Vectors

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3. Equality: $a = b$ if and only if $a_1 = b_1$ and $a_2 = b_2$.

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3. $\|a\|$, $\|b\|$, $\|3a - 4b\|$ and $\|4a\|$.
4. Find the unit vector that has same direction as a.
5. Find the vector that has the same direction as a and thrice the magnitude of of a.
6. Find the vector that has the opposite direction of a and one-third the magnitude of of a.

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Example: Use vectors to determine whether the points lie on a straight line, the points are $(1,-1,5),(0,-1,6)$ and $(3,-1,3)$.