ACTUARIAL MATHEMATICS

BY BOWERS, HICKMAN, GERBER, JONES AND NESBITT

[Published in 1986 by The Society of Actuaries]

This is a textbook which has been written for The Society of Actuaries in the U.S.A. Parts of the text have been in circulation for a number of years (and a partial review appeared in T.F.A. Vol. 39, Part 3) but the complete text was finally published only in 1986. This is a substantial work with over 600 pages (and weighing nearly 5lbs!) whose aim is to present a "modern mathematical foundation [for] actuarial science". Put more simply, the book covers the subjects of Life Contingencies and Risk Theory. (Chapters 1, 2, 11, 12 and 13 are published separately by the Institute of Faculty of Actuaries under the title "Risk Theory", and the first three of these chapters are part of the reading for the Faculty's Part IVB examination).

The contents of the book are as follows:

The decision to integrate the topics of Life Contingencies and Risk Theory into a single text is an imaginative one (which is welcomed by this reviewer). However, British actuaries brought up on a diet of Neill (1977) or Hooker and Longley-Cook (1953 & 1957) may find it hard to regard this as anything other than two separate texts unnaturally merged.

The authors have arranged the chapters so that, in their opinion, Chapters 1-13 cover material "considered basic for all actuarial students" and Chapters 14-19 are a "more in-depth treatment of selected topics for students specialising in life insurance and pensions". The choice of topics for each part provides plenty of scope for disagreement between individuals. For example, the authors have included some heavy mathematical results on the probability of ultimate ruin for a collective risk process (Chapter 12) in the first part, whereas expense loadings for life assurance premiums (Chapter 14) appear only in the second part.

Each chapter has a section entitled "Notes and References" at the end, which indicates the origin of the material in the chapter and also gives references for the reader interested in further developments of the material. The total number of references in the book is 150, of which about one half have been published within 20 years of the publication of the book. Such a large number of references is to be welcomed since they give a clear and accurate impression that actuarial mathematics is a live and growing subject, as well as being useful in themselves.

Each chapter also has a large number of exercises for the reader (there are a total of 540 in the book) as well as numerous worked examples in the text. The style and standard of the exercise in the "Risk Theory" chapters are not too dissimilar to those of Part IVB questions, but the "Life Contingencies" exercise are very different to Part III questions, being generally shorter, less convoluted, more mathematical and less numerical.

The authors' approach to Life Contingencies is uncompromisingly probabilistic. Time-until-death is regarded as a random variable and assurance and annuity values are defined as expected values of the random variables which themselves represent the present values of the respective payments. It is this feature more than any other which marks out the Life Contingencies chapters in this book as different from previous texts such as Neill and Jordan (1967). A result of this approach is that the authors are required to take, and do take, more care than is usual in such a book over the development of the subject. This firmer theoretical foundation, together with the greater care taken by the authors, will surely be welcomed by actuarial students who study Life Contingencies from this book. British actuarial students have only a rather brief Special Note by Forfar and Waters (1986) to help them in this respect. However, the Note by Forfar and Waters does include one positive feature noticeably, and regretably, missing from the book being reviewed; a (brief) discussion of the effects of variable interest rates. The authors excuse themselves in their Introduction by saying " . . . the modelling of interest rates requires ideas from economics and statistics that are not included in the

prerequisites of this volume. In addition, there are some technical problems in building models to combine random interest and random time of claim that are only in the process of being solved at the time of this publication." While there may be some force in these arguments, it is a pity that some discussion of random interest rates is not included in the book since, as is pointed out by Forfar and Waters (1986, section 7), the effects of random interest rates can be very different from those of random mortality.

The authors' attitude to numerical work is interesting. In their Introduction they say that "The issues involved in . . . computation in actuarial models are not discussed. This is a rapidly changing area, seemingly best left for readers to resolve as they choose in light of their own resources". This attitude is most clearly displayed in the exercises at the end of each chapter where the emphasis is on deriving formulae rather than calculating numerical answers. (At the end of Chapter 10 there are 21 exercises on pension funds, none of which requires a numerical answer!) Nevertheless, the content of the book has clearly been influenced by advances in computational techniques. For example, Panjer's recursion formula for the evaluation of a compound Poisson distribution can be found in Chapter 11 (formula (11.4.15)), and this is very much a product of the computer age. Traditional commutation functions do appear in this book but they are usually to be found at the end of a chapter, with little emphasis placed on them.

Two of the chapters in the book are a little disappointing for different reasons. Chapter 9 treats multiple decrement models (note that this is MOD-ELS, not TABLES) as competing risk models. This is a considerable improvement on the traditional, table-oriented, approach to this topic. However, the competing risk approach does not allow multiple decrement models to be generalised to multiple state, or increment-decrement, models. This is regretable since such models are likely to become increasingly important in actuarial work. See, for example, Hoem (1988) or Waters (1984). Chapter 13 has an enticing title, "Applications of Risk Theory", but the reader hoping to find "real world" applications may be disappointed. The applications, while they may possibly have practical value, seem to be included more on the strength of their mathematical interest. In particular, the use of Risk Theory to determine the solvency of a general insurance company, as described by Pentikainen (1982) and Rantala (1982), is not mentioned in the book.

A further criticism of the book is that occasionally the authors make rather heavy weather of relatively simple concepts. A good example of this can be found in Chapter 5, Life Annuities, where it takes nearly five pages for an annuity to be introduced. Another example can be found in Chapter 14, where we read that " . . . expenses . . .will be incorporated [in a gross premium] by modifying the loss variables".

These minor criticisms apart, this book is to be welcomed as a major step forward in the development and presentation of actuarial mathematics. The Risk Theory chapters in this book have already had an influence on the professional actuarial examinations in Britain since some of them have been

included in the reading for Part IVB. It is to be hoped that this book will have a considerable influence on the syllabus for Part III when a successor to Neill appears.

H. R. W.

REFERENCES

(1) Forfar, D. O. and Waters, H. R. (1986) "A Stochastic Approach to Life Contingencies". Special Note published by the Faculty and the Institute of Actuaries.

(2) Hoem, J. M. (1988) "The Versatility of the Markov Chain as a Tool in the Mathematics of Life Insurance". Transactions of the 23rd International Congress of Actuaries. Vol. 3.

(3) Hooker, P. F. and Longley-Cook, L. H. (1953 Vol. I, 1957 Vol. II) "Life and Other Contingencies". London. Cambridge University Press.

(4) Jordan, C. W. (1967) "Life Contingencies". Chicago. Society of Actuaries.

(5) Pentikainen, T. (1982) "Solvency of Insurers and Equalisation Reserves", Vol. I, Helsinki. Insurance Publishing Company.

(6) Rantala, J. (1982) "Solvency of Insurers and Equalisation Reserves", Vol. II. Helsinki. Insurance Publishing Company.

(7) Waters, H. R. (1984) "An Approach to the Study of Multiple State Models". *J.I.A.* **111**, pp.363-374.