Binary Search Trees (BSTs)

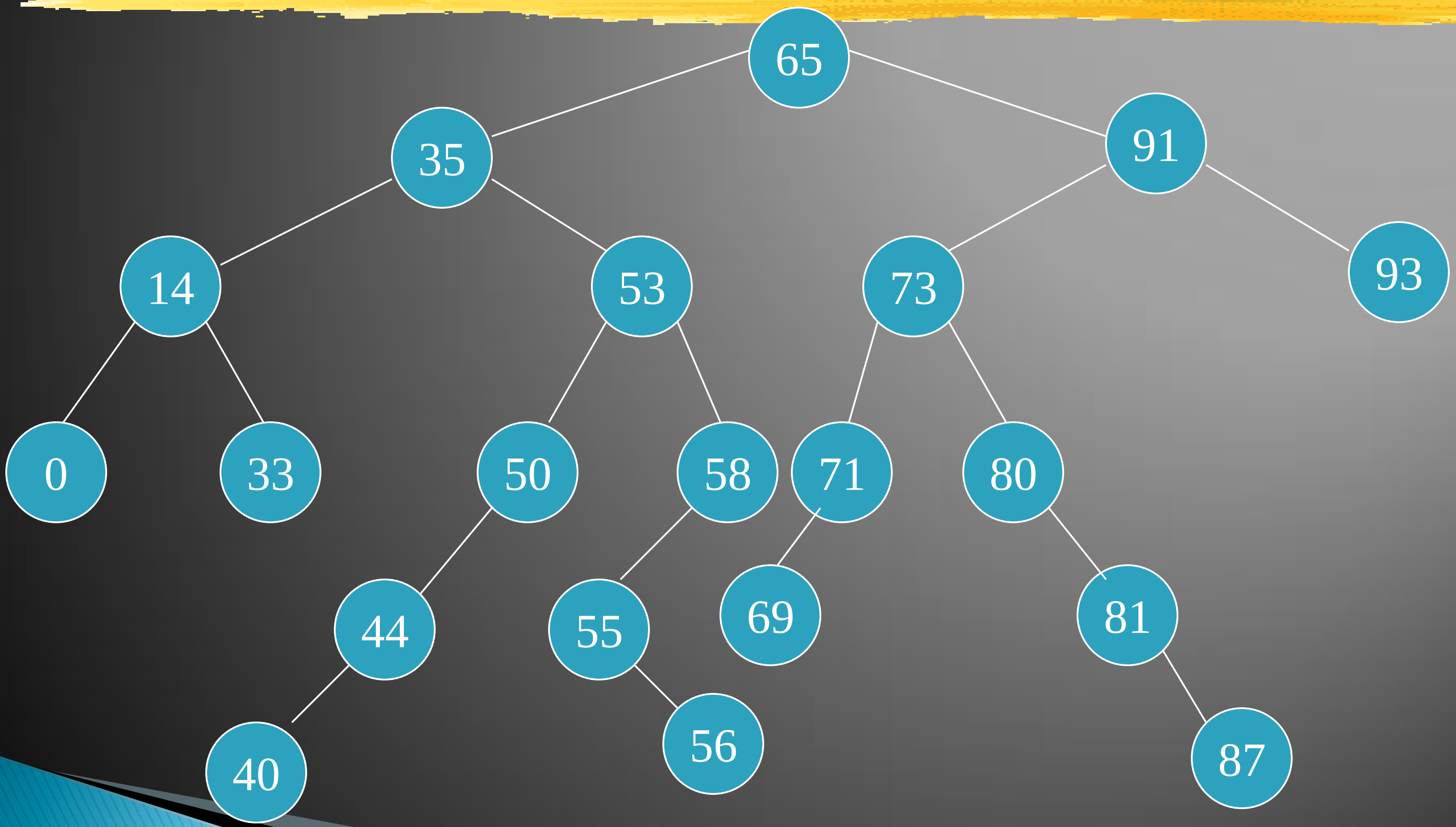
Binary Search Trees (BSTs)

- Consider the search operation FindKey (): find an element of a particular key value in a binary tree.
- This operation takes O(n) time in a binary tree.
- In a binary tree of 10^6 nodes → 10^6 steps required at least.
- In a BST this operation can be performed very efficiently: $O(\log_2 n)$.
- A binary search tree of 10^6 nodes → $\log_2(10^6) \cong 20$ steps only are required.

Binary Search Trees (BSTs)

- A binary search tree is a binary tree such that for each node, say N, the following statements are true:
  1. If L is any node in the left subtree of N, then L is less than N.
  2. If R is any node in the right subtree of N, then R is greater than N.

BST: Example.



BST: Searching

- The search operation in a binary search tree can be carried out as:

  <u>While</u> (the target element is not found <u>and</u> there is more tree to search) <u>do</u>

  <u>if</u> the target element is “less than” the current element <u>then</u> search the left subtree <u>else</u> search the right subtree.

ADT Binary Search Tree

Elements: The elements are nodes (BSTNode), each node contains the following data type: Type

Structure: hierarchical structure; each node can have two children: left or right child; there is a root node and a current node. If N is any node in the tree, nodes in the left subtree < N and nodes in the right subtree > N.

Domain: the number of nodes in a BST is bounded; type/class name is BST

ADT Binary Search Tree

<u>**Operations:**</u>

1. **Method** FindKey (int tkey, boolean found).

   results: If bst contains a node whose key value is tkey, then that node is made the current node and found is set to true; otherwise found is set to false and either the tree is empty or the current node is the node to which the node with key = tkey would be attached as a child if it were added to the BST.

ADT Binary Search Tree

2. **Method** Insert (int k, Type e, boolean inserted)
   requires: Full (bst) is false. **input**: key, e.
   results: if bst does not contain k then inserted is set to true and node with k and e is inserted and made the current element; otherwise inserted is set to false and current value does not change. **output**: inserted.
3. **Method** Remove_Key (int tkey, boolean removed) **input**: tkey
   results: Node with key value tkey is removed from the bst and removed set to true. If BST is not empty then root is made the current. **output**: removed

ADT Binary Search Tree

4. **Method** Update(Type e)
   requires: Empty(bst) is false. **results:** current node's element is replaced with e.
   These operations have the same specification as ADT Binary Tree.
5. **Method** Traverse (Order ord)
6. **Method** DeleteSub ( )
7. **Method** Retrieve (Type e)
8. **Method** Empty (boolean empty).

ADT Binary Search Tree

9. **Method** Full (boolean full)

ADT Binary Search Tree

```
public class BSTNode <T> {
    public int key;
    public T data;
    public BSTNode<T> left, right;
    /** Creates a new instance of BSTNode */
    public BSTNode(int k, T val) {
        key = k;
        data = val;
        left = right = null;
    }
```

ADT Binary Search Tree

```
public class BST <T> {
    BSTNode<T> root, current;
    /** Creates a new instance of BST */
    public BST() {
        root = current = null;
    }
   public boolean empty(){
        return root == null ? true: false;
    }
   public T retrieve () {
        return current.data;
    }
```

ADT Binary Search Tree

```
public boolean findkey(int tkey){
    BSTNode<T> p = root, q = root;
   if (empty()) return false;
    while (p != null){ q = p;
        if (p.key == tkey){
            current = p; return true;}
        else if (tkey < p.key)
            p = p.left;
        else
            p = p.right }
    current = q; return false; }
```

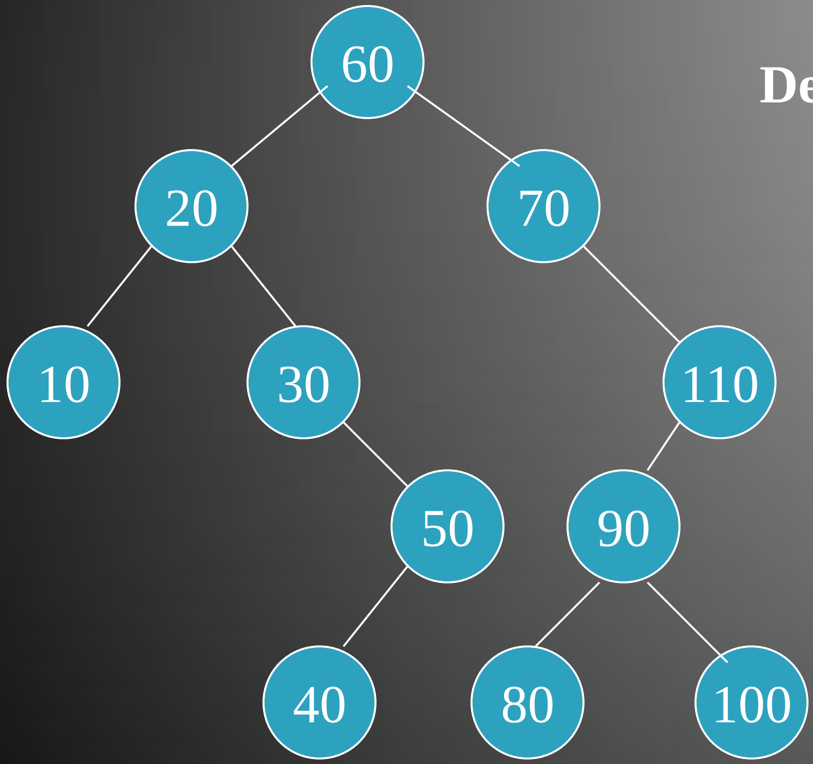
ADT Binary Search Tree

```
public boolean insert (int k, T val){
  BSTNode<T> p, q = current;
  if (findkey(k)){
      current = q; /* findkey() has modified current */
      return false; /* key already in the BST */ }
  p = new BSTNode<T>(k, val);
  if (empty()) {
      root = current = p; return true;}
  else {
      /* current is pointing to parent of the new key. */
      if (k < current.key)
          current.left = p;
      else
          current.right = p;
      current = p; return true;}}
```

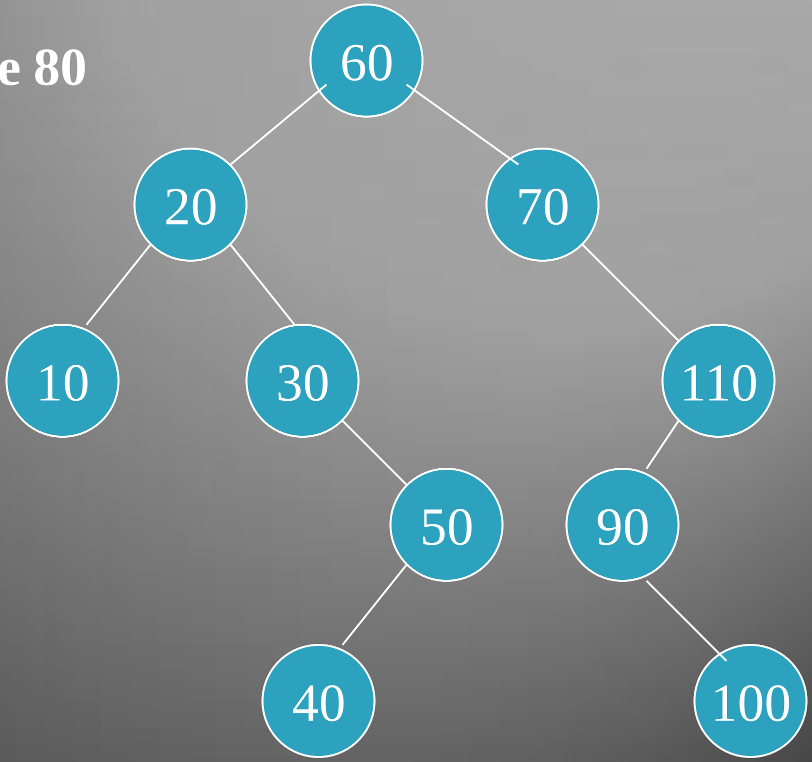
BST Node Deletion

- There are three cases:
  - Case 1: Node to be deleted has no children.
  - Case 2: Node to be deleted has one child.
  - Case 3: Node to be deleted has two children.
- In all these case it is always a leaf node that gets deleted.

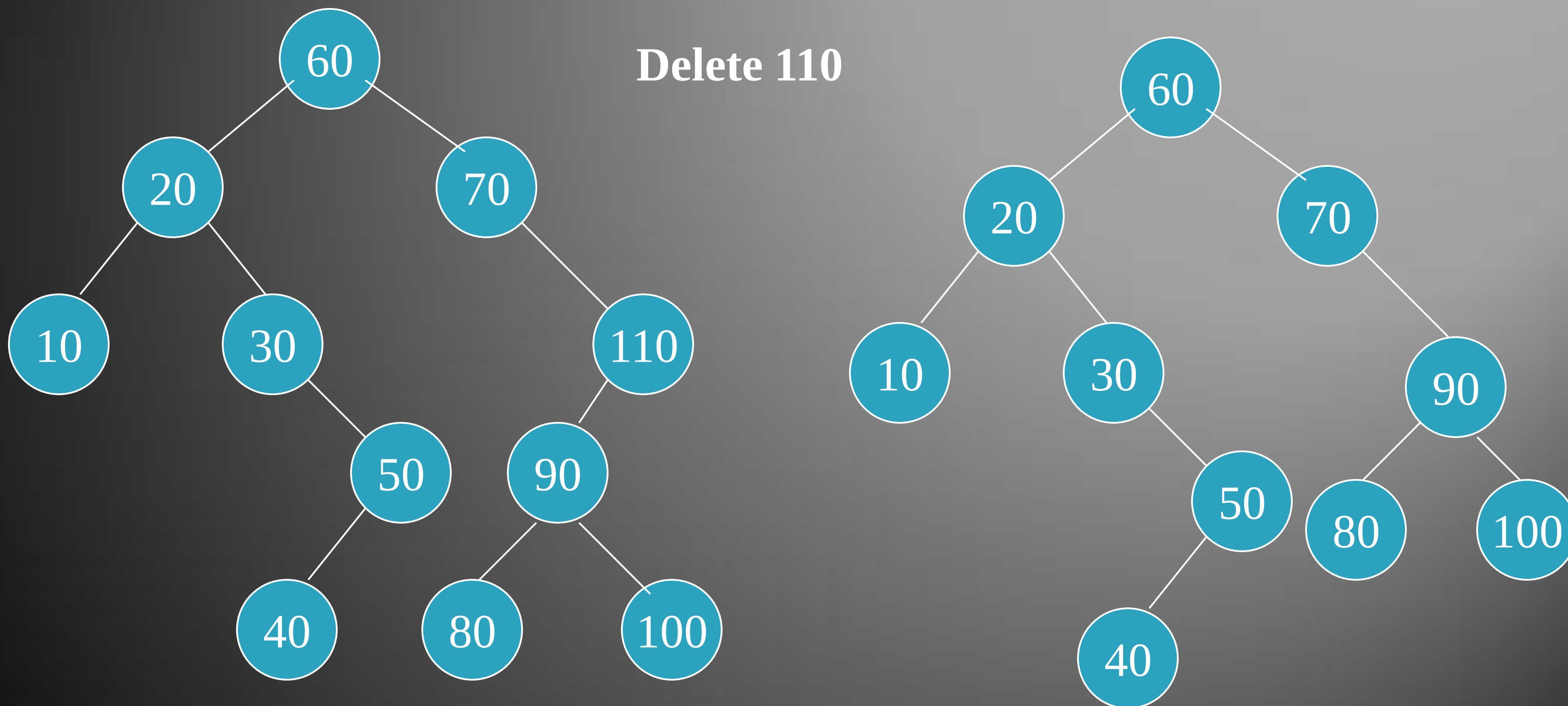
BST Deletion: Case 1



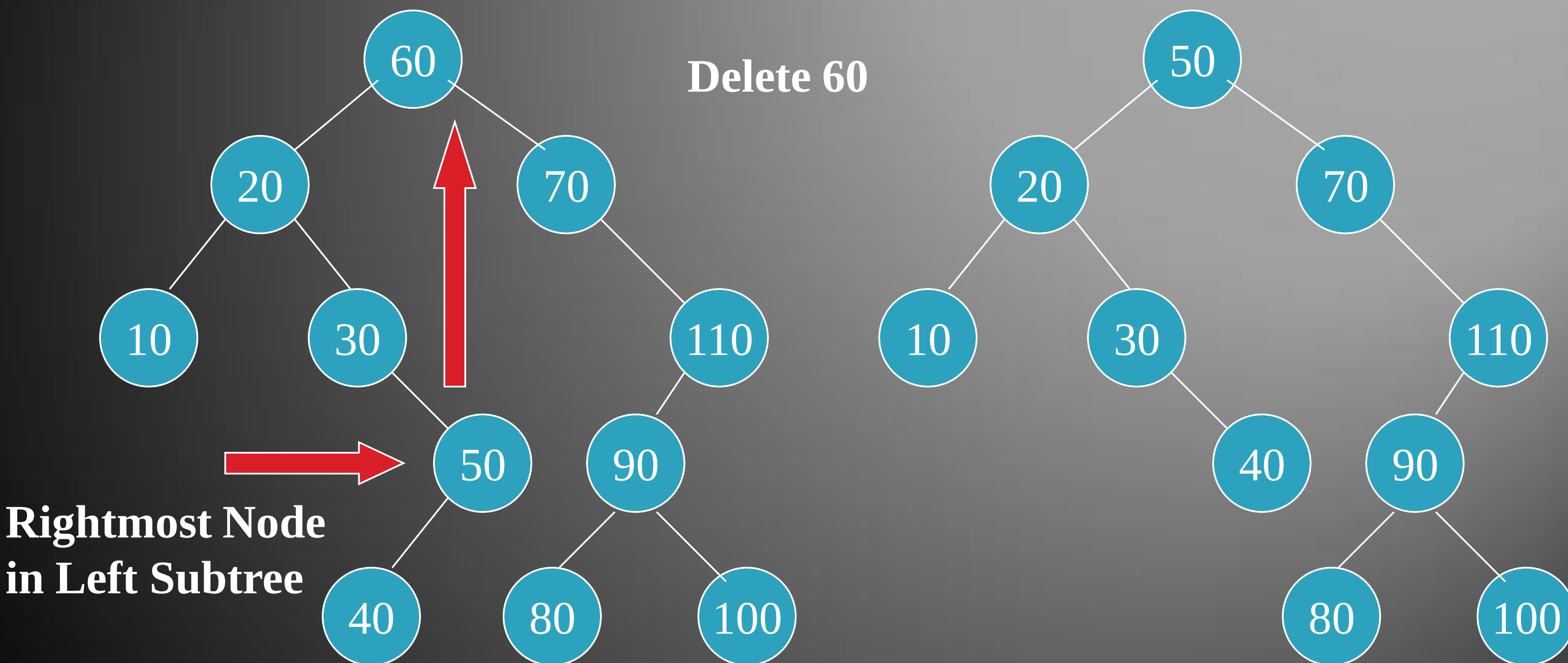
Delete 80



BST Deletion: Case 2



BST Deletion: Case 3



ADT Binary Search Tree

```
public boolean remove_key (int tkey){
    Flag removed = new Flag(false);
    BSTNode<T> p;
    p = remove_aux(tkey, root, removed);
    current = root = p;
    return removed;
}
```

ADT Binary Search Tree

```
private BSTNode<T> remove_aux(int key, BSTNode<T> p, Boolean flag) {
  BSTNode<T> q, child = null;
        if (p == null)
            return null;
        if (key < p.key)
            p.left = remove_aux(key, p.left, flag); //go left
        else if (key > p.key)
            p.right = remove_aux(key, p.right, flag); //go right
        else {
            flag = true;
            if (p.left != null && p.right != null){ //two children
                q = find_min(p.right);
                p.key = q.key; p.data = q.data;
                p.right = remove_aux(q.key, p.right, flag);}
```

ADT Binary Search Tree

```
        else {
                if (p.right == null) //one child case
                    child = p.left;
                else if (p.left == null) //one child case
                    child = p.right;
                return child;
            }
        }
        return p;
    }
```

ADT Binary Search Tree

```
private BSTNode<T> find_min(BSTNode<T> p){
        if (p == null) return null;
        while (p.left != null){
            p = p.left;
        }
        return p;
    }
```

ADT Binary Search Tree

```
    public boolean update(int key, T data){
            remove_key(current.key);
            return insert(key, data);
        }
}
```