Testing for the two populations Proportions

If we have two independent samples of size \( n_1 \) and \( n_2 \) with proportions \( p_1 \) and \( p_2 \) respectively. Thus, we will use the following steps:

1- data needed: \( x_1, \hat{p}_1 = \frac{x_1}{n_1} \) and \( x_2, \hat{p}_2 = \frac{x_2}{n_2} \)

2- the hypothesis:
\[
H_0: p_1 = p_2 \rightarrow p_1 - p_2 = 0
\]
\[
H_1: \begin{cases} 
P_1 < P_2 \rightarrow p_1 - p_2 < 0 \\
P_1 > P_2 \rightarrow p_1 - p_2 > 0 \\
P_1 \neq P_2 \rightarrow p_1 - p_2 \neq 0 
\end{cases}
\]

3- the statistic:
\[
Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}
\]
where \( \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} \)

4- Determining the rejection of \( H_0 \), that is:

i) if \( H_1: P_1 > P_2 \rightarrow P_1 - P_2 > 0 \), reject \( H_0 \) if \( Z > Z_{\alpha} \)

ii) if \( H_1: P_1 < P_2 \rightarrow P_1 - P_2 < 0 \), reject \( H_0 \) if \( Z < Z_{\alpha} \)

iii) if \( H_1: P_1 \neq P_2 \rightarrow P_1 - P_2 \neq 0 \), reject \( H_0 \) if \( Z > Z_{\frac{\alpha}{2}} \) or \( Z < -Z_{\frac{\alpha}{2}} \)

Ex (9):

Two machine A and B, a random sample of size 300 units from machine A with defective proportion 8% and another sample of size 200 units from machine B with defective proportion 4%. The manager think that the defective proportion from machine A is differ from the defective proportion from machine B, is he right?. use \( \alpha = 0.05 \)
Solu.

1-data needed: \( n_1 = 300, x_1 = 0.08 \) and \( n_2 = 200, x_2 = 0.04, \alpha = 0.05 \)

2- the hypothesis: 

\[ H_0: P_1 = P_2 \rightarrow P_1 - P_2 = 0 \]
\[ H_1: P_1 \neq P_2 \rightarrow P_1 - P_2 \neq 0 \]

3- the statistic:

\[
Z = \frac{0.08 - 0.04}{\sqrt{0.064(1 - 0.064)\left(\frac{1}{300} + \frac{1}{200}\right)}} = 0.895
\]

where \( \hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_1}{n_1 + n_2} = \frac{300(0.08) + 200(0.04)}{500} = 0.064 \)

4- reject \( H_0 \) if \( Z < Z_{\alpha/2} = Z_{0.025} = -1.96 \) or \( Z > Z_{1-\alpha/2} = 1.96 \)

Thus , we accept \( H_0 \) and reject \( H_1 \) that says there is a difference between the 
defective proportions from machines A and B.