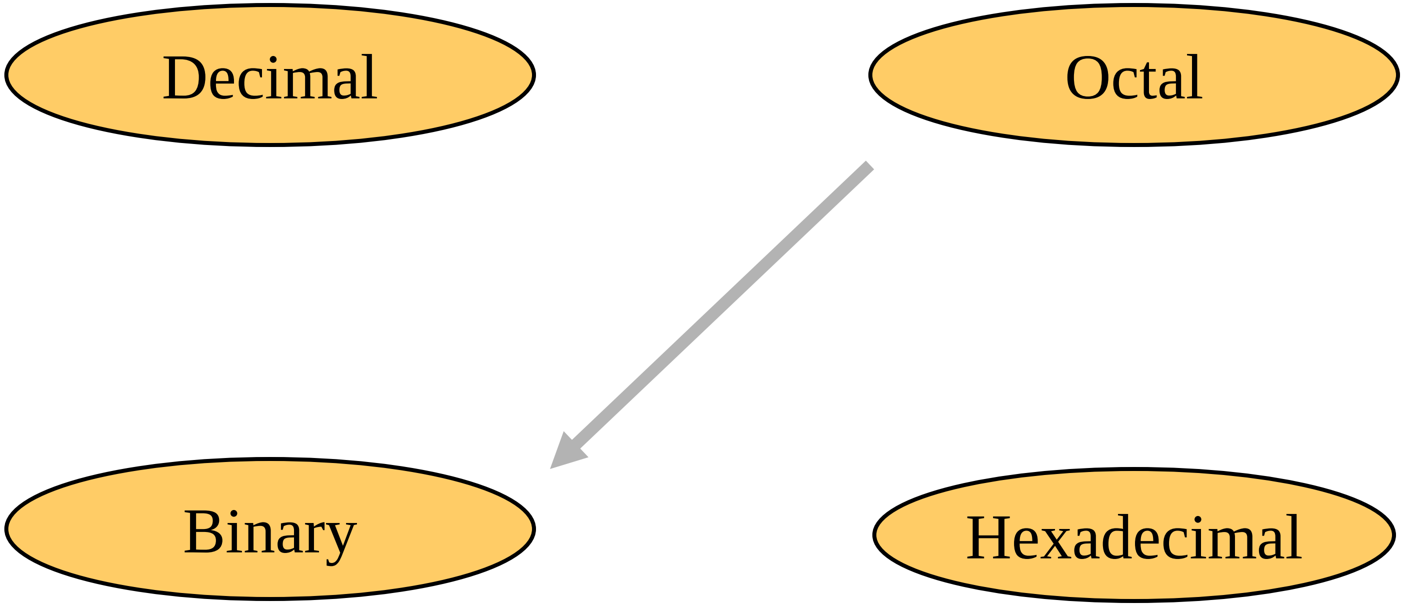
16 | 77 2

16 | 4 13 = D

0 4

$1234_{10} = 4D2_{16}$

Octal to Binary

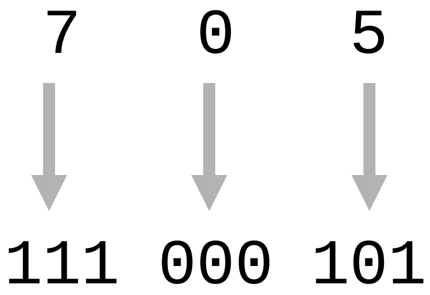


Octal to Binary

- Technique
  - Convert each octal digit to a 3-bit equivalent binary representation

Example

$705_8 = ?_2$

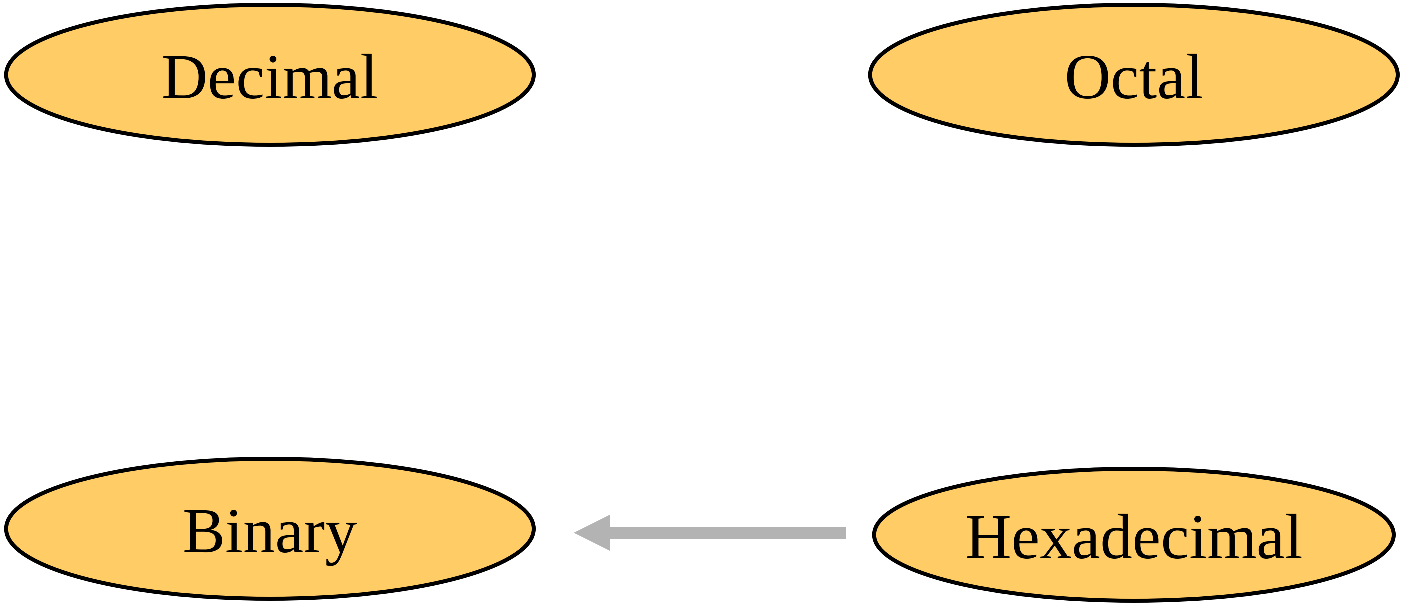


7 0 5

111 000 101

$705_8 = 111000101_2$

Hexadecimal to Binary



Hexadecimal to Binary

- Technique
  - Convert each hexadecimal digit to a 4-bit equivalent binary representation

Binary and hexadecimal conversions

- Converting from hexadecimal to binary is easy: replace each hex digit with its equivalent four-bit binary value.

$$\begin{aligned} 261.A5_{16} &= 2 \quad 6 \quad 1 \quad . \quad A \quad 5_{16} \\ &= 0010 \quad 0110 \quad 0001 \quad . \quad 1010 \quad 0101_{2} \end{aligned}$$

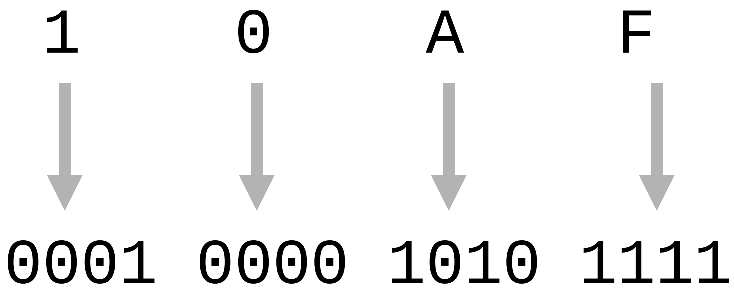
- To convert from binary to hexadecimal, partition the binary number into groups of four bits, starting from the point. (Add 0s to the ends if needed.) Then replace each four-bit group by the corresponding hex digit.

$$\begin{aligned} 10110100.001011_{2} &= 1011 \quad 0100 \quad . \quad 0010 \quad 1100_{2} \\ & \quad\; B \qquad\; 4 \quad\;\; . \quad\;\; 2 \qquad\; C_{16} \end{aligned}$$

Binary	Hex
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

Example

$10AF_{16} = ?_2$

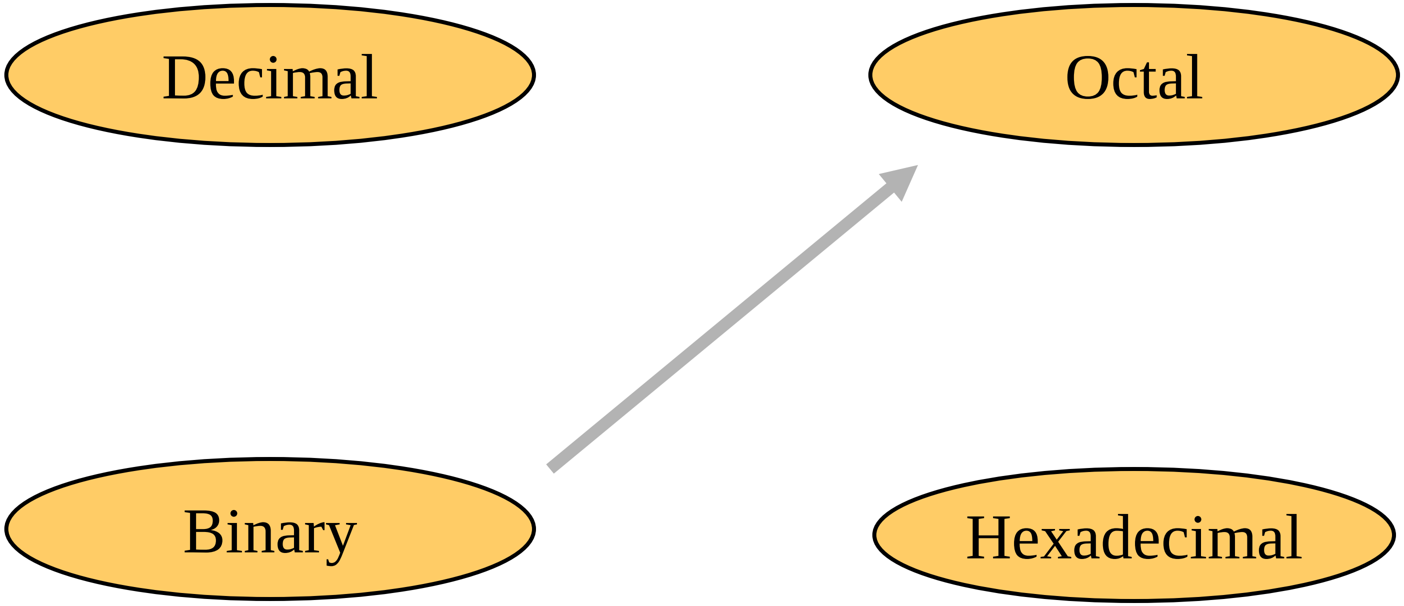


1 0 A F

0001 0000 1010 1111

$10AF_{16} = 0001000010101111_2$

Binary to Octal

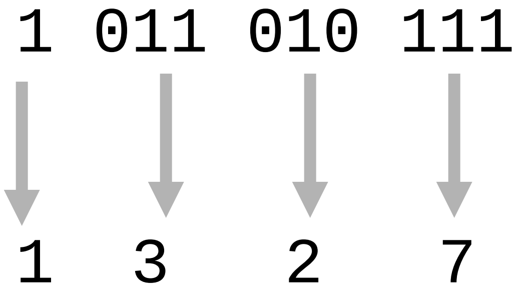


Binary to Octal

- Technique
  - Group bits in threes, starting on right
  - Convert to octal digits

Example

$1011010111_2 = ?_8$

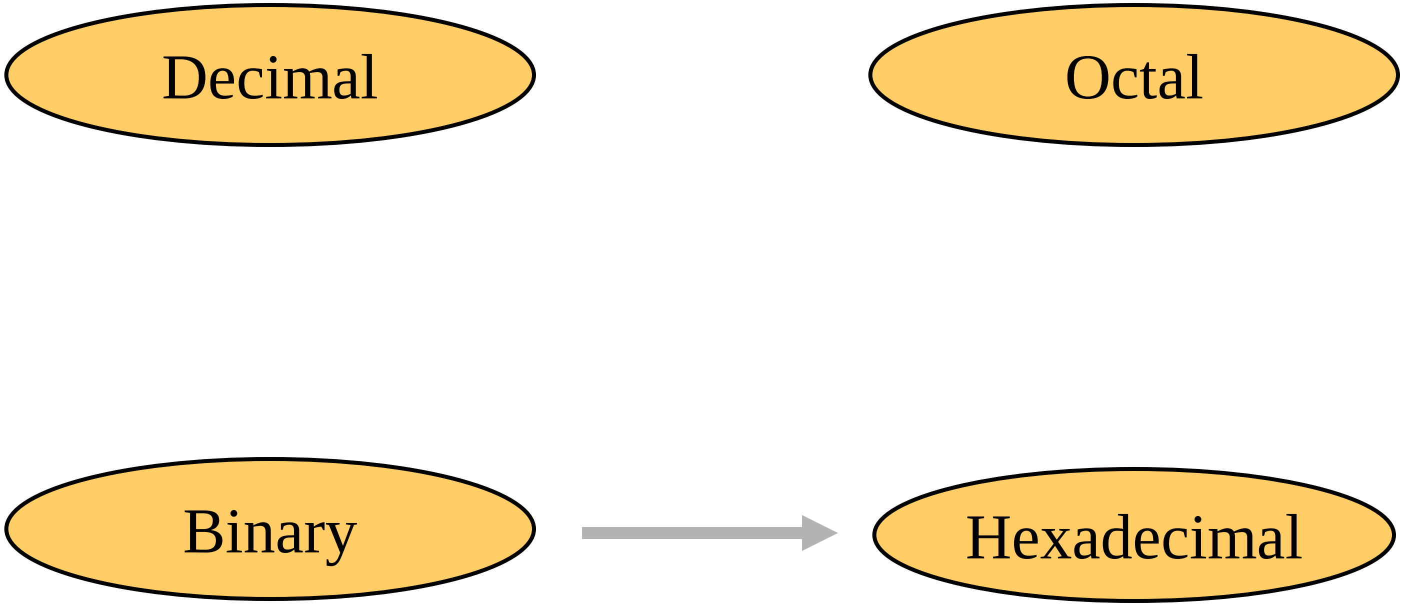


1 011 010 111

1 3 2 7

$1011010111_2 = 1327_8$

Binary to Hexadecimal

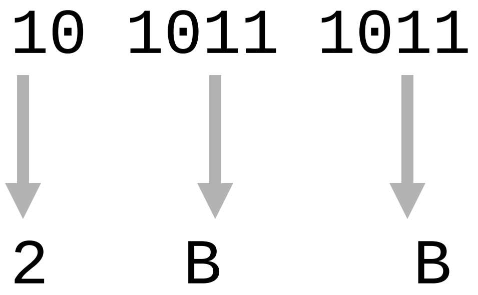


Binary to Hexadecimal

- Technique
  - Group bits in fours, starting on right
  - Convert to hexadecimal digits

Example

$1010111011_2 = ?_{16}$

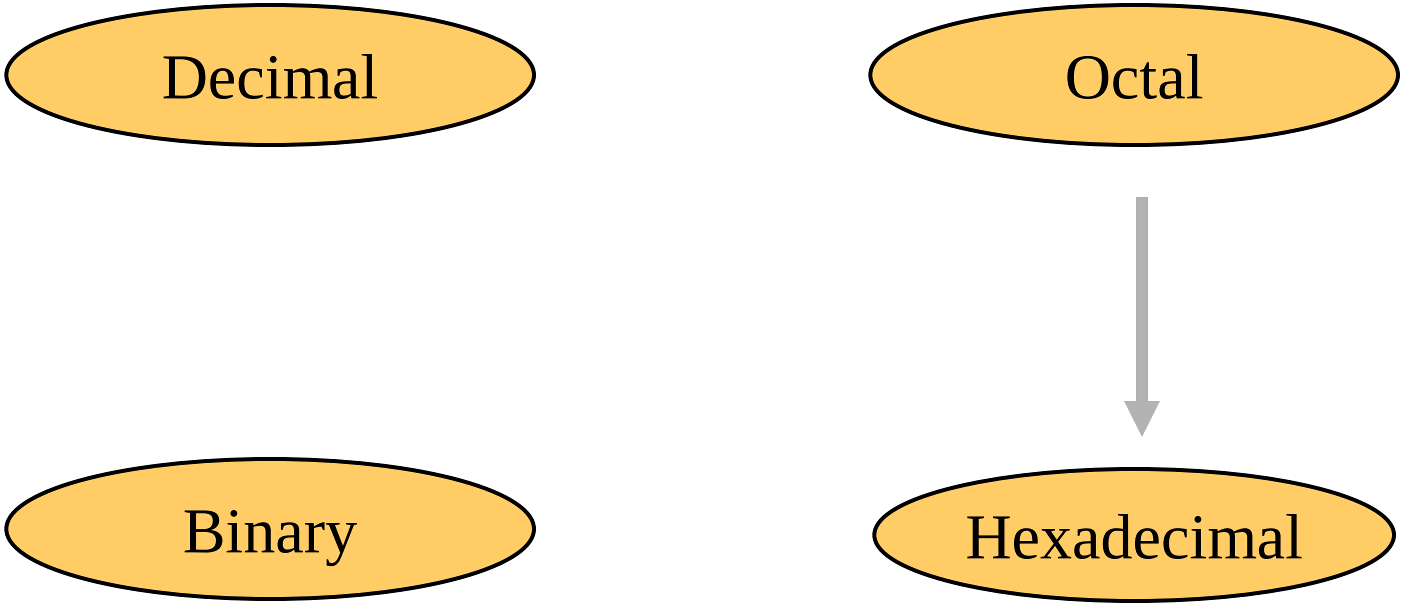


10 1011 1011

2 B B

$1010111011_2 = 2BB_{16}$

Octal to Hexadecimal

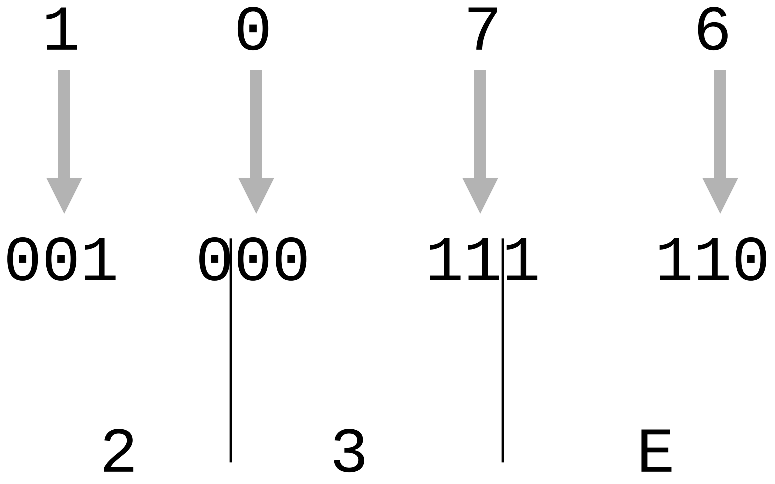


Octal to Hexadecimal

- Technique
  - Use binary as an intermediary

Example

$1076_8 = ?_{16}$



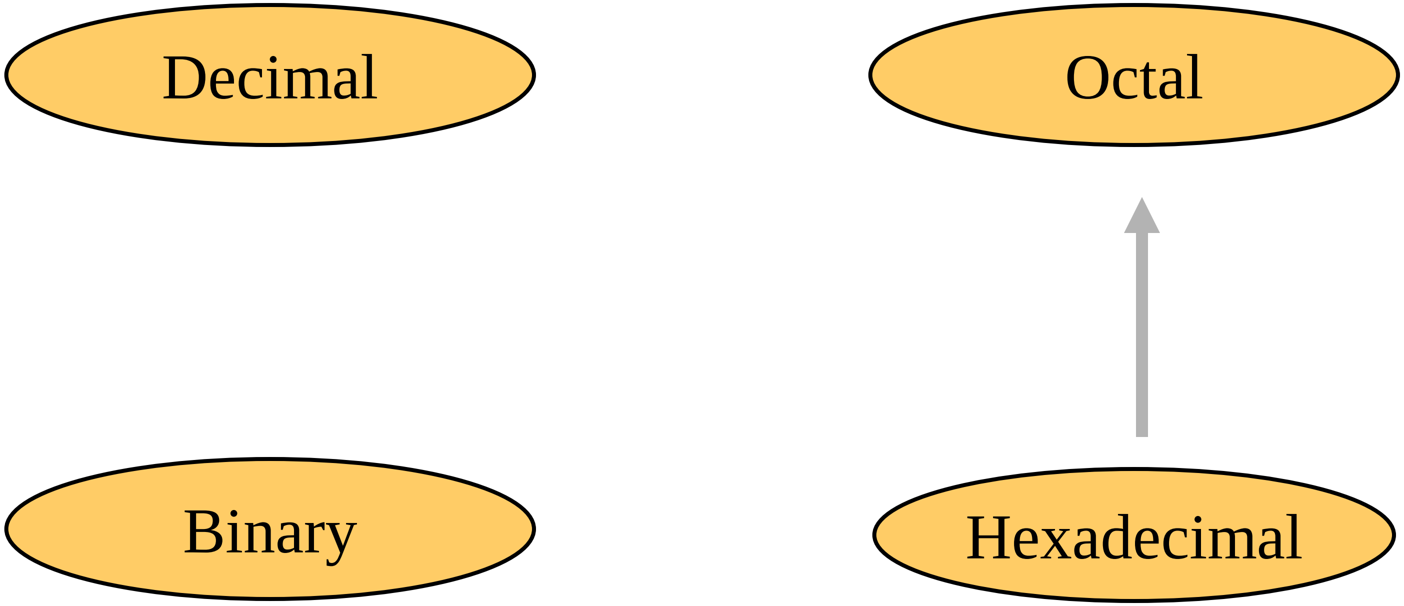
1 0 7 6

001 000 111 110

2 3 E

$1076_8 = 23E_{16}$

Hexadecimal to Octal

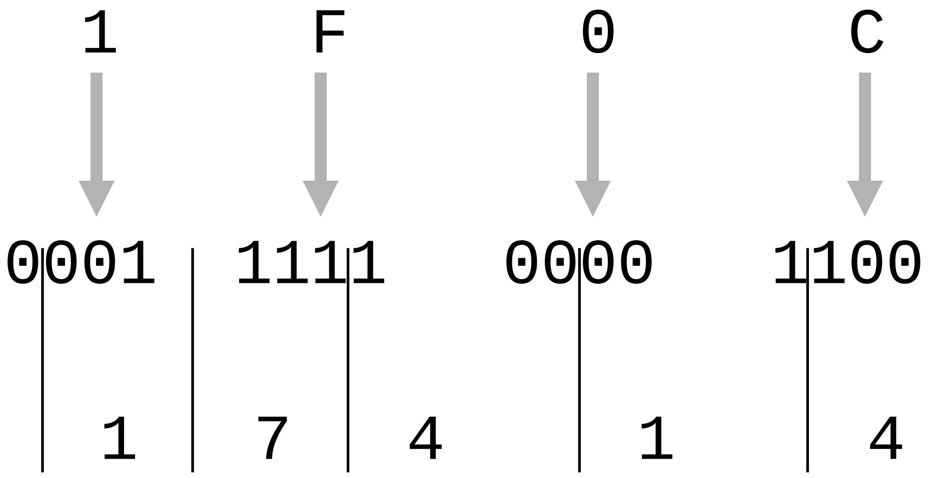


Hexadecimal to Octal

- Technique
  - Use binary as an intermediary

Example

$1F0C_{16} = ?_8$



1 F 0 C

0001 1111 0000 1100

1 7 4 1 4

$1F0C_{16} = 17414_8$

Exercise – Convert ...

Decimal	Binary	Octal	Hexa-decimal
33			
	1110101		
		703	
			1AF

Don't use a calculator!

Skip answer

Answer

Exercise – Convert …

Answer

Decimal	Binary	Octal	Hexadecimal
33	100001	41	21
117	1110101	165	75
451	111000011	703	1C3
431	110101111	657	1AF

Common Powers (1 of 2)

- Base 10

Power	Preface	Symbol	Value
10^{-12}	pico	p	.000000000001
10^{-9}	nano	n	.000000001
10^{-6}	micro	µ	.000001
10^{-3}	milli	m	.001
10^{3}	kilo	k	1000
10^{6}	mega	M	1000000
10^{9}	giga	G	1000000000
10^{12}	tera	T	1000000000000

Common Powers (2 of 2)

- Base 2

Power	Preface	Symbol	Value
2^{10}	kilo	k	1024
2^{20}	mega	M	1048576
2^{30}	Giga	G	1073741824

- What is the value of “k”, “M”, and “G”?
- In computing, particularly w.r.t. memory, the base-2 interpretation generally applies

Fractions

- Decimal to decimal (just for fun)

$$\begin{array}{lllr}
3.14 \Rightarrow & 4 \times 10^{-2} & = & 0.04 \\
 & 1 \times 10^{-1} & = & 0.1 \\
 & 3 \times 10^{0} & = & 3 \\
\cline{4-4}
 & & & 3.14
\end{array}$$

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Fractions

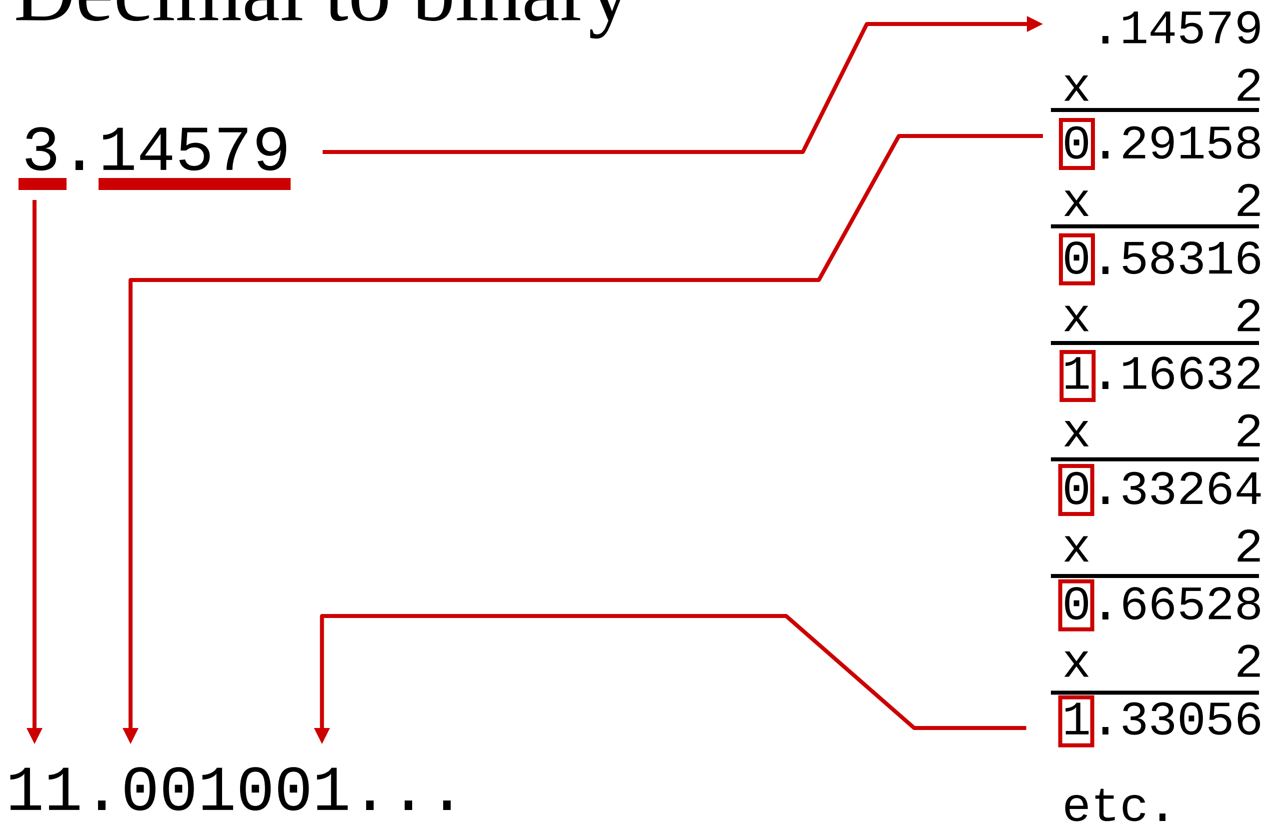
- Binary to decimal

$$
\begin{array}{llcl}
10.1011 \Rightarrow & 1 \times 2^{-4} & = & 0.0625 \\
 & 1 \times 2^{-3} & = & 0.125 \\
 & 0 \times 2^{-2} & = & 0.0 \\
 & 1 \times 2^{-1} & = & 0.5 \\
 & 0 \times 2^{0} & = & 0.0 \\
 & 1 \times 2^{1} & = & \underline{2.0} \\
 & & & 2.6875
\end{array}
$$

pp. 46-50

Fractions

- Decimal to binary



```
3.14579

   .14579
x       2
 0.29158
x       2
 0.58316
x       2
 1.16632
x       2
 0.33264
x       2
 0.66528
x       2
 1.33056

etc.

11.001001...
```

p. 50

Exercise – Convert ...

Decimal	Binary	Octal	Hexa-decimal
29.8			
	101.1101		
		3.07	
			C.82

Don’t use a calculator!

Skip answer

Answer

Exercise – Convert …

Answer

Decimal	Binary	Octal	Hexa-decimal
29.8	11101.110011…	35.63…	1D.CC…
5.8125	101.1101	5.64	5.D
3.109375	11.000111	3.07	3.1C
12.5078125	1100.10000010	14.404	C.82

4-Bit Binary Coded Decimal (BCD) Systems

- The 4-bit BCD system is usually employed by the computer systems to represent and process numerical data only. In the 4-bit BCD system, each digit of the decimal number is encoded to its corresponding 4-bit binary sequence. The two most popular 4-bit BCD systems are:

4-Bit BCD Code

Decimal digits	Weighted 4-bit BCD code
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

4-Bit BCD Code

- Represent the decimal number 5327 in BCD code.

The corresponding 4-bit BCD representation of decimal digit 5 is 0101

The corresponding 4-bit BCD representation of decimal digit 3 is 0011

The corresponding 4-bit BCD representation of decimal digit 2 is 0010

The corresponding 4-bit BCD representation of decimal digit 7 is 0111

Therefore, the BCD representation of decimal number 5327 is 0101001100100111.

Thank you