CHAPTER 8
EXERGY

Lecture slides by
Mehmet Kanoglu

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Objectives

- Examine the performance of engineering devices in light of the second law of thermodynamics.
- Define *exergy*, which is the maximum useful work that could be obtained from the system at a given state in a specified environment.
- Define *reversible work*, which is the maximum useful work that can be obtained as a system undergoes a process between two specified states.
- Define the exergy destruction, which is the wasted work potential during a process as a result of irreversibilities.
- Define the *second-law efficiency*.
- Develop the exergy balance relation.
- Apply exergy balance to closed systems and control volumes.
The useful work potential of a given amount of energy at some specified state is called **exergy**, which is also called the **availability** or **available energy**.

A system is said to be in the **dead state** when it is in thermodynamic equilibrium with the environment it is in.

**FIGURE 8–1**
A system that is in equilibrium with its environment is said to be at the dead state.
A system delivers the maximum possible work as it undergoes a reversible process from the specified initial state to the state of its environment, that is, the dead state. This represents the *useful work potential* of the system at the specified state and is called *exergy*.

Exergy represents the upper limit on the amount of work a device can deliver without violating any thermodynamic laws.

**FIGURE 8–3**
The atmosphere contains a tremendous amount of energy, but no exergy.

**FIGURE 8–2**
The immediate surroundings of a hot potato are simply the temperature gradient zone of the air next to the potato.
Exergy (Work Potential) Associated with Kinetic and Potential Energy

Exergy of kinetic energy:

\[ x_{pe} = pe = g z \quad (kJ/kg) \]

Exergy of potential energy:

\[ x_{ke} = ke = \frac{V^2}{2} \quad (kJ/kg) \]

The exergies of kinetic and potential energies are equal to themselves, and they are entirely available for work.

Unavailable energy is the portion of energy that cannot be converted to work by even a reversible heat engine.

The *work potential* or exergy of potential energy is equal to the potential energy itself.
REVERSIBLE WORK AND IRREVERSIBILITY

\[ W_{\text{surr}} = P_0(v_2 - v_1) \]

\[ W_u = W - W_{\text{surr}} = W - P_0(v_2 - v_1) \]

**Reversible work** \( W_{\text{rev}} \): The maximum amount of useful work that can be produced (or the minimum work that needs to be supplied) as a system undergoes a process between the specified initial and final states.

As a closed system expands, some work needs to be done to push the atmospheric air out of the way \( (W_{\text{surr}}) \).

The difference between reversible work and actual useful work is the irreversibility.

For constant-volume systems, the total actual and useful works are identical \( (W_u = W) \).
The Rate of Irreversibility of a Heat Engine

\[
\dot{W}_{\text{rev, out}} = \eta_{\text{th, rev}} \dot{Q}_{\text{in}} = \left( 1 - \frac{T_{\text{sink}}}{T_{\text{source}}} \right) \dot{Q}_{\text{in}}
\]

\[
= \left( 1 - \frac{300 \text{ K}}{1200 \text{ K}} \right) (500 \text{ kW}) = 375 \text{ kW}
\]

\[
\dot{i} = \dot{W}_{\text{rev, out}} - \dot{W}_{u, out} = 375 - 180 = 195 \text{ kW}
\]
Irreversibility during the Cooling of an Iron Block

\[
\delta W_{\text{rev}} = \eta_{\text{th,rev}} \delta Q_{\text{in}} = \left(1 - \frac{T_{\text{sink}}}{T_{\text{source}}}\right) \delta Q_{\text{in}} = \left(1 - \frac{T_0}{T}\right) \delta Q_{\text{in}}
\]

\[
W_{\text{rev}} = \int \left(1 - \frac{T_0}{T}\right) \delta Q_{\text{in}}
\]

\[
\frac{\delta E_{\text{in}} - \delta E_{\text{out}}}{\text{Net energy transfer by heat, work, and mass}} = \frac{dE_{\text{system}}}{\text{Change in internal, kinetic, potential, etc., energies}} = -\delta Q_{\text{out}} = dU = mc_{\text{avg}}dT
\]

\[
\delta Q_{\text{in,heat engine}} = \delta Q_{\text{out,system}} = -mc_{\text{avg}}dT
\]

\[
W_{\text{rev}} = \int_{T_1}^{T_0} \left(1 - \frac{T_0}{T}\right)(-mc_{\text{avg}}dT) = mc_{\text{avg}}(T_1 - T_0) - mc_{\text{avg}} T_0 \ln \frac{T_1}{T_0}
\]

\[
= (500 \, \text{kg})(0.45 \, \text{kJ/kg·K}) \left[(473 - 300) \, \text{K} - (300 \, \text{K}) \ln \frac{473 \, \text{K}}{300 \, \text{K}}\right]
\]

\[
= 8191 \, \text{kJ}
\]

**FIGURE 8–12**

An irreversible heat transfer process can be made reversible by the use of a reversible heat engine.
Second-law efficiency is a measure of the performance of a device relative to its performance under reversible conditions.

**Figure 8–15**

Second-law efficiency is a measure of the performance of a device relative to its performance under reversible conditions.

**Figure 8–14**

Two heat engines that have the same thermal efficiency, but different maximum thermal efficiencies.
General definition of exergy efficiency:

\[ \eta_{II} = \frac{\text{Exergy recovered}}{\text{Exergy expended}} = 1 - \frac{\text{Exergy destroyed}}{\text{Exergy expended}} \]
Second-Law Efficiency of Resistance Heaters

\[ \eta_{\text{II, electric heater}} = \frac{\dot{X}_{\text{recovered}}}{\dot{X}_{\text{expended}}} = \frac{\dot{X}_{\text{heat}}}{\dot{W}_e} \]

\[ \dot{Q}_e = \dot{W}_e \]

\[ \text{COP}_{\text{HP, rev}} = \frac{1}{1 - \frac{T_L}{T_H}} = \frac{1}{1 - \frac{10}{(21 + 273) \text{ K}}/(21 + 273 \text{ K})} = 26.7 \]

\[ \eta_{\text{II}} = \frac{\text{COP}}{\text{COP}_{\text{rev}}} = \frac{1.0}{26.7} = 0.037 \text{ or } 3.7\% \]

\[ \eta_{\text{II, electric heater}} = 1 - \frac{T_0}{T_H} = 1 - \frac{(10 + 273) \text{ K}}{(21 + 273) \text{ K}} = 0.037 \text{ or } 3.7\% \]
EXERGY CHANGE OF A SYSTEM

Exergy of a Fixed Mass: Nonflow (or Closed System) Exergy

\[ \frac{\delta E_{\text{in}} - \delta E_{\text{out}}}{\delta E_{\text{system}}} = \frac{dE_{\text{system}}}{\text{Net energy transfer by heat, work, and mass}} = \frac{-\delta Q - \delta W}{-\delta Q - \delta W} = dU \]

\[ \delta W = P \, dV = (P - P_0) \, dV + P_0 \, dV = \delta W_{b,\text{useful}} + P_0 \, dV \]

\[ \delta W_{\text{HE}} = \left(1 - \frac{T_0}{T}\right) \delta Q = \delta Q - \frac{T_0}{T} \delta Q = \delta Q - (-T_0 \, dS) \rightarrow \delta Q = \delta W_{\text{HE}} - T_0 \, dS \]

\[ \delta W_{\text{total\,useful}} = \delta W_{\text{HE}} + \delta W_{b,\text{useful}} = -dU - P_0 \, dV + T_0 \, dS \]

\[ X = (U - U_0) + P_0(V - V_0) - T_0(S - S_0) + m \frac{V^2}{2} + mgz \]

Exergy of a closed system
\[ \phi = (u - u_0) + P_0(v - v_0) - T_0(s - s_0) + \frac{V^2}{2} + gz \]
\[ = (e - e_0) + P_0(v - v_0) - T_0(s - s_0) \]

\[ \Delta X = X_2 - X_1 = m(\phi_2 - \phi_1) = (E_2 - E_1) + P_0(V_2 - V_1) - T_0(S_2 - S_1) \]
\[ = (U_2 - U_1) + P_0(V_2 - V_1) - T_0(S_2 - S_1) + m\frac{V_2^2 - V_1^2}{2} + mg(z_2 - z_1) \]

\[ \Delta \phi = \phi_2 - \phi_1 = (u_2 - u_1) + P_0(v_2 - v_1) - T_0(s_2 - s_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \]
\[ = (e_2 - e_1) + P_0(v_2 - v_1) - T_0(s_2 - s_1) \]

When the properties of a system are not uniform, the exergy of the system is

\[ X_{\text{system}} = \int \phi \, \delta m = \int \phi \rho \, dV \]
Exergy of a Flow Stream: Flow (or Stream) Exergy

Exergy of flow energy
\[ x_{\text{flow}} = P \nu - P_0 \nu = (P - P_0) \nu \]

Flow exergy
\[ \psi = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz \]

Exergy change of flow
\[ \Delta \psi = \psi_2 - \psi_1 = (h_2 - h_1) + T_0(s_2 - s_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \]

**FIGURE 8–21**
The exergy associated with flow energy is the useful work that would be delivered by an imaginary piston in the flow section.
Energy:
\[ e = u + \frac{V^2}{2} + gz \]

Exergy:
\[ \phi = (u - u_0) + P_0(v - v_0) - T_0(s - s_0) + \frac{V^2}{2} + gz \]

(a) A fixed mass (nonflowing)

Energy:
\[ \theta = h + \frac{V^2}{2} + gz \]

Exergy:
\[ \psi = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz \]

(b) A fluid stream (flowing)

**FIGURE 8–22**
The energy and exergy contents of (a) a fixed mass and (b) a fluid stream.
Work Potential of Compressed Air in a Tank

\[ X_1 = m\phi_1 \]
\[ = m \left[ (u_1 - u_0)^0 + P_0(\nu_1 - \nu_0) - T_0(s_1 - s_0) + \frac{V_{1^0}^0}{2} + g\zeta_1^0 \right] \]
\[ = m[P_0(\nu_1 - \nu_0) - T_0(s_1 - s_0)] \]

\[ P_0(\nu_1 - \nu_0) = P_0 \left( \frac{RT_1}{P_1} - \frac{RT_0}{P_0} \right) = RT_0 \left( \frac{P_0}{P_1} - 1 \right) \quad \text{(since } T_1 = T_0) \]

\[ T_0(s_1 - s_0) = T_0 \left( c_p \ln \frac{T_1}{T_0} - R \ln \frac{P_1}{P_0} \right) = -RT_0 \ln \frac{P_1}{P_0} \quad \text{(since } T_1 = T_0) \]

\[ \phi_1 = RT_0 \left( \frac{P_0}{P_1} - 1 \right) + RT_0 \ln \frac{P_1}{P_0} = RT_0 \left( \ln \frac{P_1}{P_0} + \frac{P_0}{P_1} - 1 \right) \]

\[ X_1 = m_1\phi_1 \]
Exergy Change During a Compression Process

$\Delta \psi = \psi_2 - \psi_1 = (h_2 - h_1) - T_0(s_2 - s_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)^0$

$w_{in, min} = \psi_2 - \psi_1 = 38.0 \text{ kJ/kg}$
EXERGY TRANSFER BY HEAT, WORK, AND MASS

**Exergy by Heat Transfer, \( Q \)**

\[
X_{\text{heat}} = \left( 1 - \frac{T_0}{T} \right) Q
\]

Exergy transfer by heat

\[
X_{\text{heat}} = \int \left( 1 - \frac{T_0}{T} \right) \delta Q
\]

When temperature is not constant

**FIGURE 8–25**

The Carnot efficiency \( \eta_c = 1 - T_0/T \) represents the fraction of the energy transferred from a heat source at temperature \( T \) that can be converted to work in an environment at temperature \( T_0 \).

Exergy transfereared: \( E \)

\[
\text{Exergy} = \left( 1 - \frac{T_0}{T} \right) E
\]

**FIGURE 8–26**

The transfer and destruction of exergy during a heat transfer process through a finite temperature difference.
Exergy Transfer by Work, \( W \)

\[
X_{\text{work}} = \begin{cases} 
W - W_{\text{surr}} & \text{(for boundary work)} \\
W & \text{(for other forms of work)}
\end{cases}
\]

\[
W_{\text{surr}} = P_0(V_2 - V_1)
\]

Exergy Transfer by Mass, \( m \)

\[
X_{\text{mass}} = m\psi
\]

\[
\psi = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz
\]

\[
\dot{X}_{\text{mass}} = \int_{A_c} \psi p V_n \, dA_c
\]

\[
X_{\text{mass}} = \int \psi \delta m = \int_{\Delta t} \dot{X}_{\text{mass}} \, dt
\]

**FIGURE 8–28**

Mass contains energy, entropy, and exergy, and thus mass flow into or out of a system is accompanied by energy, entropy, and exergy transfer.

**FIGURE 8–27**

There is no useful work transfer associated with boundary work when the pressure of the system is maintained constant at atmospheric pressure.
THE DECREASE OF EXERGY PRINCIPLE
AND EXERGY DESTRUCTION

Energy balance:
\[ E_{in}^0 - E_{out}^0 = \Delta E_{system} \rightarrow 0 = E_2 - E_1 \]

Entropy balance:
\[ S_{in}^0 - S_{out}^0 + S_{gen} = \Delta S_{system} \rightarrow S_{gen} = S_2 - S_1 \]

Multiplying the second relation by \( T_0 \) and subtracting it from the first one gives
\[ -T_0 S_{gen} = E_2 - E_1 - T_0(S_2 - S_1) \] (8–29)

From Eq. 8–17 we have
\[ X_2 - X_1 = (E_2 - E_1) + P_0(V_2 - V_1)^0 - T_0(S_2 - S_1) \] (8–30)
\[ = (E_2 - E_1) - T_0(S_2 - S_1) \]

since \( V_2 = V_1 \) for an isolated system (it cannot involve any moving boundary and thus any boundary work). Combining Eqs. 8–29 and 8–30 gives
\[ -T_0 S_{gen} = X_2 - X_1 \leq 0 \] (8–31)

since \( T_0 \) is the thermodynamic temperature of the environment and thus a positive quantity, \( S_{gen} \geq 0 \), and thus \( T_0 S_{gen} \geq 0 \). Then we conclude that
\[ \Delta X_{isolated} = (X_2 - X_1)_{isolated} \leq 0 \] (8–32)

The exergy of an isolated system during a process always decreases or, in the limiting case of a reversible process, remains constant. In other words, it never increases and exergy is destroyed during an actual process. This is known as the decrease of exergy principle.
Exergy destroyed is a *positive quantity* for any actual process and becomes *zero* for a reversible process.

Exergy destroyed represents the lost work potential and is also called the *irreversibility* or *lost work*.

Can the exergy change of a system during a process be negative?

Consider heat transfer from a system to its surroundings. How do you compare exergy changes of the system and the surroundings?
The exergy change of a system during a process is equal to the difference between the net exergy transfer through the system boundary and the exergy destroyed within the system boundaries as a result of irreversibilities.

**EXERGY BALANCE: CLOSED SYSTEMS**

\[
\left( \text{Total exergy entering} \right) - \left( \text{Total exergy leaving} \right) - \left( \text{Total exergy destroyed} \right) = \left( \text{Change in the total exergy of the system} \right)
\]

**General:**

\[
\frac{X_{\text{in}} - X_{\text{out}}}{\text{Net exergy transfer by heat, work, and mass}} - \frac{X_{\text{destroyed}}}{\text{Exergy destruction}} = \frac{\Delta X_{\text{system}}}{\text{Change in exergy}} \quad \text{(kJ)}
\]

**General, rate form:**

\[
\dot{X}_{\text{in}} - \dot{X}_{\text{out}} = \dot{X}_{\text{destroyed}} = \frac{dX_{\text{system}}}{dt} \quad \text{(kW)}
\]

\[
\dot{X}_{\text{heat}} = (1 - T_0/T)\dot{Q}, \quad \dot{X}_{\text{work}} = \dot{W}_{\text{useful}}, \quad \text{and} \quad \dot{X}_{\text{mass}} = \dot{m}U
\]

**General, unit-mass basis:**

\[
(x_{\text{in}} - x_{\text{out}}) - x_{\text{destroyed}} = \Delta x_{\text{system}} \quad \text{(kJ/kg)}
\]

\[
X_{\text{destroyed}} = T_0S_{\text{gen}} \quad \text{or} \quad \dot{X}_{\text{destroyed}} = T_0\dot{S}_{\text{gen}}
\]
The heat transfer to a system and work done by the system are taken to be positive quantities.

**Closed system:**

\[ X_{\text{heat}} - X_{\text{work}} - X_{\text{destroyed}} = \Delta X_{\text{system}} \]

**Closed system:**

\[ \sum \left(1 - \frac{T_0}{T_k}\right) Q_k - \left[ W - P_0 (V_2 - V_1) \right] - T_0 S_{\text{gen}} = X_2 - X_1 \]

**Rate form:**

\[ \sum \left(1 - \frac{T_0}{T_k}\right) \dot{Q}_k - \left( \dot{W} - P_0 \frac{dV_{\text{system}}}{dt} \right) - T_0 \dot{S}_{\text{gen}} = \frac{dX_{\text{system}}}{dt} \]

\( Q_k \) is the heat transfer through the boundary at temperature \( T_k \) at location \( k \).

Exergy balance for a closed system when heat transfer is to the system and the work is from the system.

Exergy destroyed outside system boundaries can be accounted for by writing an exergy balance on the extended system that includes the system and its immediate surroundings.
General Exergy Balance for Closed Systems

Energy balance:  \[ E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}} \rightarrow Q - W = E_2 - E_1 \]

Entropy balance:  \[ S_{\text{in}} - S_{\text{out}} + S_{\text{gen}} = \Delta S_{\text{system}} \rightarrow \int_{1}^{2} \left( \frac{\delta Q}{T} \right)_{\text{boundary}} + S_{\text{gen}} = S_2 - S_1 \]

\[ Q - T_0 \int_{1}^{2} \left( \frac{\delta Q}{T} \right)_{\text{boundary}} - W - T_0 S_{\text{gen}} = E_2 - E_1 - T_0(S_2 - S_1) \]

\[ \int_{1}^{2} \delta Q - T_0 \int_{1}^{2} \left( \frac{\delta Q}{T} \right)_{\text{boundary}} - W - T_0 S_{\text{gen}} = X_2 - X_1 - P_0(V_2 - V_1) \]

\[ \int_{1}^{2} \left( 1 - \frac{T_0}{T_b} \right) \delta Q - [W - P_0(V_2 - V_1)] - T_0 S_{\text{gen}} = X_2 - X_1 \]
Exergy Destruction during Heat Conduction

\[
\dot{X}_{\text{in}} - \dot{X}_{\text{out}} - \dot{X}_{\text{destroyed}} = \frac{dX_{\text{system}}}{dt}^{0 \ (\text{steady})} = 0
\]

\[
\dot{Q}\left(1 - \frac{T_0}{T}\right)_{\text{in}} - \dot{Q}\left(1 - \frac{T_0}{T}\right)_{\text{out}} - \dot{X}_{\text{destroyed}} = 0
\]

\[
(1035 \text{ W})\left(1 - \frac{273 \text{ K}}{293 \text{ K}}\right) - (1035 \text{ W})\left(1 - \frac{273 \text{ K}}{278 \text{ K}}\right) - \dot{X}_{\text{destroyed}} = 0
\]

\[
\dot{X}_{\text{destroyed}} = 52.0 \text{ W}
\]

To determine the rate of total exergy destruction during this heat transfer process, we extend the system to include the regions on both sides of the wall that experience a temperature change. Then one side of the system boundary becomes room temperature while the other side, the temperature of the outdoors. The exergy balance for this extended system (system + immediate surroundings) is the same as that given above, except the two boundary temperatures are 300 and 273 K instead of 293 and 278 K, respectively. Then the rate of total exergy destruction becomes

\[
\dot{X}_{\text{destroyed, total}} = (1035 \text{ W})\left(1 - \frac{273 \text{ K}}{300 \text{ K}}\right) - (1035 \text{ W})\left(1 - \frac{273 \text{ K}}{273 \text{ K}}\right) = 93.2 \text{ W}
\]
The exergy balance applied on the extended system (system + immediate surroundings) whose boundary is at the environment temperature of $T_0$ gives

$$\Delta X = X_2 - X_1$$

$$W_u = W - W_{\text{surr}} = W_{b,\text{out}} - P_0(v_2 - v_1) = W_{b,\text{out}} - P_0 m(v_2 - v_1)$$

$$X_{\text{destroyed}} = T_0 S_{\text{gen}} = T_0 \left[ m(s_2 - s_1) + \frac{Q_{\text{surr}}}{T_0} \right]$$

Alternative method of exergy destruction calculation:

$$E_{\text{in}} - E_{\text{out}} = \frac{\Delta E_{\text{system}}}{\text{Net energy transfer by heat, work, and mass}}$$

$$\Delta U = -Q_{\text{out}} - W_{b,\text{out}}$$

$$W_{b,\text{out}} = -Q_{\text{out}} - \Delta U = -Q_{\text{out}} - m(u_2 - u_1)$$
Dropping a Hot Iron Block into Water

\[
\frac{E_{\text{in}} - E_{\text{out}}}{\text{Net energy transfer by heat, work, and mass}} = \frac{\Delta E_{\text{system}}}{\text{Change in internal, kinetic, potential, etc., energies}}
\]

\[
0 = \Delta U
\]

\[
0 = (\Delta U)_{\text{iron}} + (\Delta U)_{\text{water}}
\]

\[
0 = [mc(T_f - T_i)]_{\text{iron}} + [mc(T_f - T_i)]_{\text{water}}
\]

\[
X = (U - U_0) - T_0(S - S_0) + P_0(V - V_0)
\]

\[
= mc(T - T_0) - T_0 mc \ln \frac{T}{T_0} + 0
\]

\[
= mc\left(T - T_0 - T_0 \ln \frac{T}{T_0}\right)
\]

\[
\frac{X_{\text{in}} - X_{\text{out}}}{\text{Net exergy transfer by heat, work, and mass}} - \frac{X_{\text{destroyed}}}{\text{Exergy destruction}} = \frac{\Delta X_{\text{system}}}{\text{Change in exergy}}
\]

\[
0 - X_{\text{destroyed}} = X_2 - X_1
\]

\[
X_{\text{destroyed}} = X_1 - X_2
\]
Work Potential of Heat Transfer Between Two Tanks

\[
\frac{S_{\text{in}} - S_{\text{out}}}{\text{Net entropy transfer by heat and mass}} + \frac{S_{\text{gen}}^0}{\text{Entropy generation}} = -\Delta S_{\text{system}}^{\text{Change in entropy}}
\]

\[
0 + S_{\text{gen}}^0 = \Delta S_{\text{tank,source}} + \Delta S_{\text{tank,sink}} + \Delta S_{\text{heat engine}}^0
\]

\[
\Delta S_{\text{tank,source}} + \Delta S_{\text{tank,sink}} = 0
\]

\[
\left( mc_v \ln \frac{T_2}{T_1} + mR \ln \frac{V_2^{0}}{V_1} \right)_{\text{source}} + \left( mc_v \ln \frac{T_2}{T_1} + mR \ln \frac{V_2^{0}}{V_1} \right)_{\text{sink}} = 0
\]

\[
\ln \frac{T_2}{T_{1,A}} \frac{T_2}{T_{1,B}} = 0 \rightarrow T_2^2 = T_{1,A} T_{1,B}
\]

\[
Q_{\text{source, out}} = mc_v(T_{1,A} - T_2)
\]

\[
Q_{\text{sink, in}} = mc_v(T_2 - T_{1,B})
\]

\[
W_{\text{max, out}} = Q_H - Q_L = Q_{\text{source, out}} - Q_{\text{sink, in}}
\]
EXERGY BALANCE: CONTROL VOLUMES

\[ X_{\text{heat}} - X_{\text{work}} + X_{\text{mass,in}} - X_{\text{mass,out}} - X_{\text{destroyed}} = (X_2 - X_1)_{CV} \]

\[ \sum \left(1 - \frac{T_0}{T_k}\right)Q_k - [W - P_0(V_2 - V_1)] + \sum_{\text{in}} m\psi - \sum_{\text{out}} m\psi - X_{\text{destroyed}} = (X_2 - X_1)_{CV} \]

\[ \sum \left(1 - \frac{T_0}{T_k}\right)\dot{Q}_k - \left(\dot{W} - P_0 \frac{dV_{CV}}{dt}\right) + \sum_{\text{in}} \dot{m}\psi - \sum_{\text{out}} \dot{m}\psi - \dot{X}_{\text{destroyed}} = \frac{dX_{CV}}{dt} \]

The rate of exergy change within the control volume during a process is equal to the rate of net exergy transfer through the control volume boundary by heat, work, and mass flow minus the rate of exergy destruction within the boundaries of the control volume.

![Diagram of a control volume and its surroundings. Exergy is transferred into or out of a control volume by mass as well as heat and work transfer.](image)
Exergy Balance for Steady-Flow Systems

Most control volumes encountered in practice such as turbines, compressors, nozzles, diffusers, heat exchangers, pipes, and ducts operate steadily, and thus they experience no changes in their mass, energy, entropy, and exergy contents as well as their volumes. Therefore, $dV_{CV}/dt = 0$ and $dX_{CV}/dt = 0$ for such systems.

**Steady-flow:**
\[ \sum \left( 1 - \frac{T_0}{T_k} \right) Q_k - W + \sum_{in} m \psi_{in} - \sum_{out} m \psi_{out} - \dot{X}_{destroyed} = 0 \]

**Single-stream:**
\[ \sum \left( 1 - \frac{T_0}{T_k} \right) Q_k - W + m(\psi_1 - \psi_2) - \dot{X}_{destroyed} = 0 \]

\[ \psi_1 - \psi_2 = (h_1 - h_2) - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \]

**Per-unit mass:**
\[ \sum \left( 1 - \frac{T_0}{T_k} \right) q_k - w + (\psi_1 - \psi_2) - x_{destroyed} = 0 \]

**FIGURE 8–42**

The exergy transfer to a steady-flow system is equal to the exergy transfer from it plus the exergy destruction within the system.
Reversible Work

The exergy balance relations presented above can be used to determine the reversible work $W_{rev}$ by setting the exergy destroyed equal to zero. The work $W$ in that case becomes the reversible work.

**General:** \[ W = W_{rev} \quad \text{when} \quad X_{\text{destroyed}} = 0 \]

**Single stream:** \[ \dot{W}_{rev} = \dot{m}(\psi_1 - \psi_2) + \sum \left(1 - \frac{T_0}{T_k}\right)\dot{Q}_k \quad (\text{kW}) \]

**Adiabatic, single stream:** \[ \dot{W}_{rev} = \dot{m}(\psi_1 - \psi_2) \]

The exergy destroyed is zero only for a reversible process, and reversible work represents the maximum work output for work-producing devices such as turbines and the minimum work input for work-consuming devices such as compressors.
Second-Law Efficiency of Steady-Flow Devices

The second-law efficiency of various steady-flow devices can be determined from its general definition, \( \eta_\text{II} = \frac{\text{Exergy recovered}}{\text{Exergy expended}} \). When the changes in kinetic and potential energies are negligible and the devices are adiabatic:

\[
\eta_{\text{II,turb}} = \frac{w}{w_{\text{rev}}} = \frac{h_1 - h_2}{\psi_1 - \psi_2} \quad \text{or} \quad \eta_{\text{II,turb}} = 1 - \frac{T_0 S_{\text{gen}}}{\psi_1 - \psi_2}
\]

\[
\eta_{\text{II,comp}} = \frac{w_{\text{rev, in}}}{w_{\text{in}}} = \frac{\psi_2 - \psi_1}{h_2 - h_1} \quad \text{or} \quad \eta_{\text{II,comp}} = 1 - \frac{T_0 S_{\text{gen}}}{h_2 - h_1}
\]

\[
\eta_{\text{II,HX}} = \frac{\dot{m}_{\text{cold}} (\psi_4 - \psi_3)}{\dot{m}_{\text{hot}} (\psi_1 - \psi_2)}
\]

\[
\eta_{\text{II,HX}} = 1 - \frac{T_0 \dot{S}_{\text{gen}}}{\dot{m}_{\text{hot}} (\psi_1 - \psi_2)}
\]

\[
\dot{S}_{\text{gen}} = \dot{m}_{\text{hot}} (s_2 - s_1) + \dot{m}_{\text{cold}} (s_4 - s_3)
\]

\[
\eta_{\text{II,mix}} = \frac{\dot{m}_3 \psi_3}{\dot{m}_1 \psi_1 + \dot{m}_2 \psi_2}
\]

\[
\eta_{\text{II,mix}} = 1 - \frac{T_0 \dot{S}_{\text{gen}}}{\dot{m}_1 \psi_1 + \dot{m}_2 \psi_2}
\]

\[
\dot{S}_{\text{gen}} = \dot{m}_3 s_3 - \dot{m}_2 s_2 - \dot{m}_1 s_1
\]

Turbine

Compressor

Heat exchanger

Mixing chamber
Second-law analysis of a steam turbine

\[ \dot{X}_{\text{in}} - \dot{X}_{\text{out}} - \dot{X}_{\text{destroyed}}^{0\text{ (reversible)}} = \frac{dX_{\text{system}}}{dt}^{0\text{ (steady)}} = 0 \]

Rate of net exergy transfer by heat, work, and mass
Rate of exergy destruction
Rate of change in exergy

\[ \dot{X}_{\text{in}} = \dot{X}_{\text{out}} \]

\[ \dot{m}\psi_1 = \dot{W}_{\text{rev, out}} + \dot{X}_{\text{heat}}^{0} + \dot{m}\psi_2 \]

\[ \dot{W}_{\text{rev, out}} = \dot{m}(\psi_1 - \psi_2) \]

\[ = \dot{m}[(h_1 - h_2) - T_0(s_1 - s_2) - \Delta ke^{0} - \Delta pe^{0}] \]

\[ \eta_{\text{II}} = \frac{\dot{W}_{\text{out}}}{\dot{W}_{\text{rev, out}}} \]

\[ \dot{X}_{\text{destroyed}} = \dot{W}_{\text{rev, out}} - \dot{W}_{\text{out}} \]

\[ \dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \frac{dE_{\text{system}}}{dt}^{0\text{ (steady)}} = 0 \]

Rate of net energy transfer by heat, work, and mass
Rate of change in internal, kinetic, potential, etc., energies

\[ \dot{E}_{\text{in}} = \dot{E}_{\text{out}} \]

\[ \dot{m}h_1 = \dot{W}_{\text{out}} + \dot{Q}_{\text{out}} + \dot{m}h_2 \quad \text{(since ke \equiv pe \equiv 0)} \]

\[ \dot{W}_{\text{out}} = \dot{m}(h_1 - h_2) - \dot{Q}_{\text{out}} \]
Exergy Destroyed During Mixing of Fluid Streams

\[
\dot{X}_{\text{in}} - \dot{X}_{\text{out}} - \dot{X}_{\text{destroyed}} = \frac{dX_{\text{system}}}{dt} = 0
\]

- Rate of net exergy transfer by heat, work, and mass
- Rate of exergy destruction
- Rate of change in exergy

\[
\dot{X}_{\text{in}} = \dot{X}_{\text{out}}
\]

\[
\dot{m}_1 \psi_1 + \dot{m}_2 \psi_2 = \dot{W}_{\text{rev, out}} + \dot{X}_{\text{heat}} + \dot{m}_3 \psi_3
\]

\[
\dot{W}_{\text{rev, out}} = \dot{m}_1 \psi_1 + \dot{m}_2 \psi_2 - \dot{m}_3 \psi_3
\]

Mass balance:
\[
\dot{m}_\text{in} - \dot{m}_\text{out} = \frac{dm_{\text{system}}}{dt} = 0 \rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3
\]

Energy balance:
\[
\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \frac{dE_{\text{system}}}{dt} = 0
\]

- Rate of net energy transfer by heat, work, and mass
- Rate of change in internal, kinetic, potential, etc., energies

\[
\dot{E}_{\text{in}} = \dot{E}_{\text{out}}
\]

\[
\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{Q}_{\text{out}} \quad \text{(since } \dot{W} = 0, \text{ ke } \equiv \text{ pe } \equiv 0\text{)}
\]

\[
\dot{Q}_{\text{out}} = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3
\]
Charging a Compressed Air Storage System

\[ X_{\text{in}} - X_{\text{out}} - X_{\text{destroyed}} = \Delta X_{\text{system}} \]

\[ W_{\text{rev,in}} + m_1 \phi_1'\cdot 0 = m_2 \phi_2 - m_1 \phi_1'\cdot 0 \]

\[ W_{\text{rev,in}} = m_2 \phi_2 \]

\[ \phi_2 = (u_2 - u_0)'\cdot 0 (\text{since } T_2 = T_0) + P_0 (\nu_2 - \nu_0) - T_0 (s_2 - s_0) + \frac{V_2^2\cdot 0}{2} + gz_2'\cdot 0 \]

\[ = P_0 (\nu_2 - \nu_0) - T_0 (s_2 - s_0) \]

\[ P_0 (\nu_2 - \nu_0) = P_0 \left( \frac{RT_2}{P_2} - \frac{RT_0}{P_0} \right) = RT_0 \left( \frac{P_0}{P_2} - 1 \right) \quad (\text{since } T_2 = T_0) \]

\[ T_0 (s_2 - s_0) = T_0 \left( c_p \ln \frac{T_2}{T_0} - R \ln \frac{P_2}{P_0} \right) = -RT_0 \ln \frac{P_2}{P_0} \quad (\text{since } T_2 = T_0) \]

\[ \phi_2 = RT_0 \left( \frac{P_0}{P_2} - 1 \right) + RT_0 \ln \frac{P_2}{P_0} = RT_0 \left( \ln \frac{P_2}{P_0} + \frac{P_0}{P_2} - 1 \right) \]
Summary

• Exergy: Work potential of energy
  ✓ Exergy (work potential) associated with kinetic and potential energy
• Reversible work and irreversibility
• Second-law efficiency
• Exergy change of a system
  ✓ Exergy of a fixed mass: Nonflow (or closed system) exergy
  ✓ Exergy of a flow stream: Flow (or stream) exergy
• Exergy transfer by heat, work, and mass
• The decrease of exergy principle and exergy destruction
• Exergy balance: Closed systems
• Exergy balance: Control volumes
  ✓ Exergy balance for steady-flow systems
  ✓ Reversible work
  ✓ Second-law efficiency of steady-flow devices