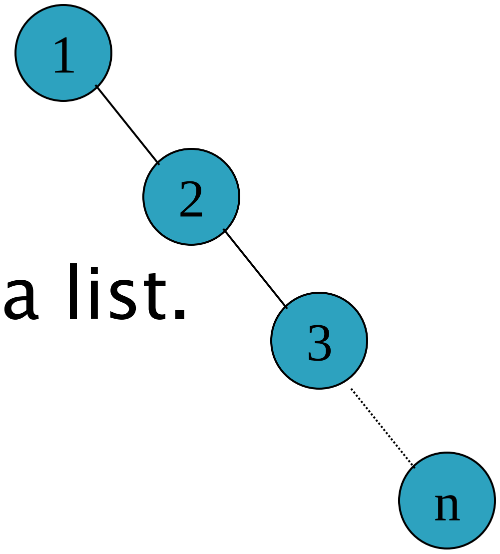
AVL Trees

AVL Trees

- Consider a situation when data elements are inserted in a BST in sorted order: 1, 2, 3, …
- BST becomes a degenerate tree.
- Search operation FindKey takes O(n), which is as inefficient as in a list.

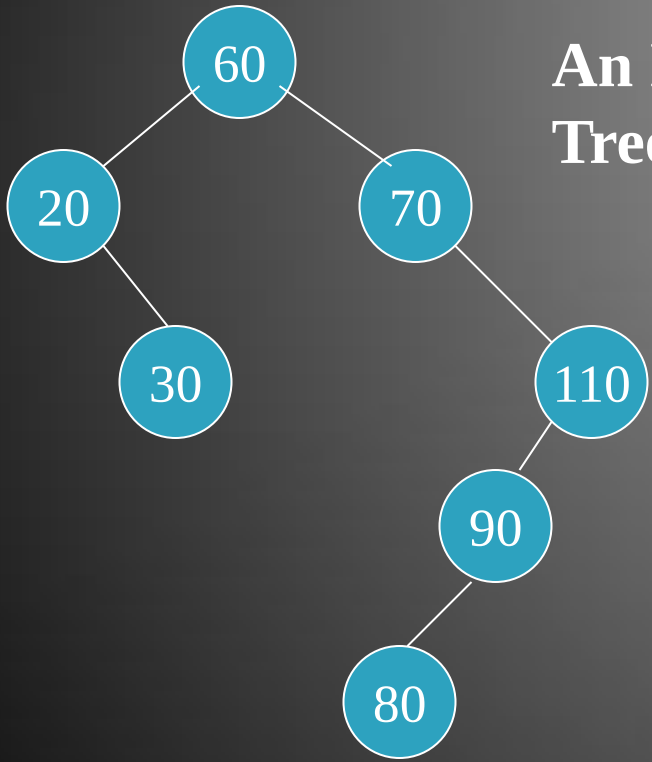


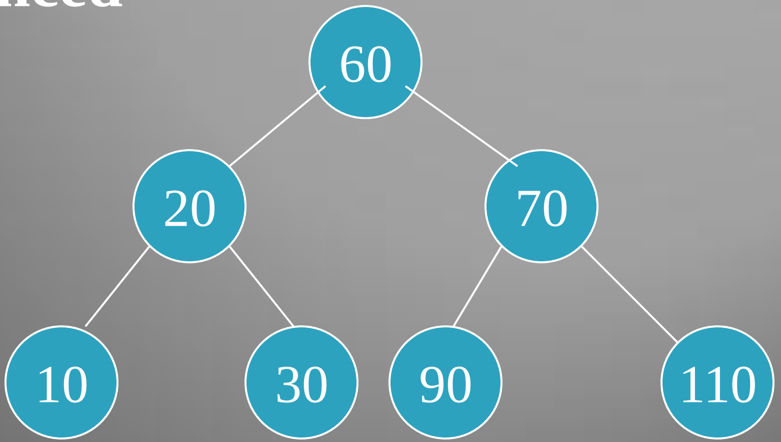
AVL Trees

- It is possible that after a number of insert and delete operations a binary tree may become imbalanced and increase in height.
- Can we insert and delete elements from BST so that its height is guaranteed to be O(log n)? → Yes, AVL Tree ensures this.
- Named after its two inventors: Adelson-Velski and Landis.

Imbalanced Tree

An Imbalanced Tree



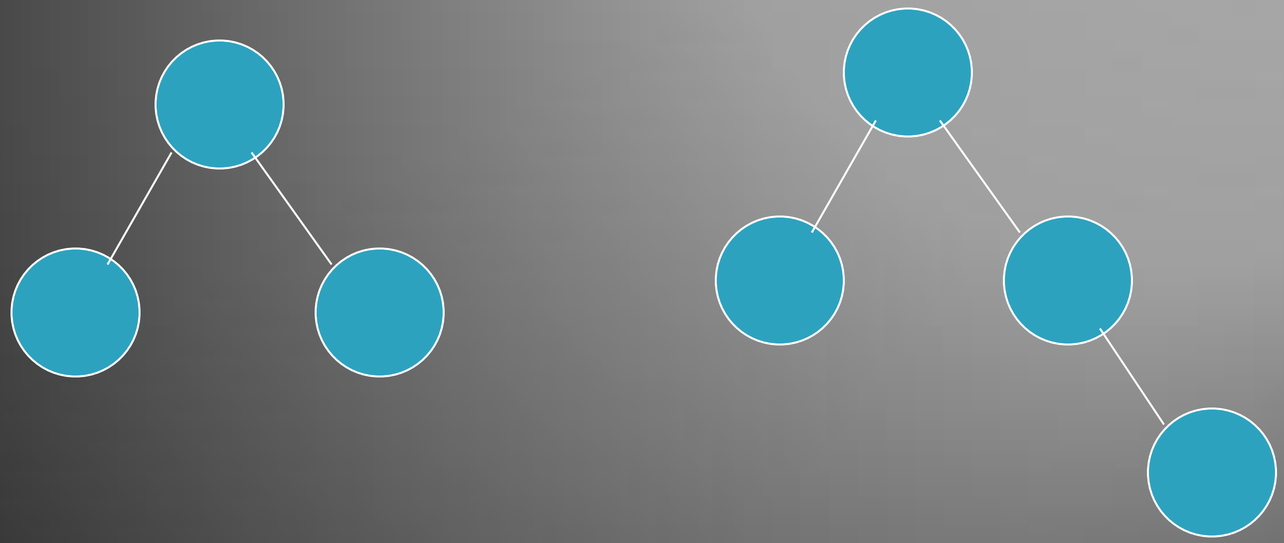


A Balanced Tree

AVL Tree: Definition

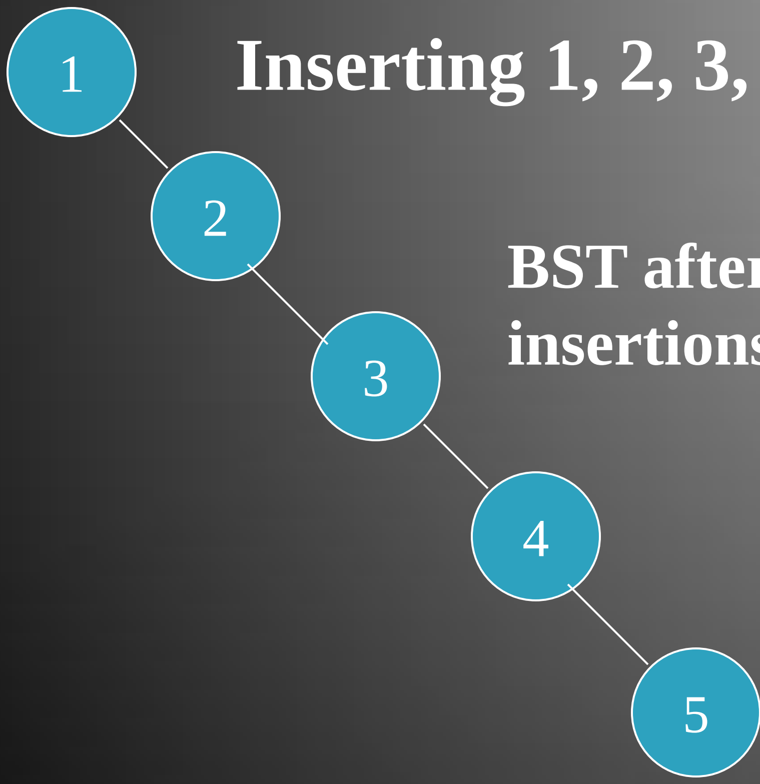
- Height-balanced tree: A binary tree is a height-balanced-p-tree if for each node in the tree, the difference in height of its two subtrees is at the most p.
- AVL tree is a BST that is height-balanced-1-tree.

AVL Trees: Examples

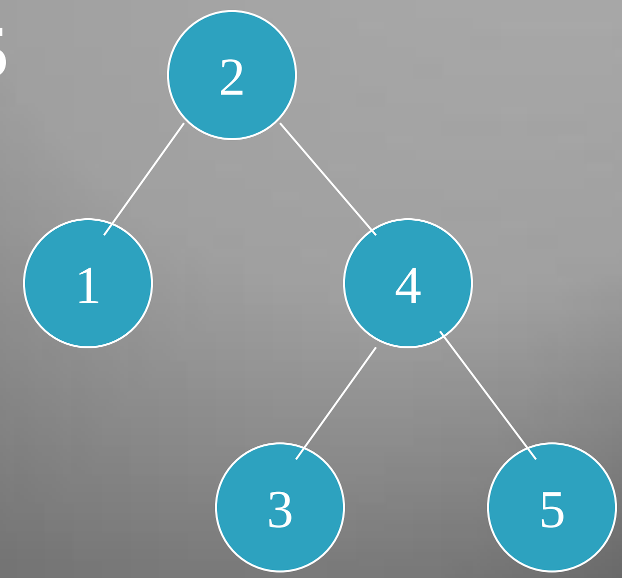


AVL Trees

Inserting 1, 2, 3, 4 and 5



BST after insertions



AVL Tree after insertions

ADT AVL Tree

Elements: The elements are nodes, each node contains the following data type: Type

Structure: Same as for the BST; in addition the height difference of the two subtrees of any node is at the most one.

Domain: the number of nodes in a AVL is bounded; type AVLTree

ADT AVL Tree

<u>Operations:</u>

1. **Method** FindKey (int tkey, boolean found).
2. **Method** Insert (int k, Type e, boolean inserted).
3. **Method** Remove_Key (int tkey, boolean deleted)
4. **Method** Update(Type e)

ADT AVL Tree

5. **Method** Traverse (Order ord)
6. **Method** DeleteSub ( )
7. **Method** Retrieve (Type e)
8. **Method** Empty (boolean empty).
9. **Method** Full (boolean full)

ADT AVL Tree

Representation:

```
public class <Type> AVLNode  // AVL Tree Node {
  private:
      int key
      Type data;
      Balance bal; //Balance is enum +1, 0, -1
      AVLNode<Type> *left, *right;
  public AVLNode(int, Type);       // constructors
};
```

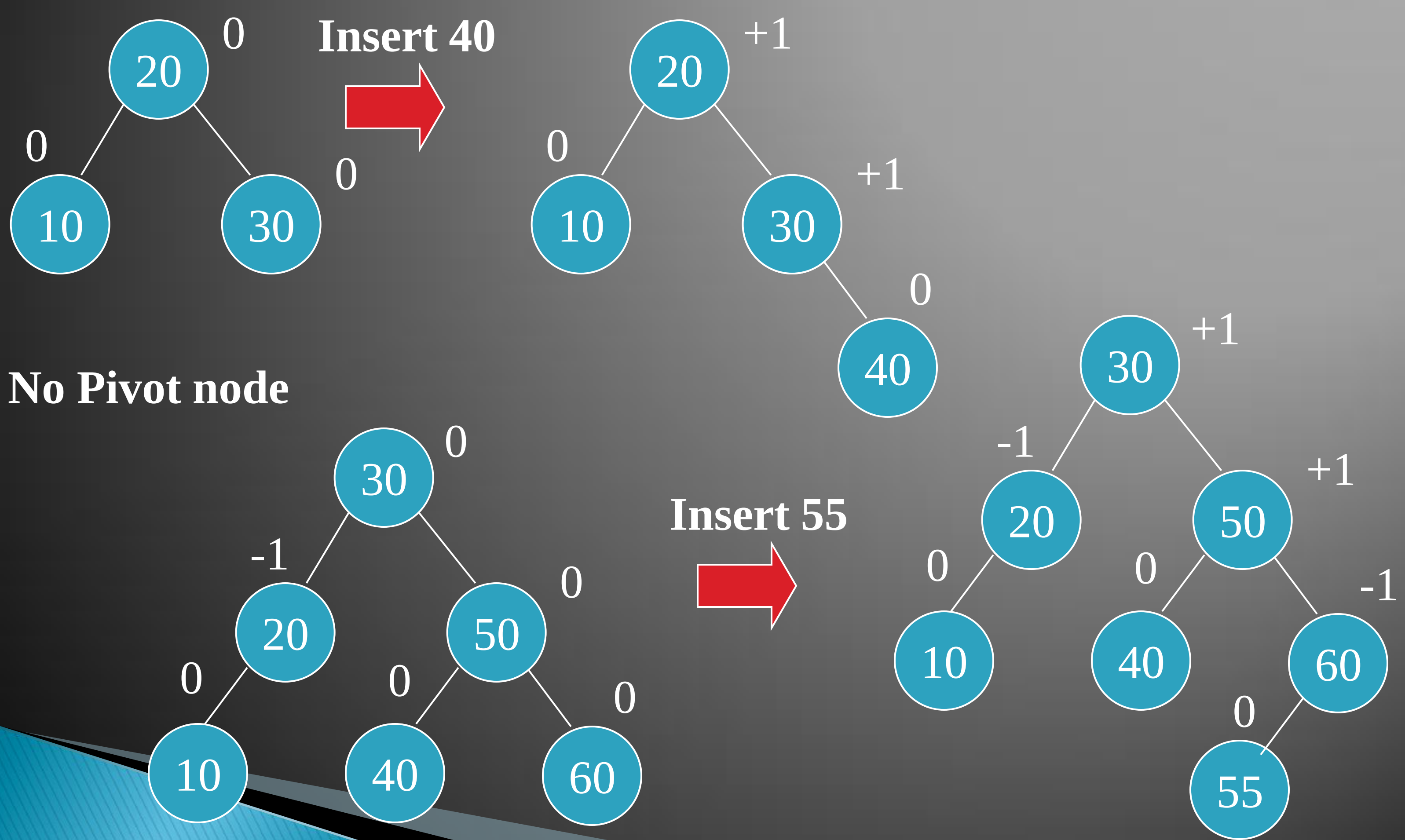
AVL Tree: Insert

- Step 1: A node is first inserted into the tree as in a BST.
- There is always a unique path from the root to the new node called the search path.
- Step 2: Nodes in the search path are examined to see if there is a pivot node. Three cases arise.
- A pivot node is a node closest to the new node on the search path, whose balance is either –1 or +1.

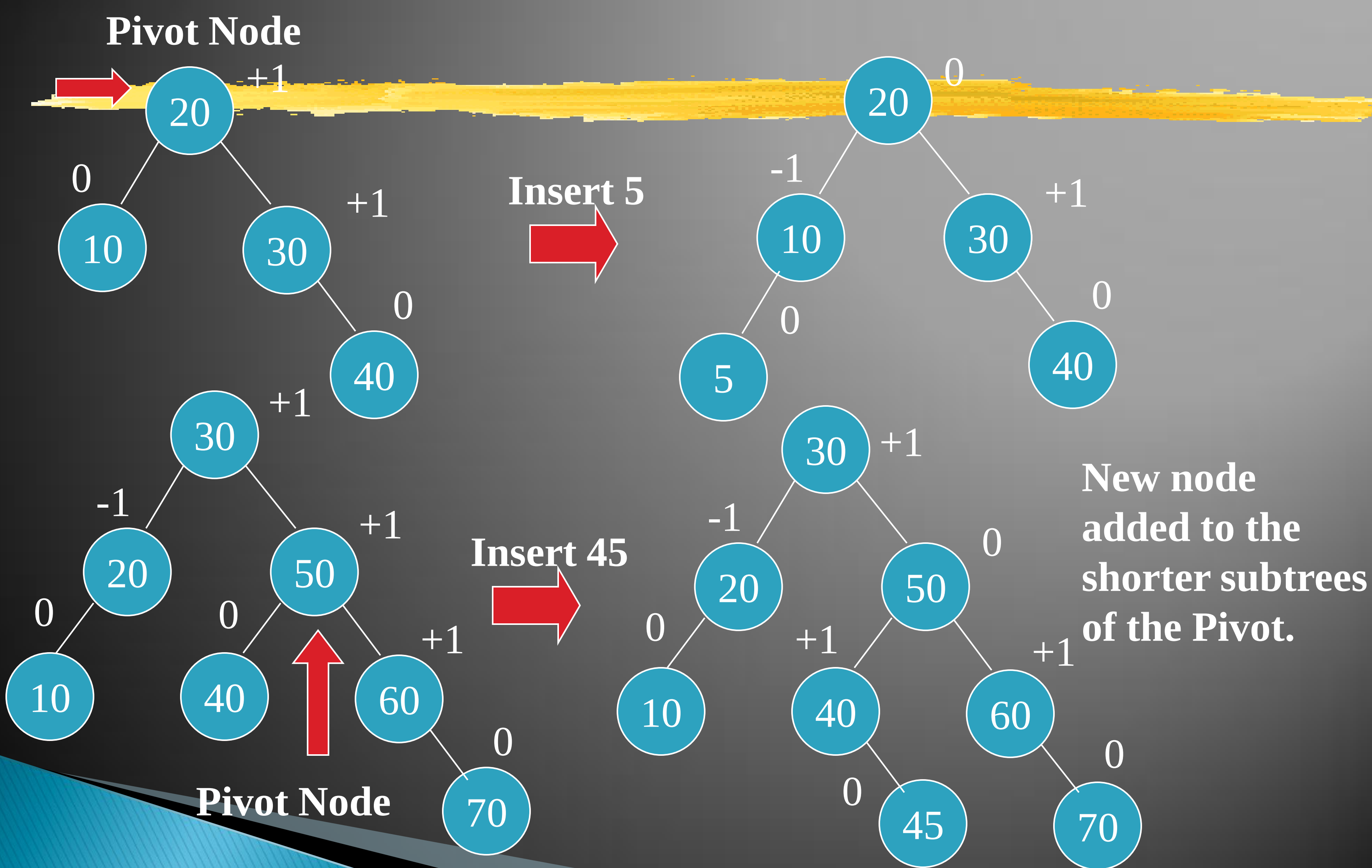
AVL Tree: Insert

- Case 1: There is no pivot node. No adjustment required.
- Case 2: The pivot node exists and the subtree to which the new node is added has smaller height. No adjustment required.
- Case 3: The pivot node exists and the subtree to which the new node is added has the larger height. Adjustment required.

Insert: Case 1



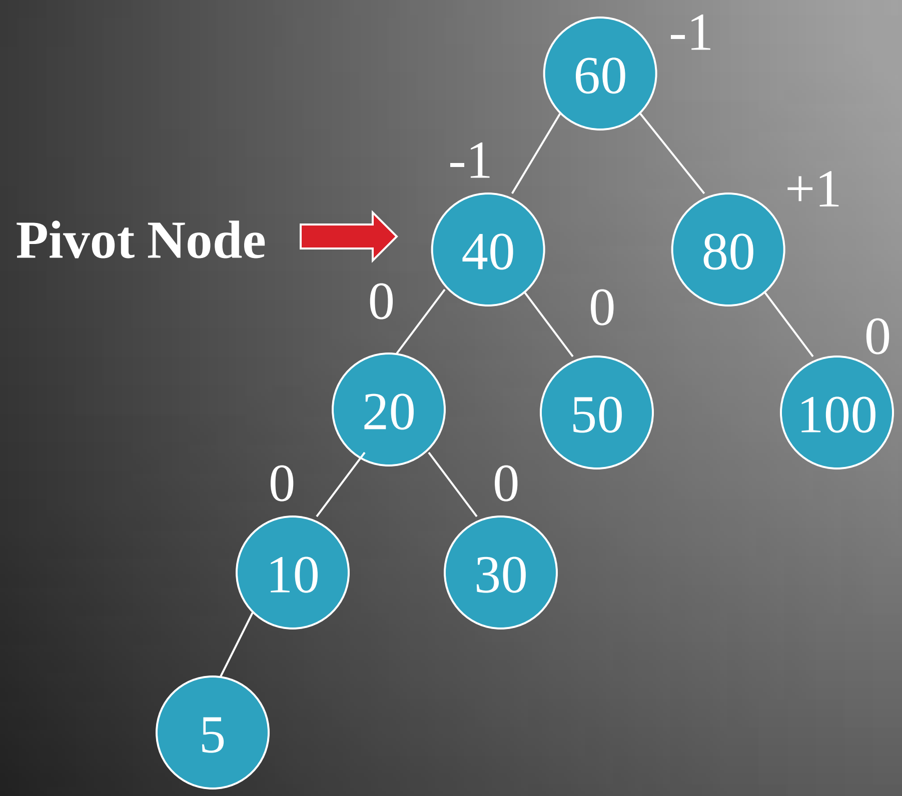
Insert: Case 2



New node added to the shorter subtrees of the Pivot.

Insert: Case 3

Insert 5

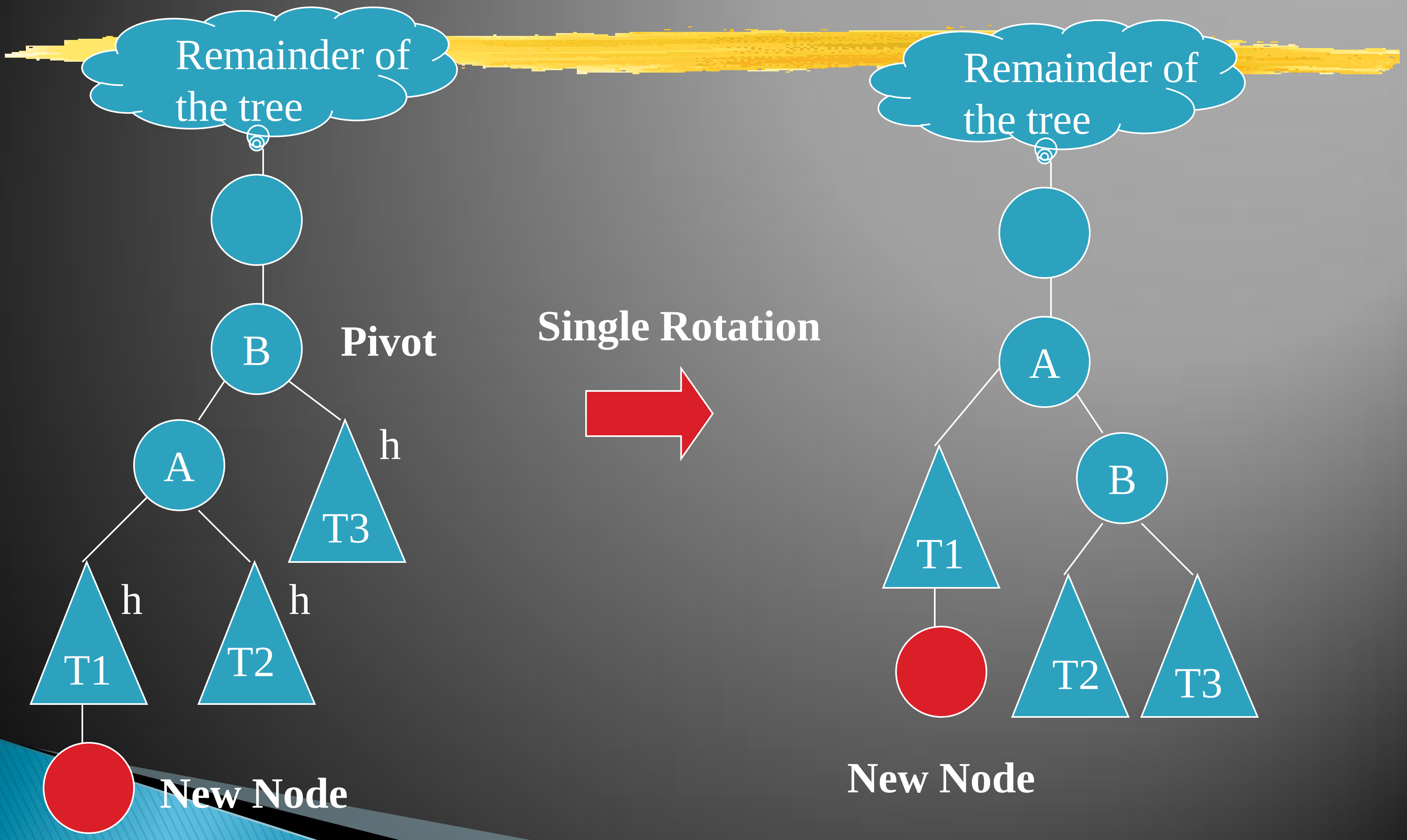


AVL Tree is no more an AVL Tree after insertion.

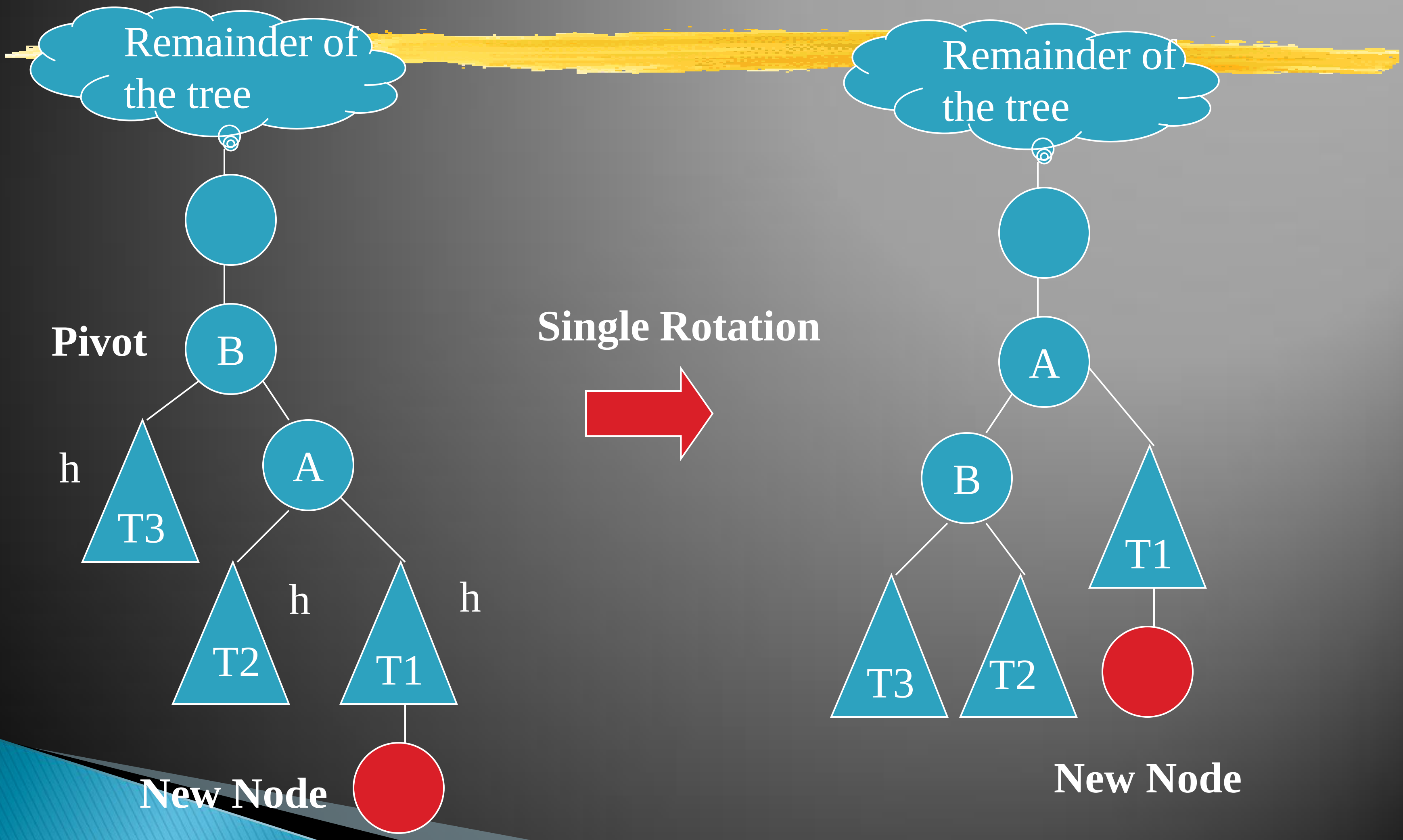
Insert: Case 3

- When after an insertion or a deletion an AVL tree becomes imbalanced, adjustments must be made to the tree to change it back into an AVL tree.
- These adjustments are called <u>rotations</u>.
- Rotations are either <u>single</u> or <u>double</u> rotations.
- For Case 3 there are 4 sub-cases (2+2)

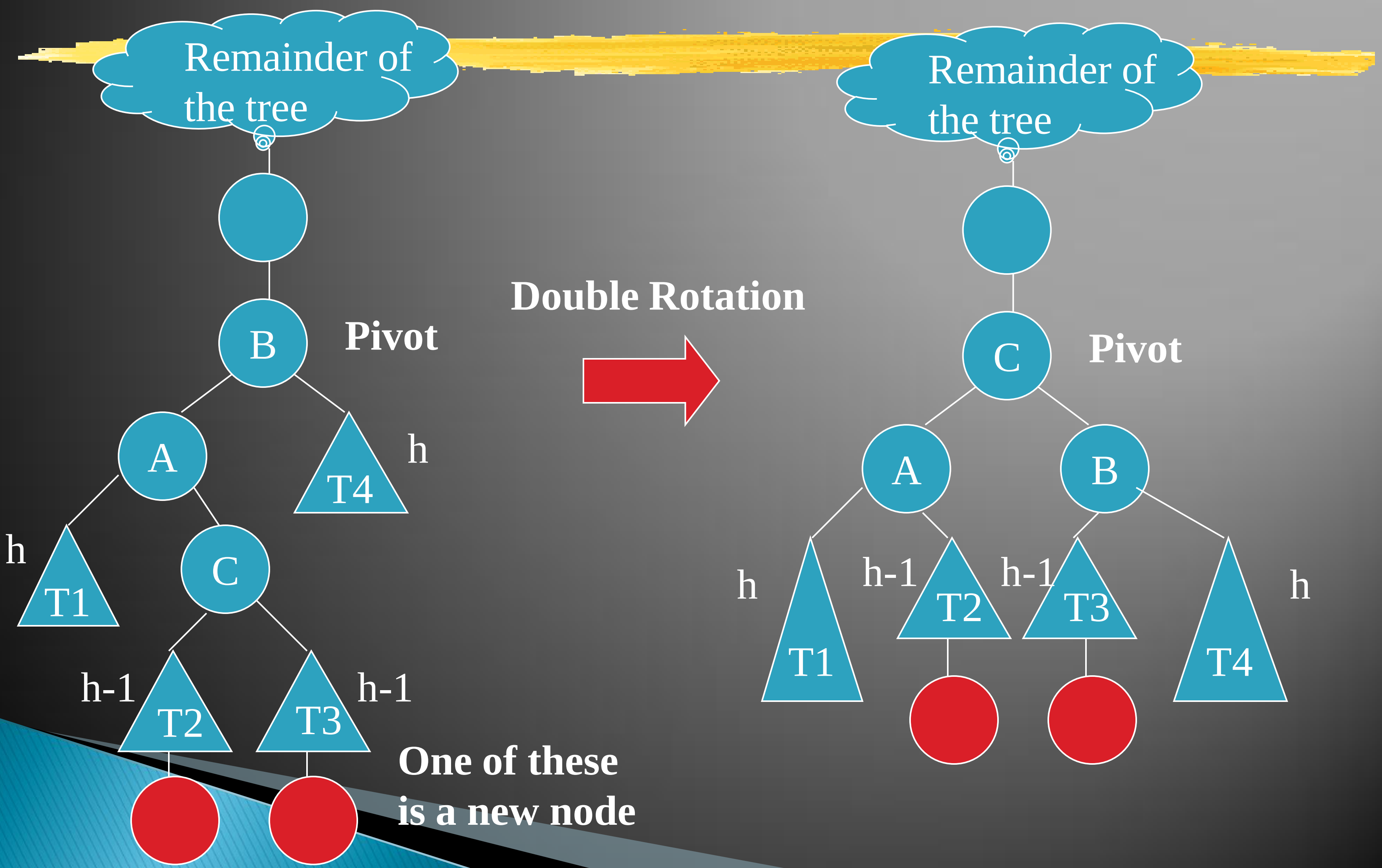
Insert: Case3 (Sub-Case 1)



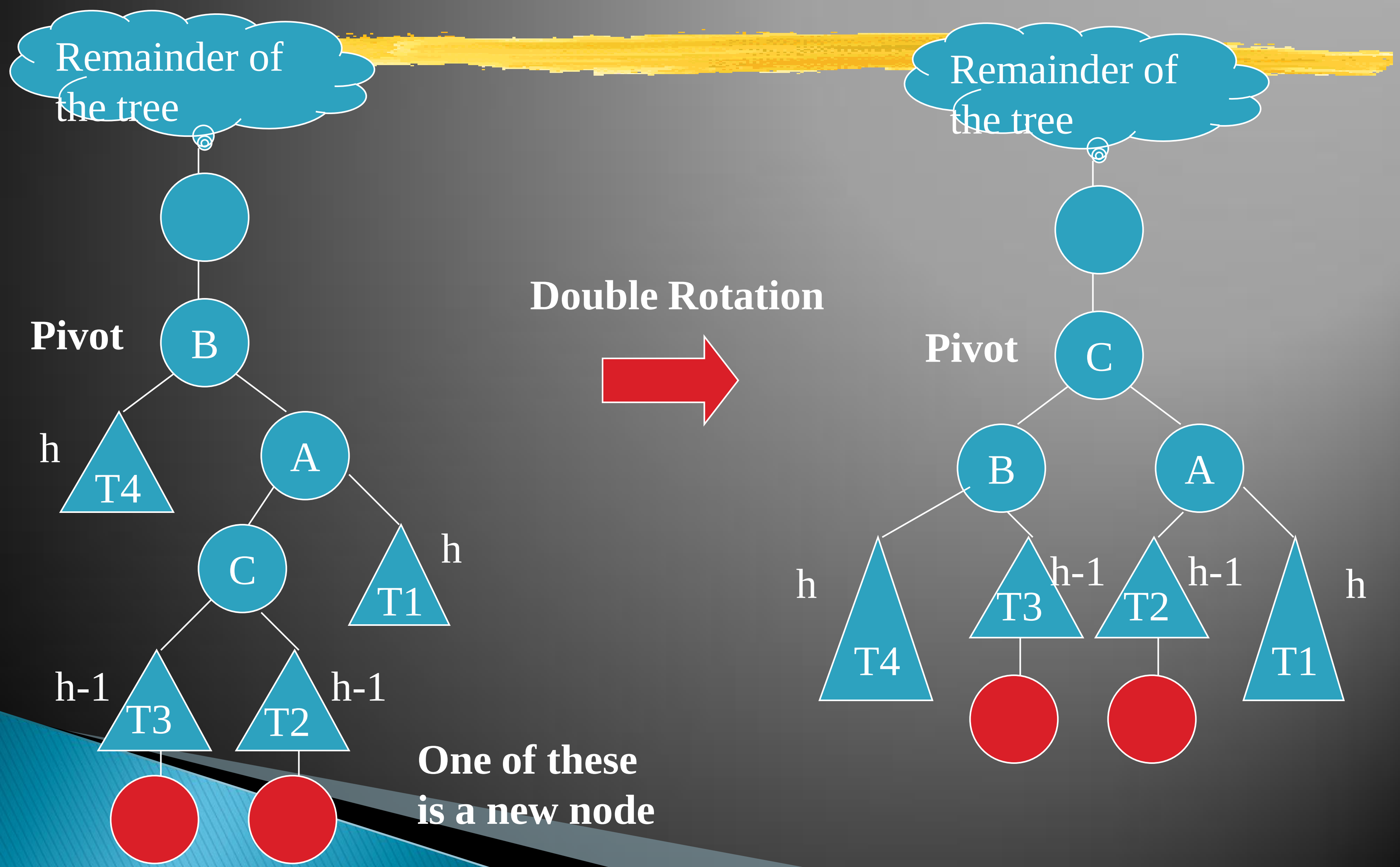
Insert: Case 3 (Sub-Case 2)



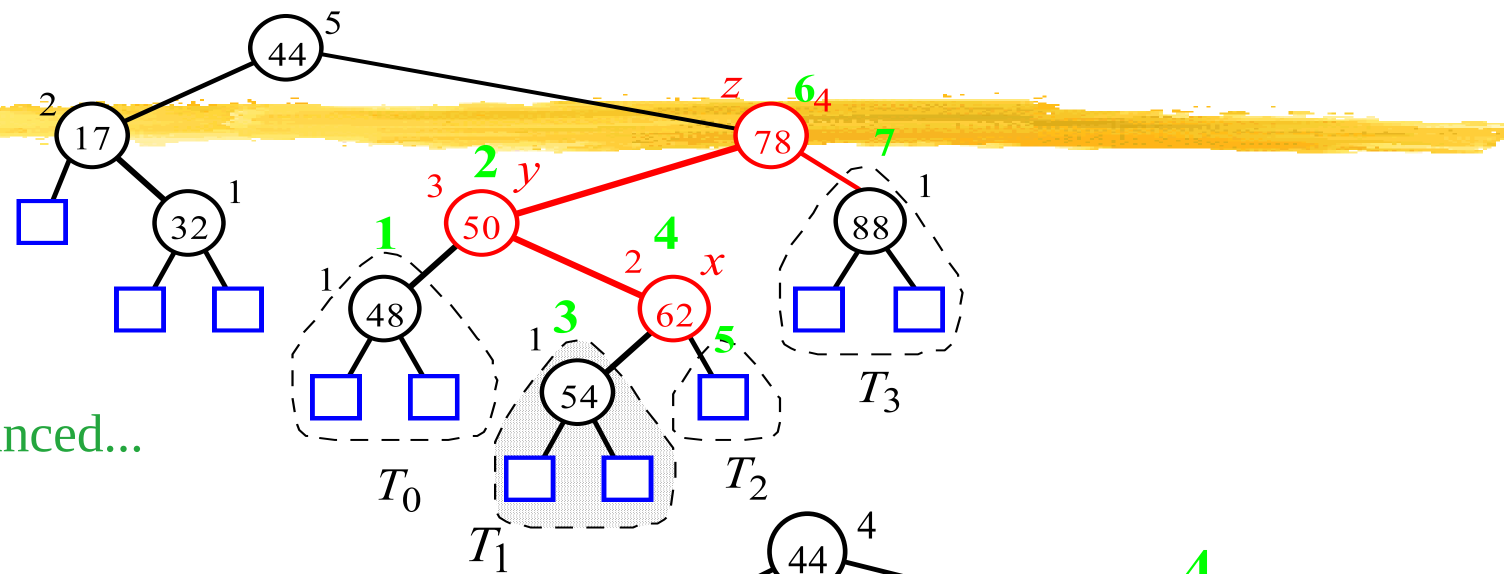
Insert: Case 3 (Sub-Case 3)



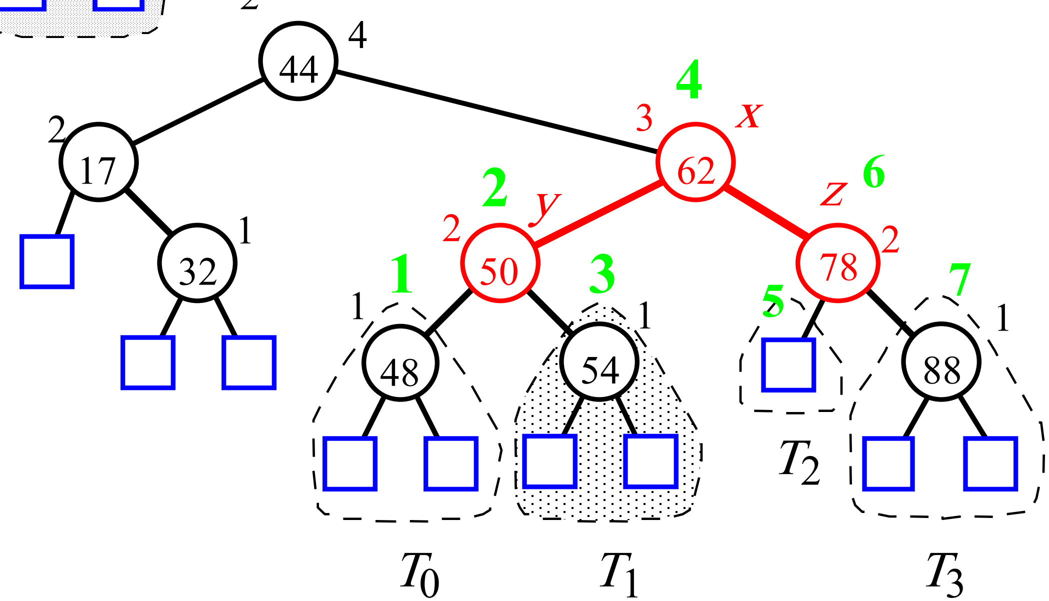
Insert: Case 3 (Sub-Case 4)



Insertion Example



unbalanced...



...balanced

AVL Tree: Delete

- Step 1: Delete the node as in BSTs. Leaf or node with one child, will always be deleted.
- Step 2: For each node on the path from the root to deleted node, check if the node has become imbalanced; if yes perform rotation operations otherwise update balance factors and exit. ➔ Three cases can arise for each node p, in the path.

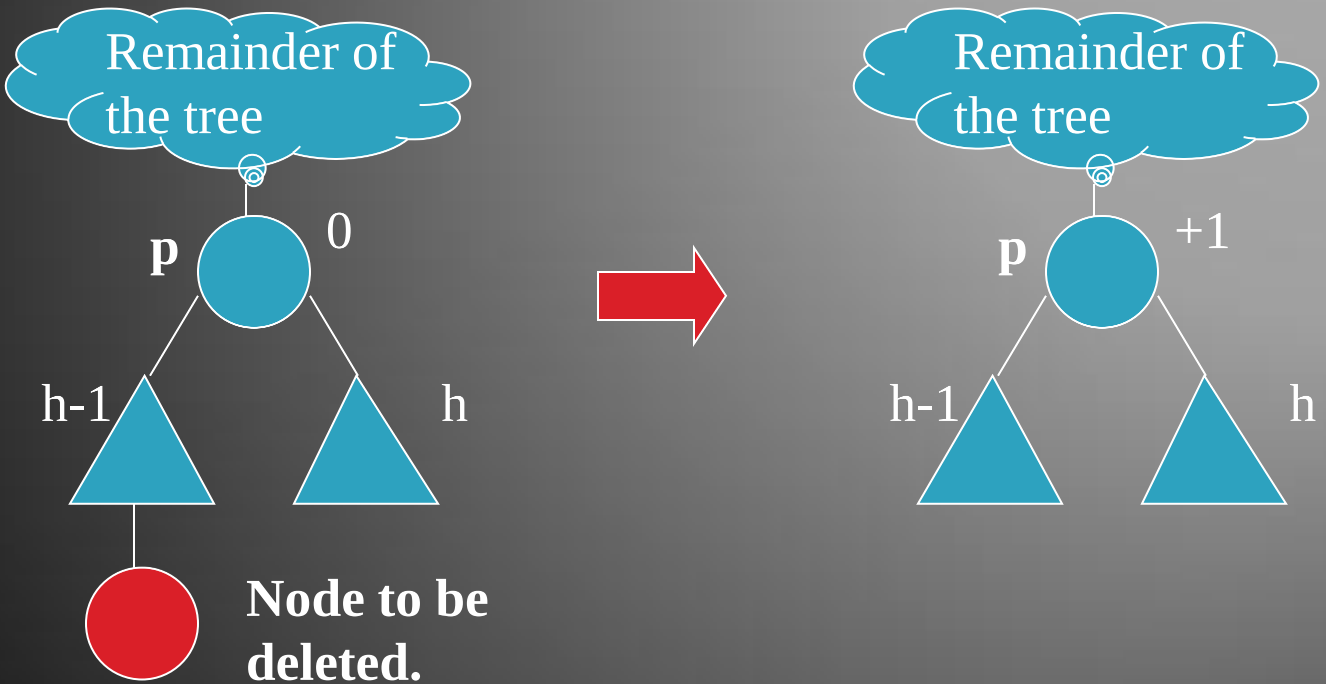
AVL Tree: Delete

- Step 2 (contd.): Case 1: Node p has balance factor 0. No rotation needed.

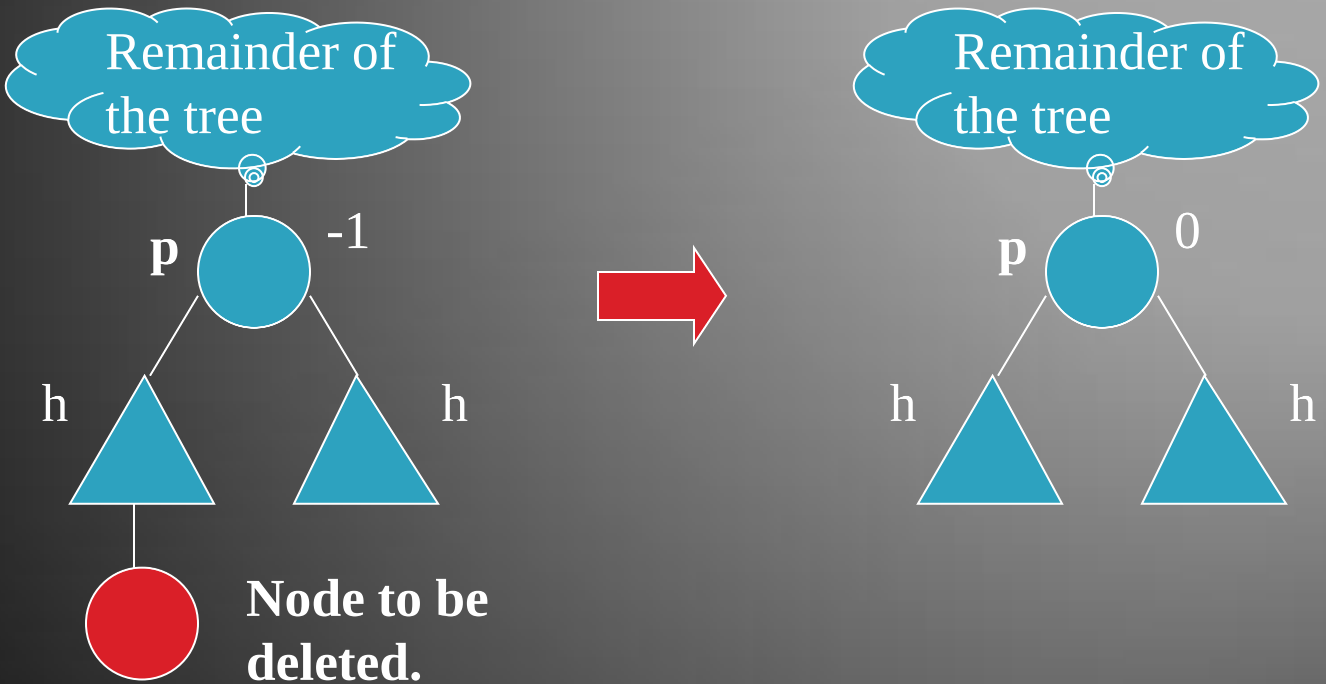
  Case 2: Node p has balance factor of +1 or –1 and a node was deleted from the taller sub-trees. No rotation needed.

  Case 3: Node p has balance factor of +1 or –1 and a node was deleted from the shorter sub-trees. Rotation needed. Eight sub-cases. (4 + 4)

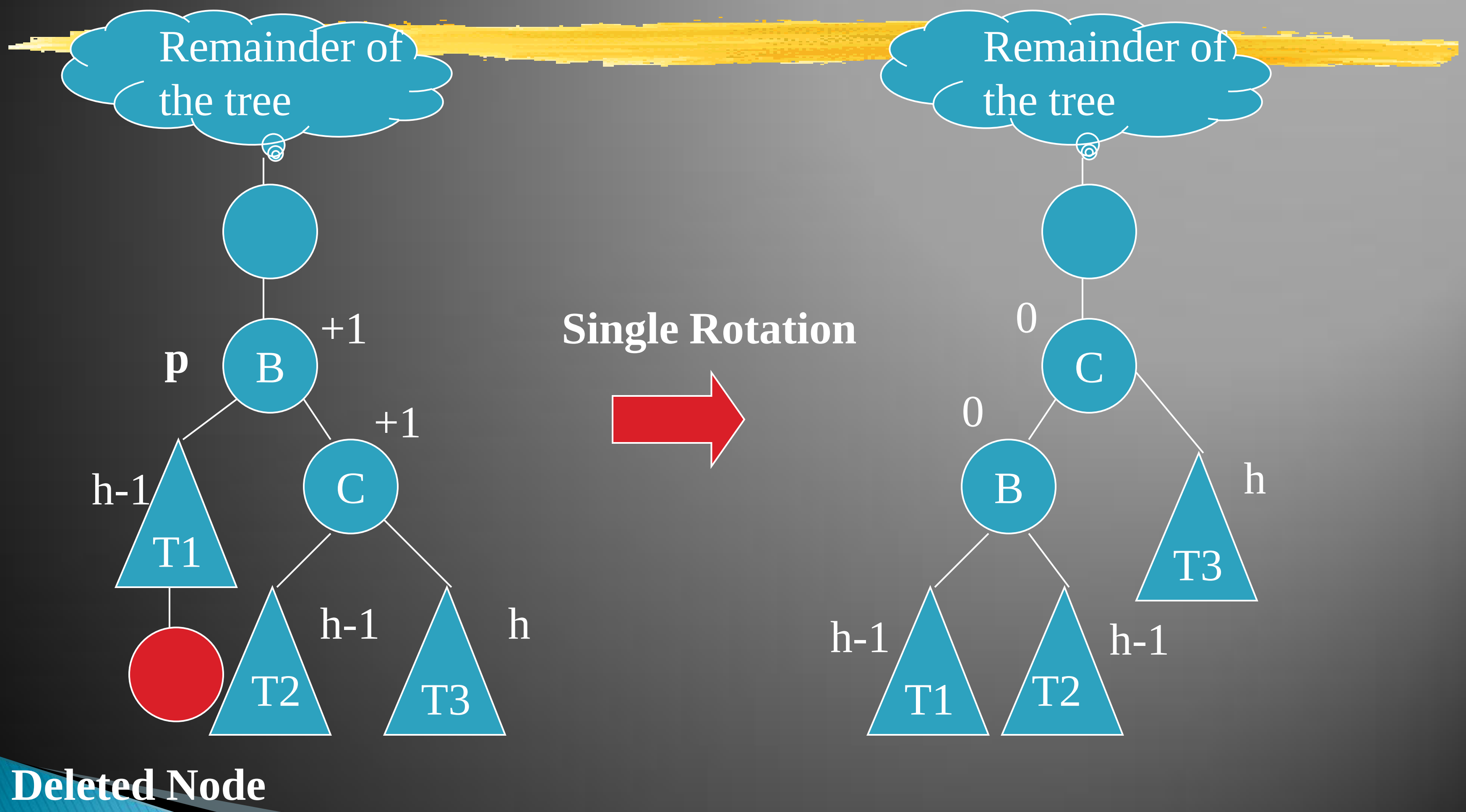
Delete: Case 1



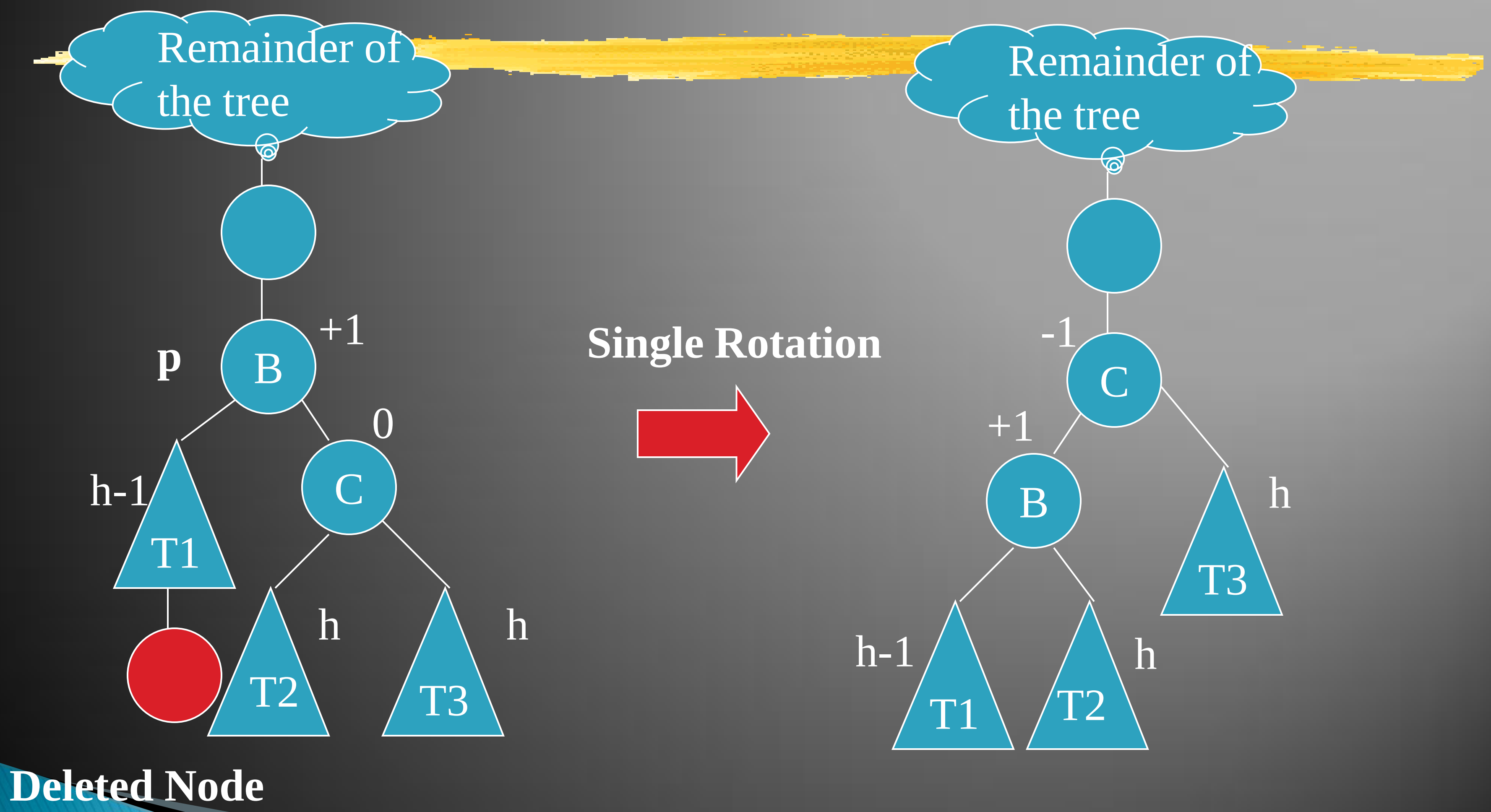
Delete: Case 2



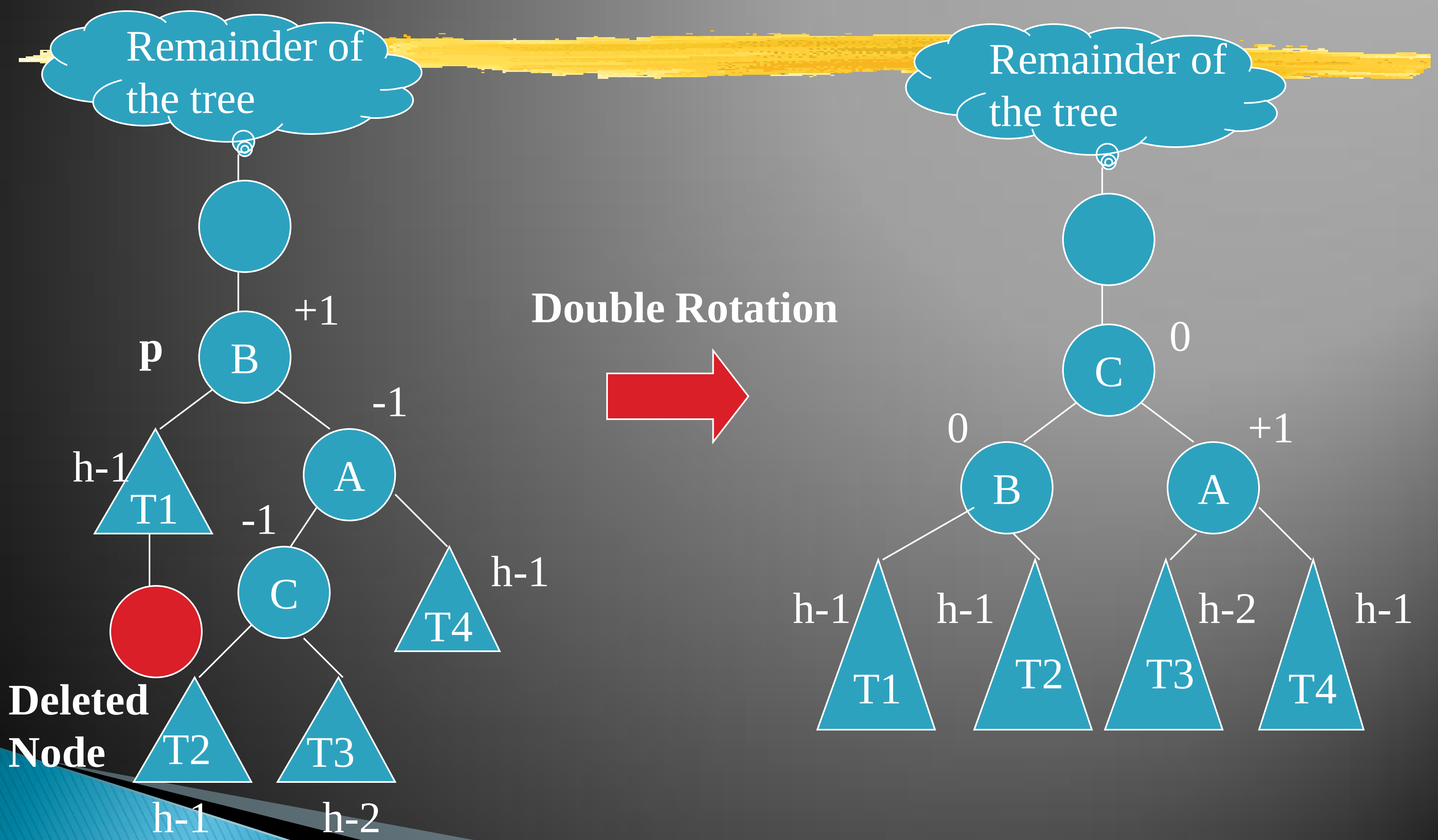
Delete: Case 3 (Sub-Case 1)



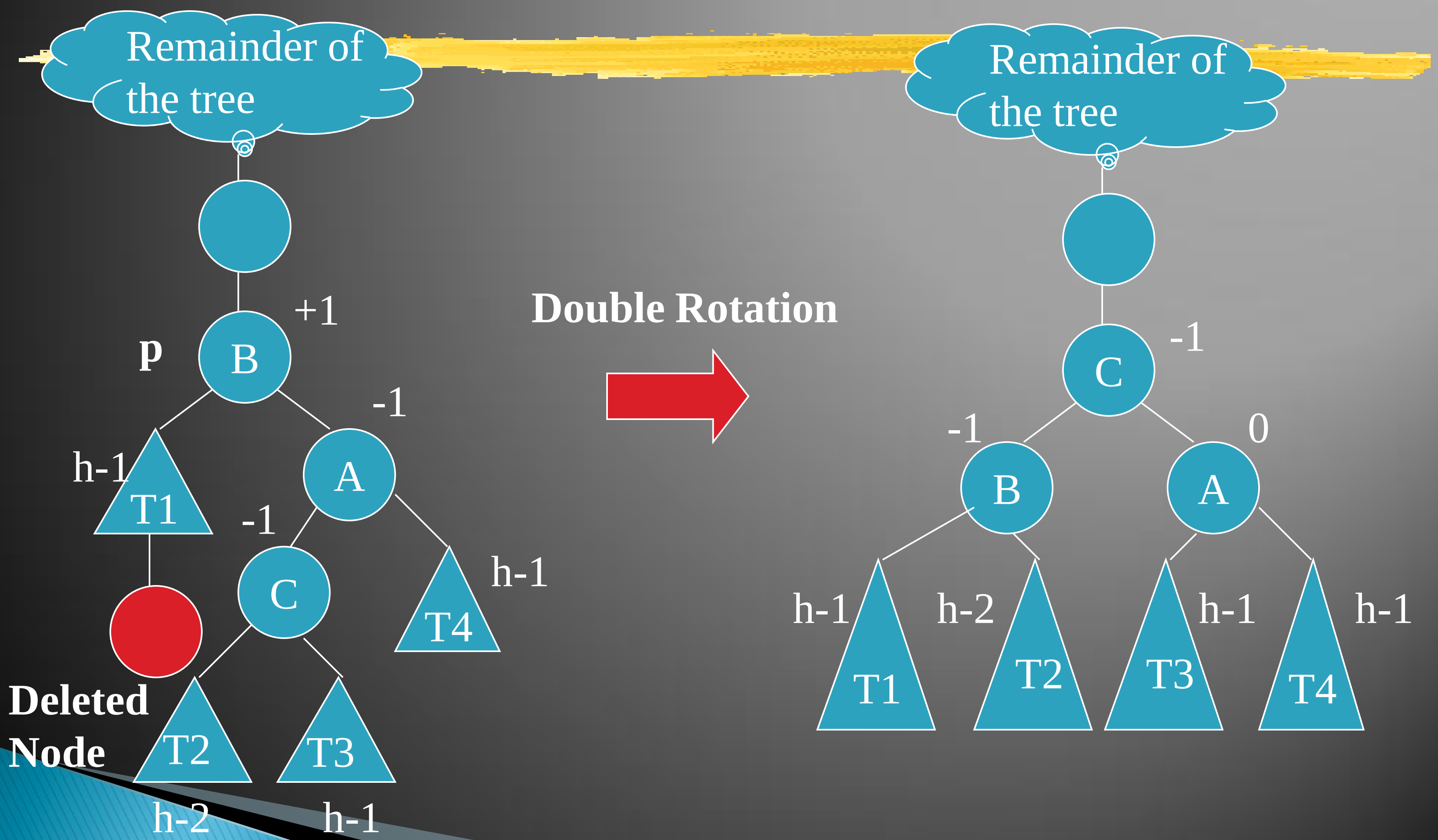
Delete: Case 3 (Sub-Case 2)



Delete: Case 3 (Sub-Case 3)



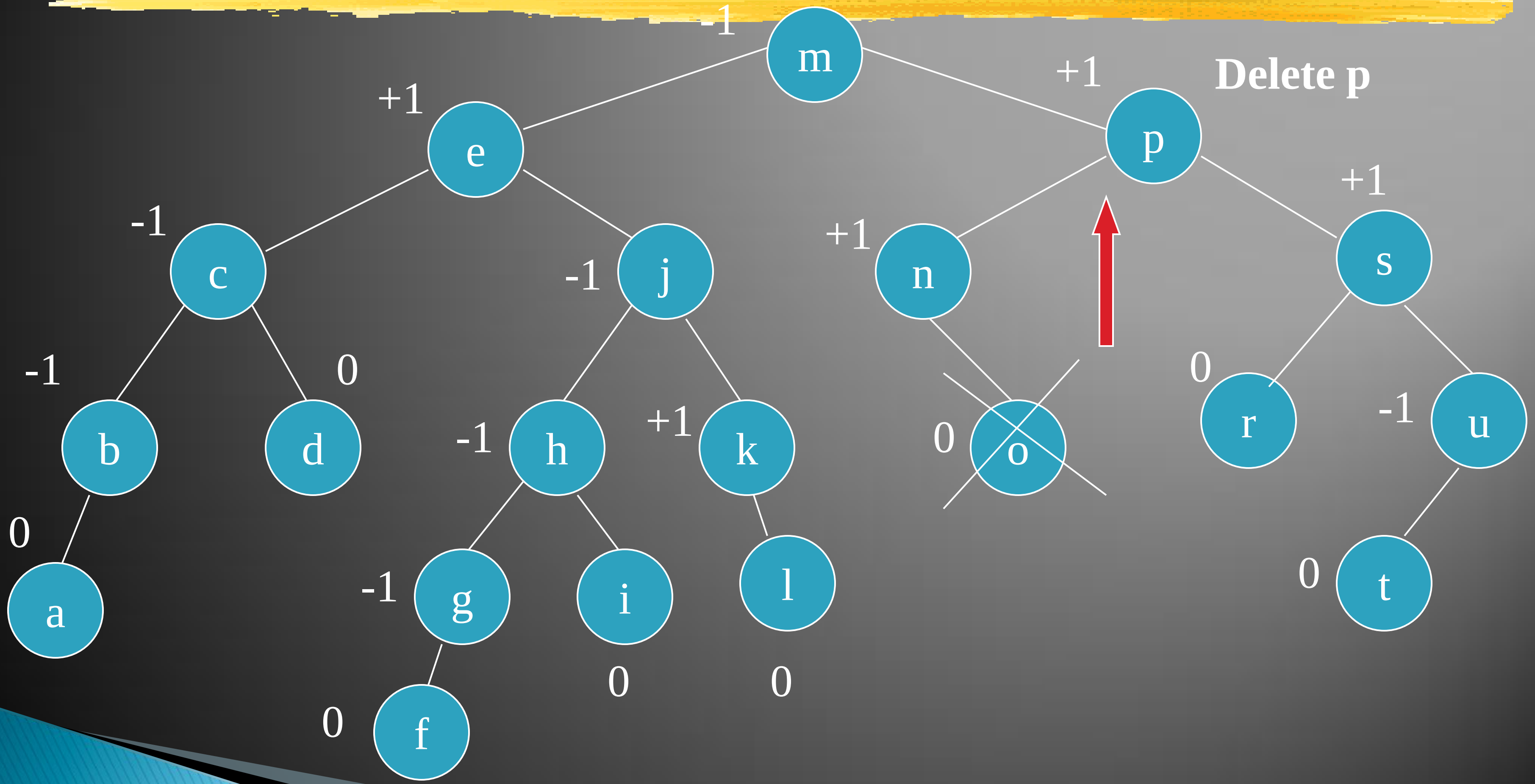
Delete: Case 3 (Sub-Case 4)



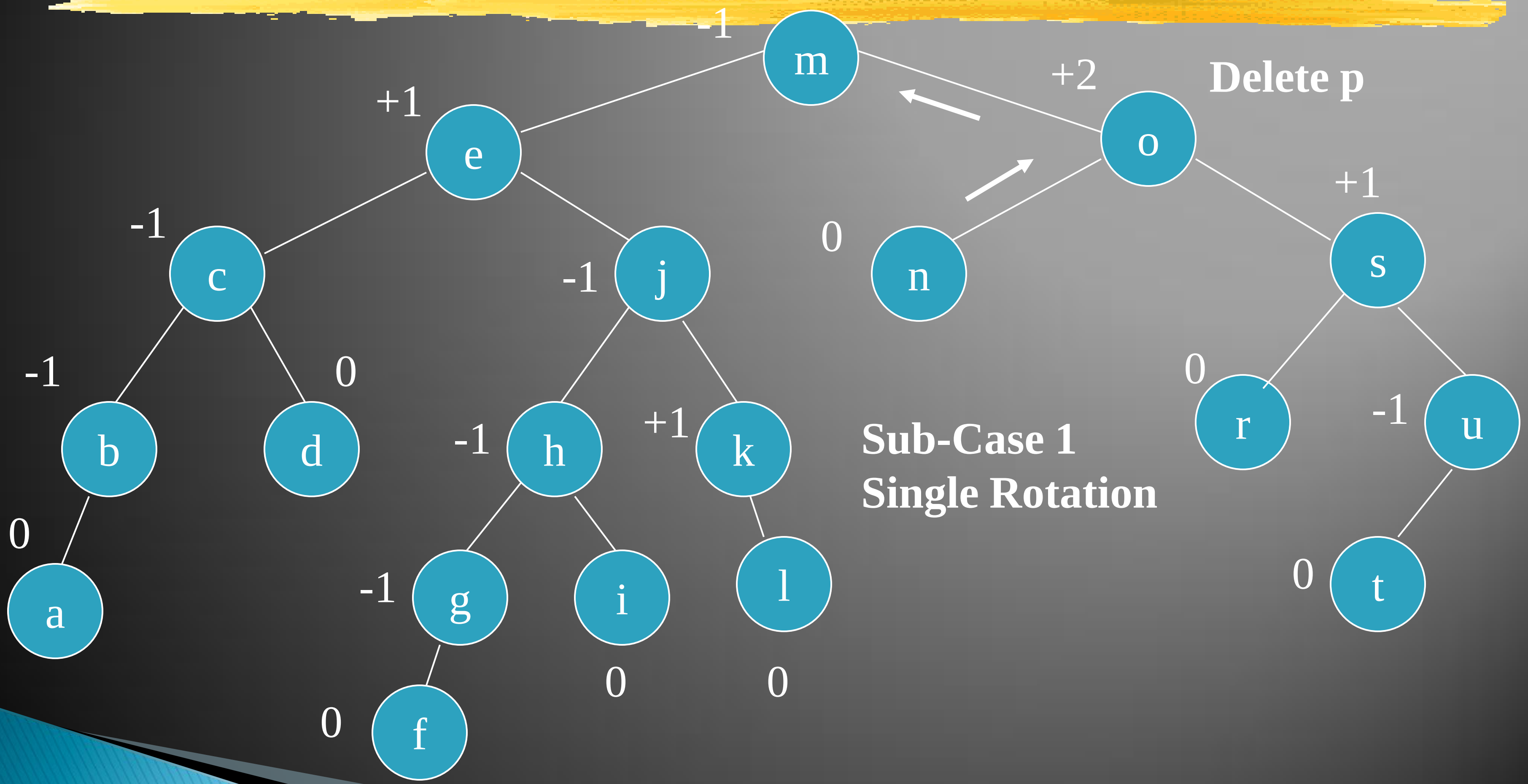
Delete: Case 3 (Other Sub-Cases)

- Sub-Case 5: mirror image of Sub-Case 1.
- Sub-Case 6: mirror image of Sub-Case 2.
- Sub-Case 7: mirror image of Sub-Case 3.
- Sub-Case 8: mirror image of Sub-Case 4.

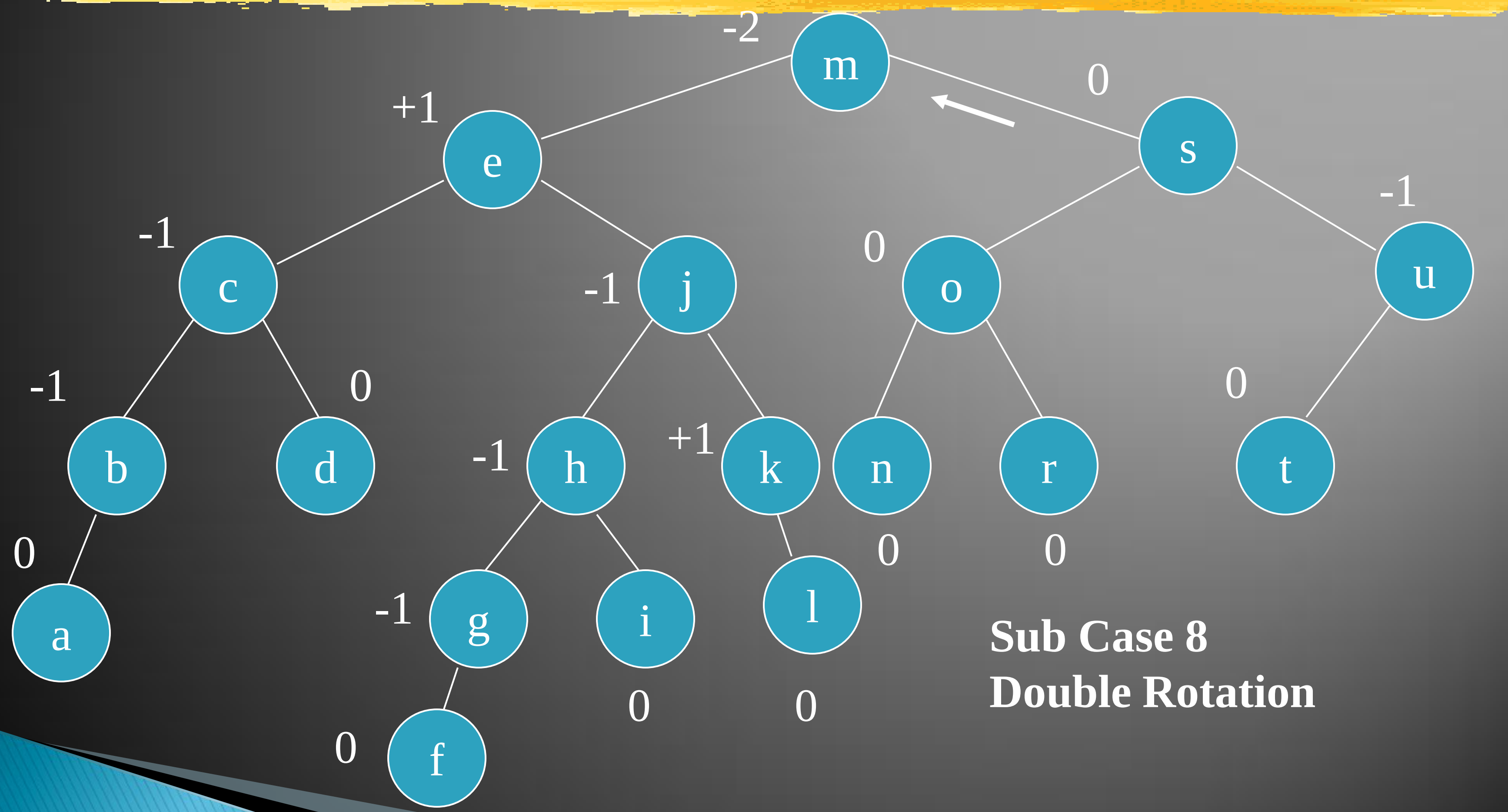
Deletion: Example



Deletion: Example



Deletion: Example



Sub Case 8
Double Rotation

Deletion: Example

After
Double Rotation