Vectors

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1. Revision
2. Vector Valud Functins
3. Graph
4. Limits and Derivaties

Equation of a Line:

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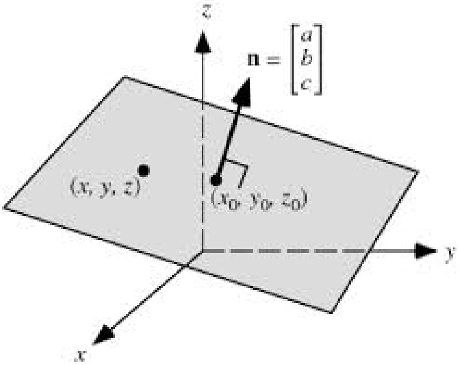

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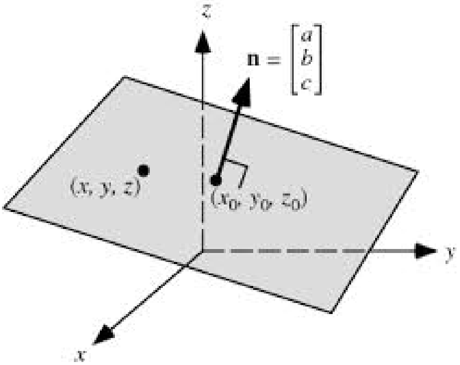

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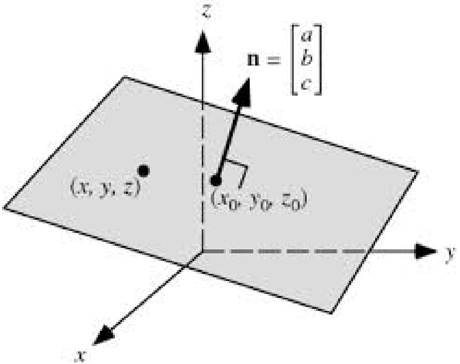

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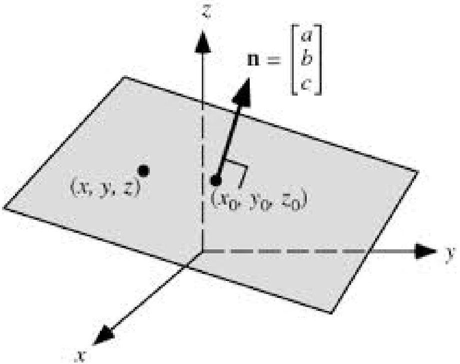

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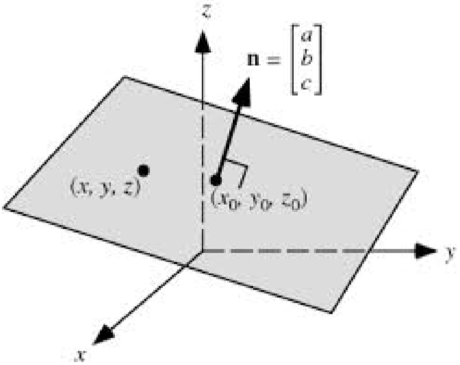

Let $P_0(x_0, y_0, z_0)$ be a point in the plane and $\mathbf{n} = \langle a, b, c\rangle$. Let $P(x, y, z)$ be any point in the plane, then $\mathbf{n}.\overrightarrow{P_0P} = 0$. The equation of the plane is:

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*Vector functions can also be referred to in a different notation:
$\mathbf{r}(t) = \langle f(t), g(t)\rangle$ or
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These are the parametric equation of a line passing through the point $(2, 3, -5)$ and parallel to the vector $\langle 1, -1, -4 \rangle$.

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which is equatin of ellipse in yz-plane and $x = 5$.

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Q: Represent the curve by a vector valued function:
[a] $y = x^2 + 4$. [b] $x^2 + y^2 = 16$.

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Example(3): Find parametric equations for the tangent line to C, which given parametrically by

$$x = 2t^3 - 1, \quad y = -5t^2 + 3, \quad z = 8t + 2 \quad \text{at } P(1, -2, 10).$$

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