

ME 476

Solar Energy

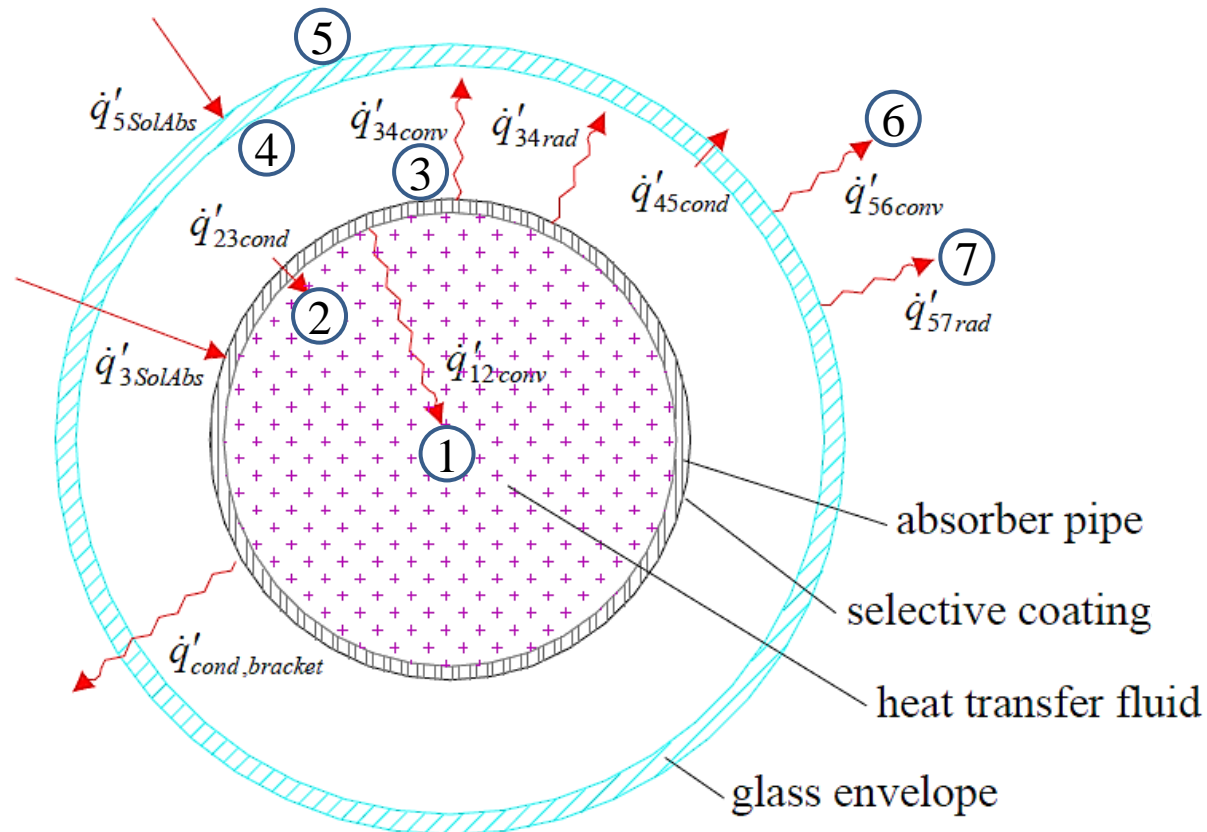
UNIT FIVE

THERMAL ANALYSIS OF PARABOLIC TROUGH COLLECTORS

- The objective of this unit is to formulate a simplified analysis of the useful energy gain of a parabolic trough collector.
- This analysis will be only one-dimensional.
- It will look at the heat loss and energy gain in the radial direction only.
 - **The temperature of the fluid will be assumed to be constant along the absorber tube.**
- This analysis is suitable for a short section of a parabolic trough receiver.

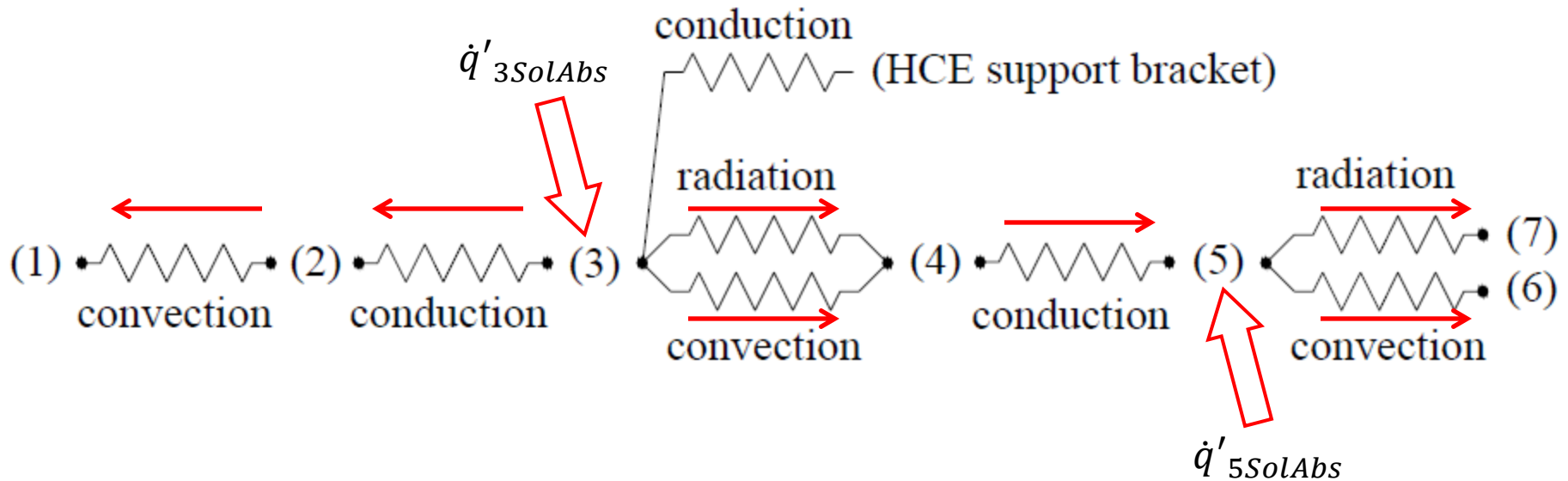
ENERGY GAIN AND LOSS MECHANISMS

- (1) heat transfer fluid
- (2) absorber inner surface
- (3) absorber outer surface
- (4) glass envelope inner surface
- (5) glass envelope outer surface
- (6) surrounding air
- (7) sky



- The prime symbol (') denotes “per unit length”

THERMAL RESISTANCE MODEL

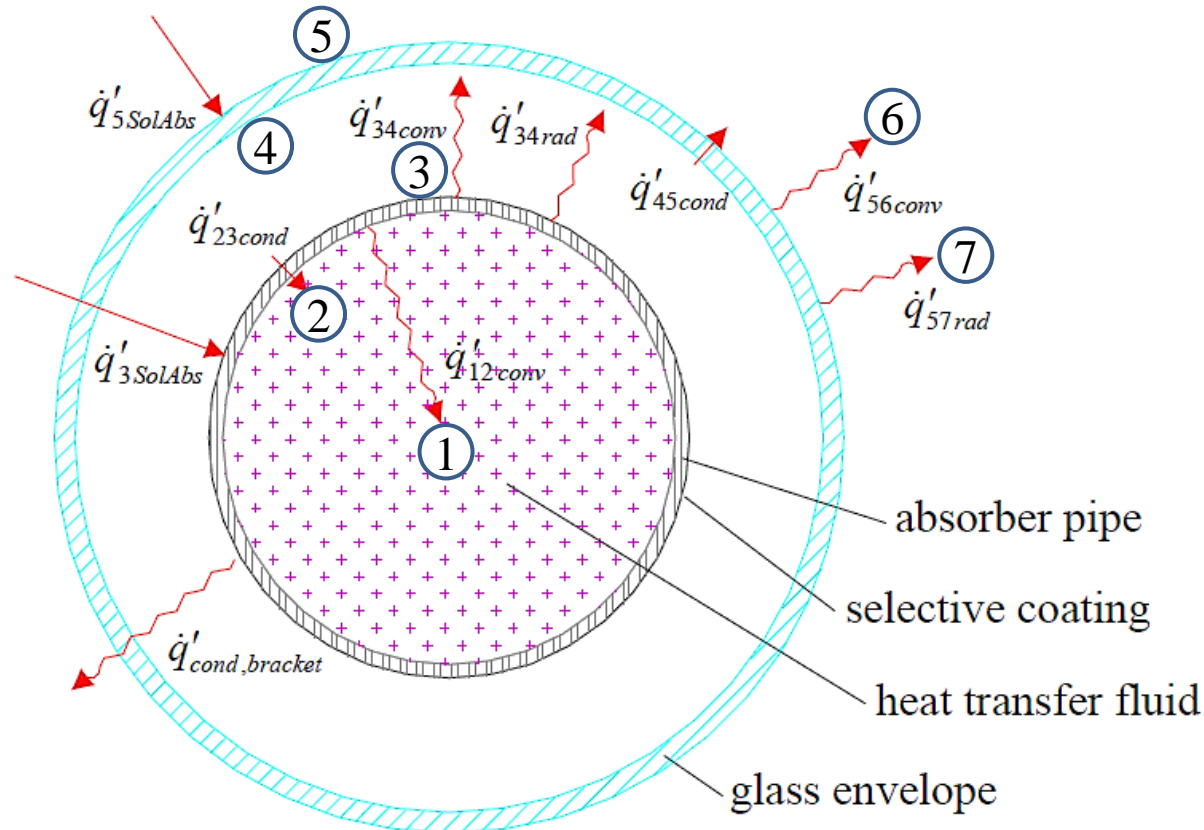


- (1) heat transfer fluid
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- (4) glass envelope inner surface

- (5) glass envelope outer surface
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- (7) sky

ENERGY BALANCE ON SURFACE (2)

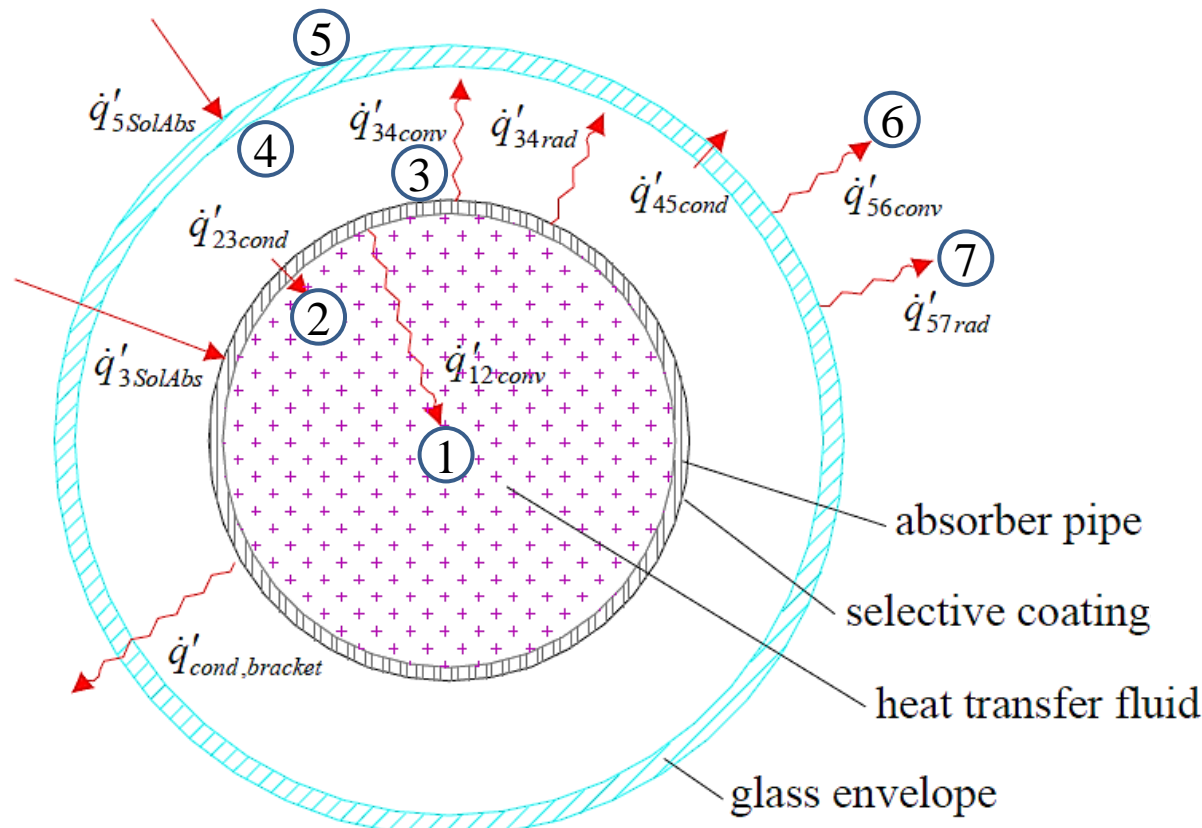
$$\dot{q}'_{12conv} = \dot{q}'_{23cond}$$



- Either \dot{q}'_{12conv} or \dot{q}'_{23cond} are the **useful energy gain**.

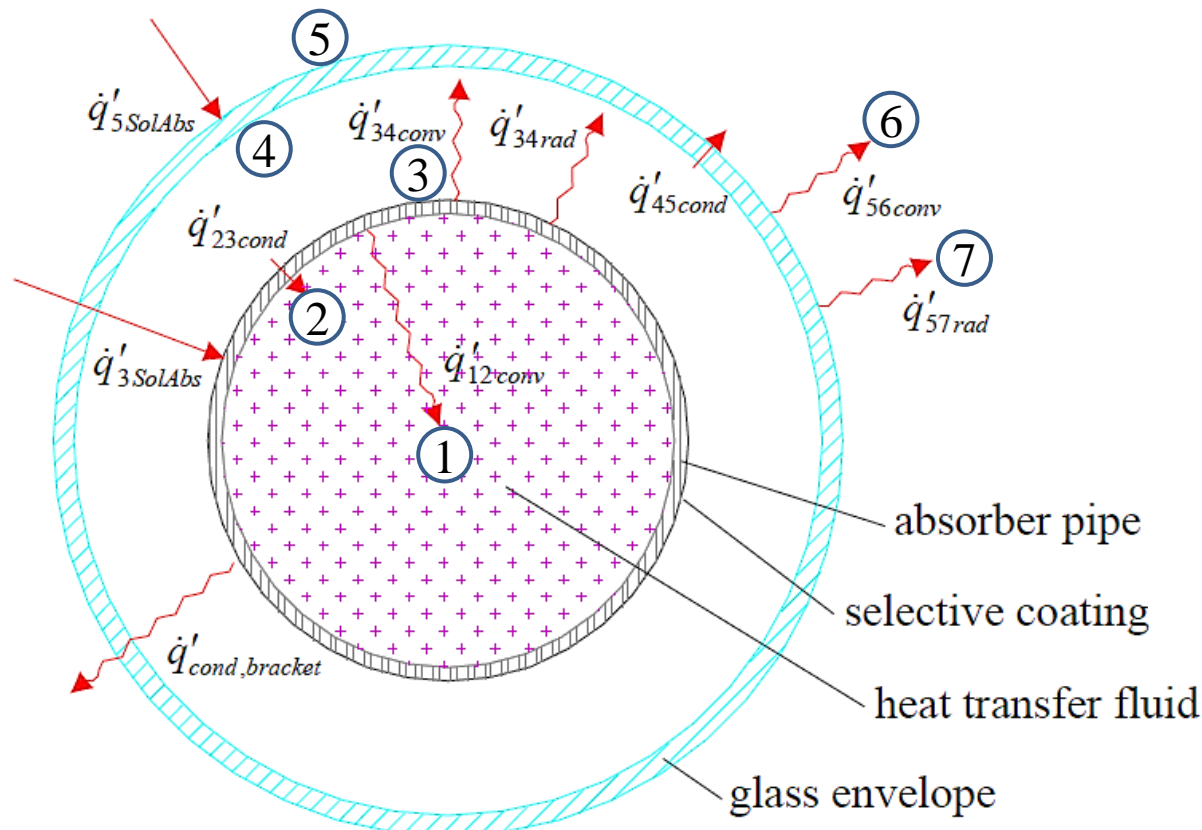
ENERGY BALANCE ON SURFACE (3)

$$\dot{q}'_{3SolAbs} = \dot{q}'_{34conv} + \dot{q}'_{34rad} + \dot{q}'_{23cond} + \dot{q}'_{cond, bracket}$$



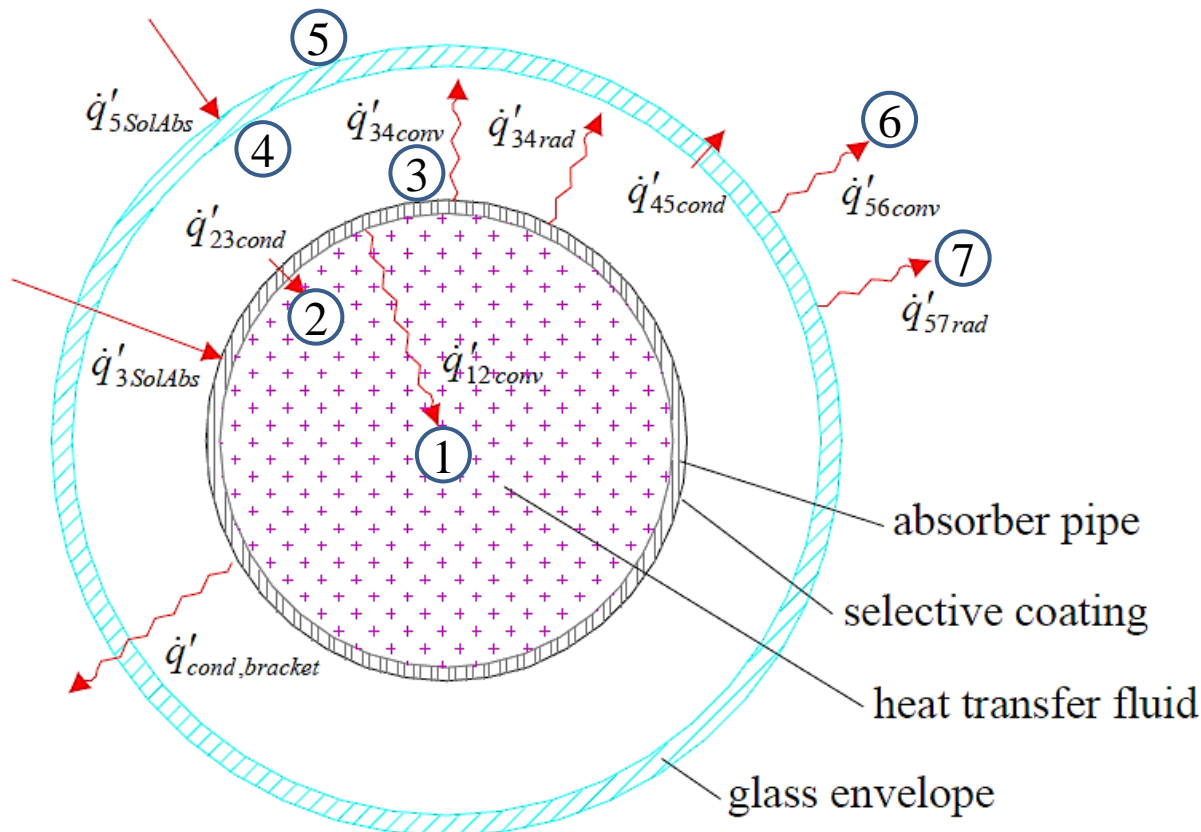
ENERGY BALANCE ON SURFACE (4)

$$\dot{q}'_{34conv} + \dot{q}'_{34rad} = \dot{q}'_{45cond}$$



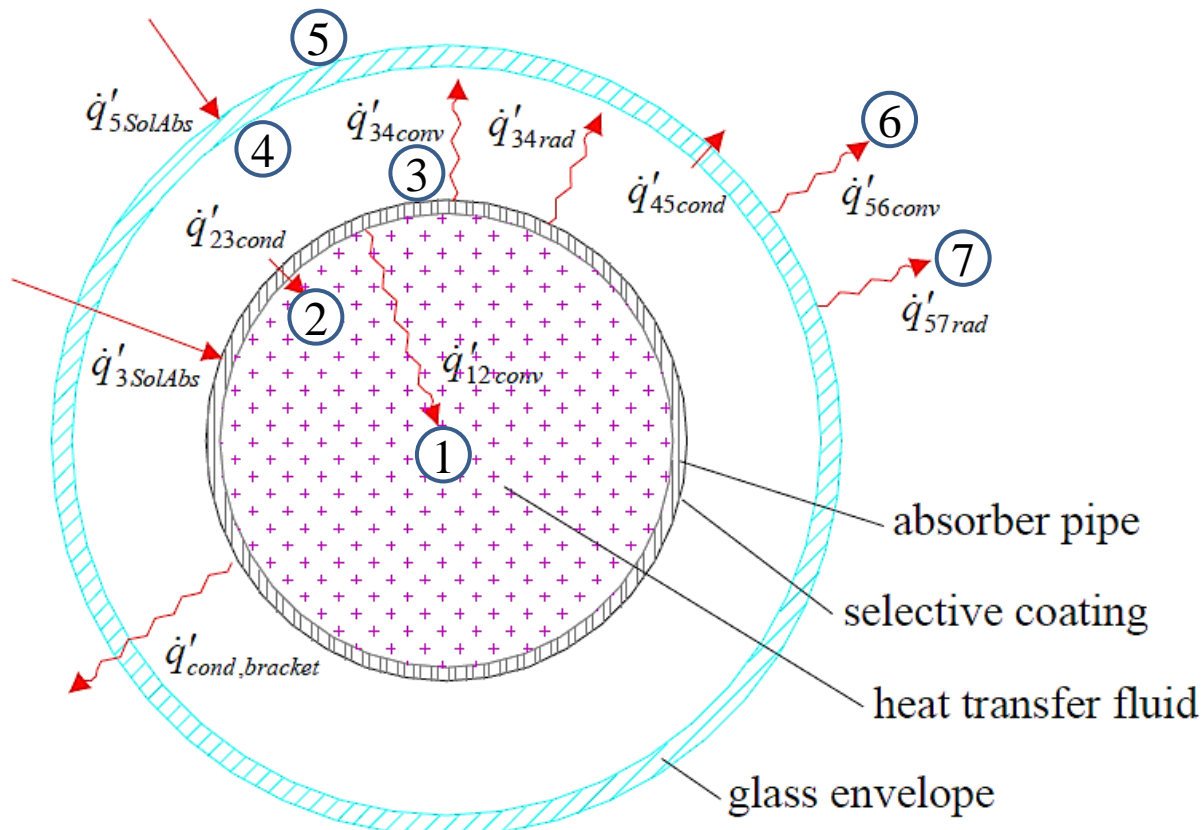
ENERGY BALANCE ON SURFACE (5)

$$\dot{q}'_{45\text{cond}} + \dot{q}'_{5\text{SolAbs}} = \dot{q}'_{56\text{conv}} + \dot{q}'_{57\text{rad}}$$



HEAT LOSS

$$\dot{q}'_{HeatLoss} = \dot{q}'_{56conv} + \dot{q}'_{57rad} + \dot{q}'_{cond,bracket}$$



SUMMARY OF EQUATIONS

$$\dot{q}'_{12conv} = \dot{q}'_{23cond}$$

$$\dot{q}'_{3SolAbs} = \dot{q}'_{34conv} + \dot{q}'_{34rad} + \dot{q}'_{23cond} + \dot{q}'_{cond,bracket}$$

$$\dot{q}'_{34conv} + \dot{q}'_{34rad} = \dot{q}'_{45cond}$$

$$\dot{q}'_{45cond} + \dot{q}'_{5SolAbs} = \dot{q}'_{56conv} + \dot{q}'_{57rad}$$

$$\dot{q}'_{HeatLoss} = \dot{q}'_{56conv} + \dot{q}'_{57rad} + \dot{q}'_{cond,bracket}$$

- Each term has to be determined from the principles of heat transfer.

Convection Heat Transfer Between the Fluid and Absorber

$$\dot{q}'_{12\text{conv}} = h_1 D_2 \pi (T_2 - T_1)$$

$$h_1 = Nu_{D2} \frac{k_1}{D_2}$$

h_1 = HTF convection heat transfer coefficient at T_1 (W/m²-K)

D_2 = inside diameter of the absorber pipe (m)

T_1 = mean (bulk) temperature of the HTF (°C)

T_2 = inside surface temperature of absorber pipe (°C)

Nu_{D2} = Nusselt number based on D_2

k_1 = thermal conductance of the HTF at T_1 (W/m-K)

Convection Heat Transfer Between the Fluid and Absorber

TURBULENT FLOW

$$Nu_{D_2} = \frac{f_2/8(Re_{D_2} - 1000)Pr_1}{1 + 12.7\sqrt{f_2/8}(Pr_1^{2/3} - 1)} \left(\frac{Pr_1}{Pr_2}\right)^{0.11} \quad f_2 = (1.82 \log_{10}(Re_{D_2}) - 1.64)^{-2}$$

- f_2 = friction factor for the inner surface of the absorber pipe
 Pr_1 = Prandtl number evaluated at the HTF temperature, T_1
 Pr_2 = Prandtl number evaluated at the absorber inner surface temperature, T_2

- This correlation was developed by Gnielinski.
- It is valid for $0.5 < Pr_1 < 2000$ and $2300 < Re_{D_2} < 5 \times 10^6$

LAMINAR FLOW

- Nusselt number is always equal to 4.36

Conduction Heat Transfer through the Absorber Wall

$$\dot{q}'_{23cond} = 2\pi k_{23} (T_2 - T_3) / \ln(D_3 / D_2)$$

k_{23} = absorber thermal conductance at the average absorber temperature $(T_2 + T_3)/2$ (W/m-K)

T_2 = absorber inside surface temperature (K)

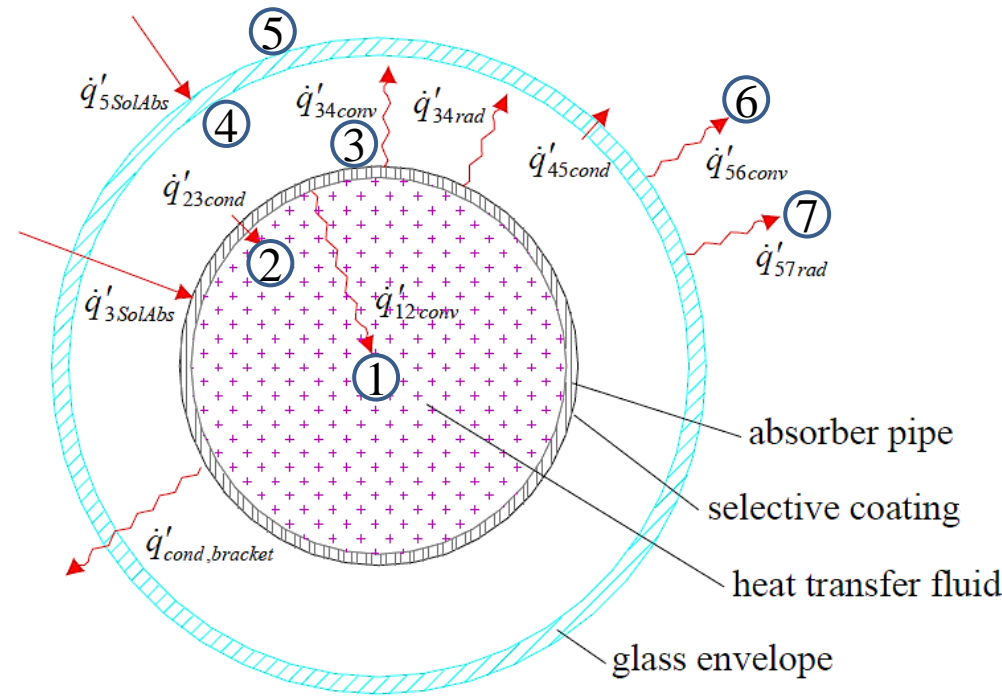
T_3 = absorber outside surface temperature (K)

D_2 = absorber inside diameter (m)

D_3 = absorber outside diameter (m)

Heat Transfer from the Absorber to the Glass Envelope

- There are two possible mechanisms for heat transfer:
 - **Convection**
 - **Radiation**
- If the receiver is under vacuum, convection losses are negligible.
- If the receiver is filled with a fluid, convection losses must be calculated.



Heat Transfer from the Absorber to the Glass Envelope

CONVECTION FROM AIR-FILLED ANNULUS

$$q'_{34conv} = \frac{2.425k_{34}(T_3 - T_4)(Pr Ra_{D3}/(0.861 + Pr_{34}))^{1/4}}{(1 + (D_3/D_4)^{3/5})^{5/4}}$$

$$Ra_{D3} = \frac{g\beta(T_3 - T_4)D_3^3}{\alpha\nu} \quad \beta = \frac{1}{T_{avg}}$$

- k_{34} = thermal conductance of annulus gas at T_{34} (W/m-K)
- T_3 = outer absorber surface temperature ($^{\circ}\text{C}$)
- T_4 = inner glass envelope surface temperature ($^{\circ}\text{C}$)
- D_3 = outer absorber diameter (m)
- D_4 = inner glass envelope diameter (m)
- Pr_{34} = Prandtl number
- Ra_{D3} = Rayleigh number evaluated at D_3
- β = volumetric thermal expansion coefficient (1/K)
- T_{34} = average temperature, $(T_3 + T_4)/2$ ($^{\circ}\text{C}$)

**All physical properties
are evaluated at the
average temperature
 $(T_3 + T_4)/2$**

Heat Transfer from the Absorber to the Glass Envelope

RADIATION

$$\dot{q}'_{34rad} = \frac{\sigma \pi D_3 (T_3^4 - T_4^4)}{(1/\varepsilon_3 + (1 - \varepsilon_4)D_3/(\varepsilon_4 D_4))}$$

- σ = Stefan-Boltzmann constant ($\text{W/m}^2\cdot\text{K}^4$)
- D_3 = outer absorber diameter (m)
- D_4 = inner glass envelope diameter (m)
- T_3 = outer absorber surface temperature (K)
- T_4 = inner glass envelope surface temperature (K)
- ε_3 = Absorber selective coating emissivity
- ε_4 = glass envelope emissivity

Conduction Heat Transfer through the Glass Envelope

$$\dot{q}'_{45cond} = 2\pi k_{45}(T_4 - T_5)/\ln(D_5/D_4)$$

k_{45} = glass envelope thermal conductance at the average glass temperature $(T_4 + T_5)/2$ (W/m-K)

T_4 = glass inside surface temperature (K)

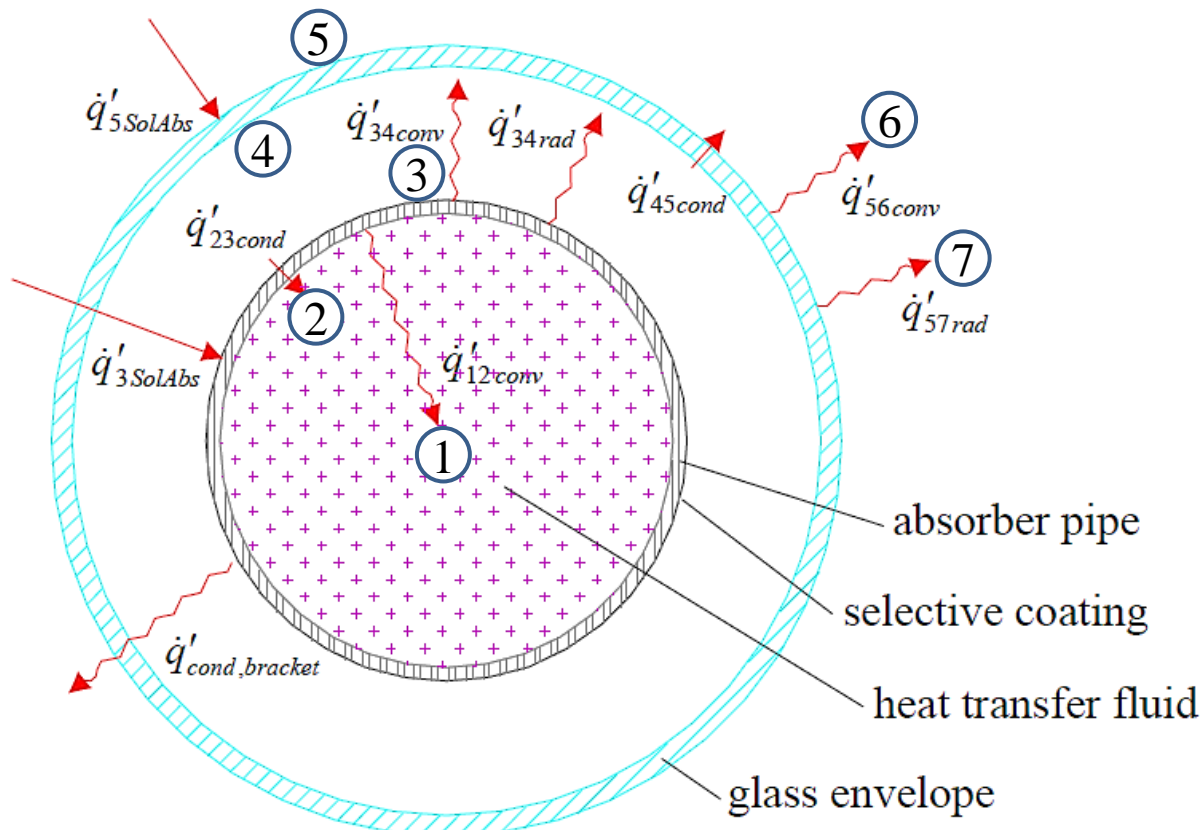
T_5 = glass outside surface temperature (K)

D_4 = glass inside diameter (m)

D_5 = glass outside diameter (m)

Heat Transfer from the Glass Envelope to the Atmosphere

- There are two possible mechanisms for heat transfer:
 - Convection
 - Radiation



Heat Transfer from the Glass Envelope to the Atmosphere CONVECTION

$$\dot{q}'_{56conv} = h_{56} \pi D_5 (T_5 - T_6)$$

$$h_{56} = \frac{k_{56}}{D_5} Nu_{D5}$$

T_5 = glass envelope outer surface temperature ($^{\circ}\text{C}$)

T_6 = ambient temperature ($^{\circ}\text{C}$)

h_{56} = convection heat transfer coefficient for air at $(T_5 - T_6)/2$ ($\text{W}/\text{m}^2\text{-K}$)

k_{56} = thermal conductance of air at $(T_5 - T_6)/2$ ($\text{W}/\text{m-K}$)

D_5 = glass envelope outer diameter (m)

Nu_{D5} = average Nusselt number based on the glass envelope outer diameter

Heat Transfer from the Glass Envelope to the Atmosphere CONVECTION

$$\overline{Nu}_{D5} = C Re_{D5}^m Pr_6^n \left(\frac{Pr_6}{Pr_5} \right)^{1/4}$$

$n = 0.37$, for $Pr \leq 10$
 $n = 0.36$, for $Pr > 10$

Re_D	C	m
1-40	0.75	0.4
40-1000	0.51	0.5
1000-200000	0.26	0.6
200000-1000000	0.076	0.7

- Valid for $0.7 < Pr_6 < 500$.
- All fluid properties are evaluated at atmospheric temperature, T_6 , except Pr_5 .

Heat Transfer from the Glass Envelope to the Atmosphere RADIATION

$$\dot{q}'_{57rad} = \sigma D_5 \pi \varepsilon_5 (T_5^4 - T_7^4)$$

- σ = Stefan-Boltzmann constant (5.670E-8) (W/m²-K⁴)
 D_5 = glass envelope outer diameter (m)
 ε_5 = emissivity of the glass envelope outer surface
 T_5 = glass envelope outer surface temperature (K)
 T_7 = effective sky temperature (K)

- Sky temperature depends on many variables.
- A simplified model is to assume that it is less than ambient temperature by 8°C.

OPTICAL EFFICIENCY OF THE PTC

- Optical efficiency is defined as the ratio of irradiation incident on the outside of the receiver to the irradiation incident on the collector (mirrors).
- There are many sources of “error” that cause the optical efficiency to be less than 1.

SOURCES OF LOSSES IN OPTICAL EFFICIENCY

$\varepsilon'_1 = \text{HCE Shadowing (bellows, shielding, supports)}$	0.974
$\varepsilon'_2 = \text{Tracking Error}$	0.994
$\varepsilon'_3 = \text{Geometry Error (mirror alignment)}$	0.98
$\rho_{cl} = \text{Clean Mirror Reflectance}$	0.935
$\varepsilon'_4 = \text{Dirt on Mirrors}^*$	reflectivity/ ρ_{cl}
$\varepsilon'_5 = \text{Dirt on HCE}$	$(1 + \varepsilon'_4)/2$
$\varepsilon'_6 = \text{Unaccounted}$	0.96
* reflectivity is a user input (typically between 0.88 and 0.93)	

- Values in the table are typical values.
- They can vary from one PTC model to another and from one time to another.
- $[\varepsilon'_1 \ \varepsilon'_2 \ \varepsilon'_3 \ \varepsilon'_4 \ \varepsilon'_5 \ \varepsilon'_6 \ \rho_{cl}]$ is the optical efficiency.
- The optical efficiency is denoted by η_{opt}

Solar Irradiation Absorption in the Absorber

$$\dot{q}'_{3SolAbs} = G_{ND} W_a [\varepsilon'_1 \varepsilon'_2 \varepsilon'_3 \varepsilon'_4 \varepsilon'_5 \varepsilon'_6 \rho_{cl}] K \tau_{env} \alpha_{abs}$$

OR

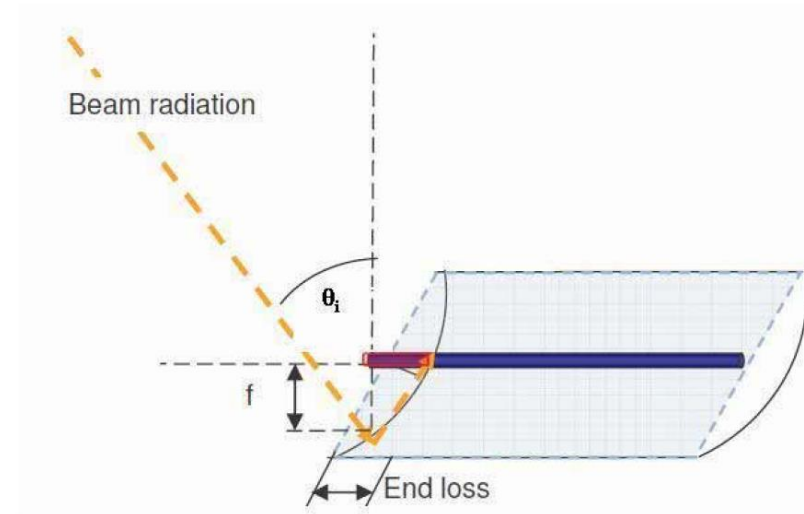
$$\dot{q}'_{3SolAbs} = G_{ND} W_a \eta_{opt} K \tau_{env} \alpha_{abs}$$

Where,

- W_a is the aperture width
- K is the incidence angle modifier (explained in next slide)
- τ_{env} is the glass tube's transmittance.
- α_{abs} is the absorber tube's absorptance.

Incidence Angle Modifier (K)

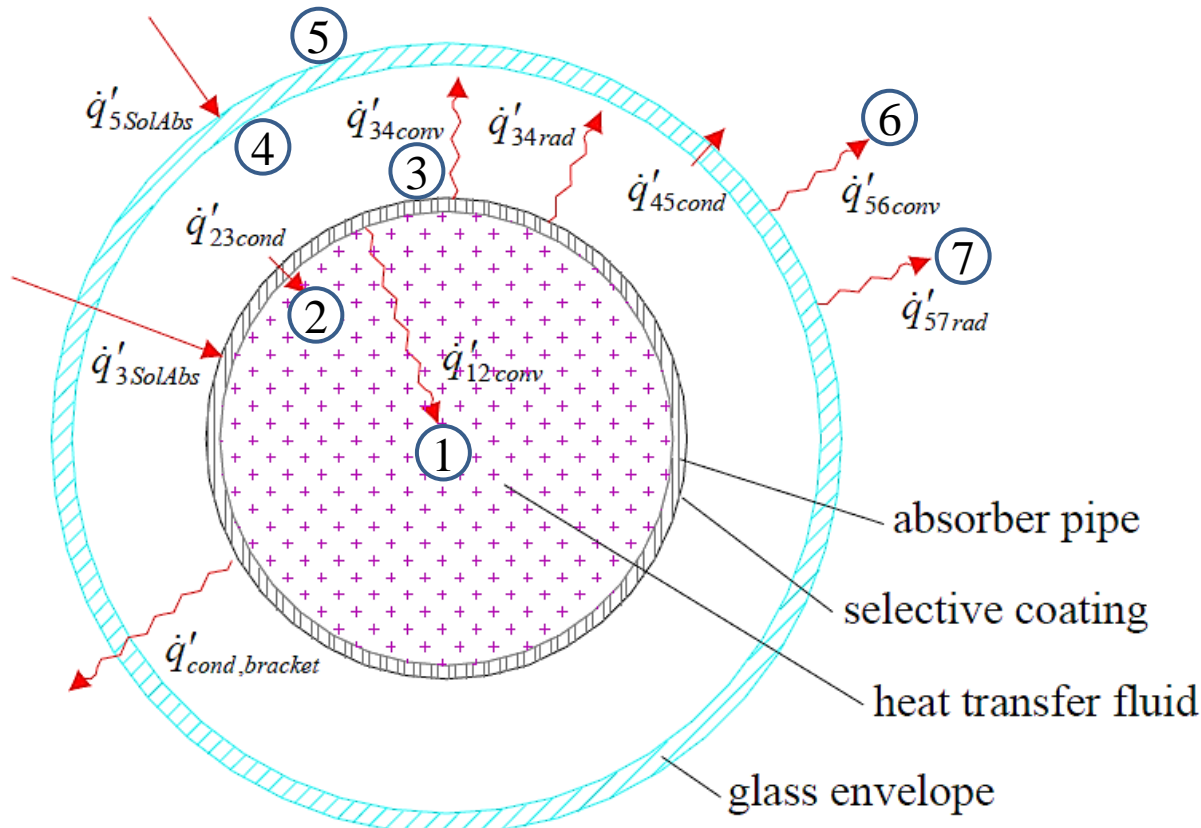
- K accounts for cases when the incidence angle is not 0° .
- It also accounts for the “end loss”.
- K depends on the exact geometry of the PTC.
- The following is an example of the formula for calculating K :



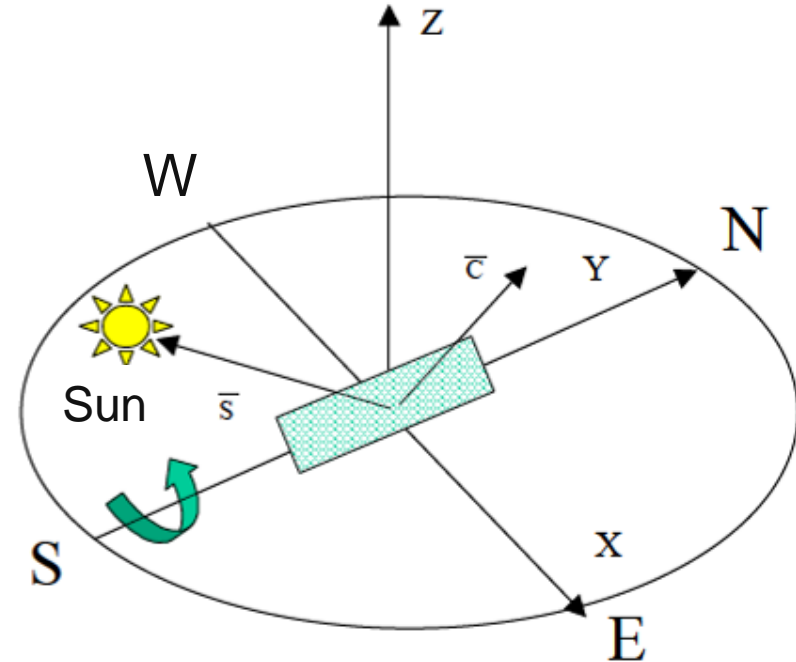
$$K = \cos(\theta) + 0.000884\theta - 0.00005369\theta^2$$

Solar Irradiation Absorption in the Glass Envelope

$$\dot{q}'_{5SolAbs} = G_{ND} W_a [\varepsilon'_1 \varepsilon'_2 \varepsilon'_3 \varepsilon'_4 \varepsilon'_5 \varepsilon'_6 \rho_{cl}] K \alpha_{env}$$



- The tilt angle (α_{PTC}) varies with the time of the day since PTCs track the sun continuously.
- For PTC systems with a north-south tracking axis, the tilt angle can be given by:



$$\sin \alpha_{PTC} = \cos \beta \cos |\phi - 90| \text{ if } h \leq 0$$

$$\sin \alpha_{PTC} = \cos \beta \cos |\phi - 270| \text{ if } h > 0$$