جـــامـــعـــة الملك سعود
King Saud University

College of Science.
Department of Statistics & Operations Research

كلية العلوم
قسم الإحصاء وبحوث العمليات

First Midterm Exam
Academic Year 1443-1444 Hijri-First Semester

Exam Information معلومات الامتحان			
Course name	Modeling and Simulation	النمذجة والمحاكاة	اسم المقرر
Course Code	OPER 441	441 بحث	رمز المقرر
Exam Date	5-10-2021	9-3-1444	تاريخ الامتحان
Exam Time	1:30pm		وقت الامتحان
Exam Duration	2.5 hours	ساعتان ونصف	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of ☐ PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان ☐ صفحة. (باستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	Understanding the processes and steps for building a simulation model			
2	Implement an inverse cumulative distribution function based random variate generation algorithm			
3	Explain and implement the convolution algorithm for random variate generation			
4	Explain and implement the acceptance rejection algorithm for random variate generation			
5	Compute statistical quantities from simulation output			
6	Generate random numbers from any given distribution discrete or continuous			
7	Building simulation models from basic applications			
8				

EXAM COVER PAGE

Question #1: Answer the following with *True* or *False*: (15 points)

F	***False***	1. The sample space in a random experiment is always determined and unique to everyone.
F	***False***	2. Validation step is to make sure that the simulation program is running correctly.
F	***False***	3. In call center model with two lines it is impossible to lose any incoming call.
T	***True***	4. Triangular distribution is used when there is lack of data
F	***False***	5. Simulating flight distance for an airplane is a discrete system simulation.
T	***True***	6. In Bank simulation, the variable (X = number of customers in in service) is a state variable for the system.
F	***False***	7. In Bank simulation, the variable (X = number of credit cards with customer) is a state variable for the system.
T	***True***	8. Every simulation run for the same model give the different results but same statistical properties.
F	***False***	9. The Geometric ang Gamma distribution both has memory less property
F	***False***	10. If a used computer didn't fail for the past 6 months then the probability that it will not fail the next month is always the same as buying a new computer now for any distribution.
T	***True***	11. The normal distribution with parameters μ and σ^2 always cover more than 80% of the distribution within $\pm 2\sigma$
T	***True***	12. The beta distribution between (0,1) can be rescaled for any real values because it has 4 parameters
T	***True***	13. The random variable with Exponential distribution is always has mean value equals to the standard deviation.
T	***True***	14. If the random variable has mean value equals to the variance then it must have a Poisson distribution.
F	***False***	15. The number of trials until first 2 success and stop is a binomial distribution.
T	***True***	16. The normal distribution always has the mean equals to the median
F	***False***	17. The Triangular distribution is used if there is no data except the range of the values for the random variable.
T	***True***	18. The Exponential distribution is a special case of Gamma distribution
F	***False***	19. Erlang Distribution is a special case of the Exponential distribution
F	***False***	20. The Uniform distribution is good when there is no data about the random process except the range of the possible values and the most likely value.
F	***False***	21. In building simulation model, we always have to start data collection after validation.
F	***False***	22. To use the Empirical distribution for the data, we must first estimate the parameters of the distribution for the data.
T	***True***	23. T is possible to create a Triangular distribution with exactly two parameters

T	***True***	24. The Weibull distribution is always having a closed form function for the CDF.
F	***False***	25. We can directly apply the inverse transform on The chi-Square distribution because has a closed form function for the CDF .

Question #2: (12 points)

Given the following functions:

A	$np(1-p)$	**G**	$\Gamma(\beta) = \int_0^\infty x^{\beta-1}e^{-x}\,dx$	**M**	$a+(b-a)\left(\frac{\beta_1}{\beta_1+\beta_2}\right)$
B	$\frac{q}{p^2}$	**H**	$\frac{\lambda^k x^{k-1}e^{-\lambda x}}{(k-1)!}$ for $x, \lambda \geq 0$,	**N**	$e^{\mu+\sigma^2/2}$
C	p	**I**	$\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$	**O**	$1-e^{-\lambda x}$,
D	$p(1-p)$	**J**	$\frac{\beta}{\alpha}\left(\frac{x-\nu}{\alpha}\right)^{\beta-1}\exp\left[-\left(\frac{x-\nu}{\alpha}\right)^{\beta}\right]$, $x \geq \nu$	**P**	$\frac{1}{k\theta^2}$
E	$\frac{e^{-\alpha}\alpha^k}{k!}$	**K**	$\frac{\beta\theta}{\Gamma(\beta)}(\beta\theta x)^{\beta-1}e^{-\beta\theta x}$, $x>0$	**Q**	$\binom{n}{x}p^x q^{n-x}$,
F	$\frac{kq}{p^2}$	**L**	$\frac{1}{\sqrt{2\pi}\sigma x}\exp\left[-\frac{(\ln x-\mu)^2}{2\sigma^2}\right]$,		

Complete the following by choosing the correct function from above

10	The random variable X has ***Poisson*** distribution if X has the pdf	**E**
11	The random variable X has ***Bernoulli*** distribution (p), then X has the expected value	**C**
1	The random variable X has ***Exponential*** distribution if X has the CDF	**O**
5	The random variable X has ***Lognormal*** distribution if X has the pdf	**L**
7	The random variable X has ***Beta*** distribution if X has the pdf	**K**
8	The random variable X has ***Lognormal*** distribution, Then X has expected value	**N**
9	The random variable X has ***Beta*** distribution, then X has the expected value	**M**
12	The random variable X is has ***Erlang*** distribution (k) , then it has variance	**P**
6	A random variable X has ***Negative binomial*** distribution(p) then X has the a variance	**F**
2	The random variable X is has ***Erlang*** distribution if X has the pdf	**H**
4	The random variable X has ***Geometric*** distribution (p) then X has the a variance	**B**
3	The random variable X has ***Beta*** distribution if X has the pdf	**K**

Question #3: (12 points)

An airport has a counter of three servers and a single waiting line. Passenger arrive at random to the airport for check-in of their luggage's. Also, the airport has three run ways for airplanes to land or departing the airport. run servers serve customers in the order in which they arrive. Customers may leave the system without service due to long waiting time. The service time of the customers changes according to their gender and the type of service they request. The service facility provide four types of services.

1. **(3 points)** Define the three different entities in the system.
2. **(3 points)** For **each entity in (1)** define two attributes
3. **(3 points)** Define three different activities in the system
4. **(3 points)** Define two state variables for **each entity in (1).**

<u>Question #4:</u> (8 points)

Consider the following LCG generator: Xn = (13 Xn-1 + 13) mod (16),   X0 = 14

Answer the following:

a) **(4 points)** Prove that this LCG has Full Period
b) **(4 points)** Generate all possible uniform pseudo-random numbers from the above LCG.

(a) all possible uniform pseudo-random is listed below:

n	X_{n-1}	$X_n = (13\,X_{n-1} + 13)\ mod\ (16)$	$U_n = X_n/16$
1	14	X_1 = (13(14)+13) mod 16 = 3	0.1875
2	3	X_2 = (13(3)+13) mod 16 =4	0.2500
3	4	X_3 = (13(4)+13) mod 16 = 1	0.0625
4	1	X_4 = (13(1)+13) mod 16 =10	0.6250
5	10	X_5 = (13(10)+13) mod 16 = 15	0.9375
6	15	X_6 = (13(15)+13) mod 16 = 0	0.0000
7	0	X_7 = (13(0)+13) mod 16 = 13	0.8125
8	13	X_8 = (13(13)+13) mod 16 = 6	0.3750
9	6	X_9 = (13(16)+13) mod 16 = 11	0.6875
10	11	X_{10} = (13(11)+13) mod 16 = 12	0.7500
11	12	X_{11} = (13(12)+13) mod 16 = 9	0.5625
12	9	X_{12} = (13(9)+13) mod 16 = 2	0.1250
13	2	X_{13} = (13(2)+13) mod 16 = 7	0.4375
14	7	X_{14} = (13(7)+13) mod 16 = 8	0.5000
15	8	X_{15} = (13(8)+13) mod 16 = 5	0.3125
16	5	X_{16} = (13(5)+13) mod 16 = 14	0.8750

The starting value X_n = 14 is repeated. Then STOP calculations

(b) The given LCG has a FULL CYCLE because number of generated pseudo-numbers is equal to m = 16 number.

Question #5: (12 points)

n	1	2	3	4	5	6	7	8
$U_x(0,1)$	0.304	0.696	0.171	0.339	0.981	0.125	0.842	0.656
X	**1.329**	**1.557**	**1.2174**	**1.355**	**1.675**	**1.171**	**1.621**	**1.538**
$U_y(0,1)$	0.124	0.842	0.304	0.697	0.842	0.172	0.338	0.697
Y	**0.058**	**0.305**	**0.133**	**0.264**	**0.305**	**0.07**	**0.146**	**0.264**

(a) **(4 points)** Consider the following probability function:

$$f(x) = \begin{cases} \frac{4}{7}x^3 ; & 1 \le x \le 2 \\ 0 \quad ; & otherwise \end{cases}$$

Find the inverse transform of the probability and generate random numbers from f(x) using the table of U(0,1) random numbers above.

c) **(4 points)** Consider the following probability function:

$$f(y) = \begin{cases} 2e^{2y} ; & 0 \le y \\ 0 \quad ; & otherwise \end{cases}$$

Find the inverse transform of the probability and generate random numbers from f(y) using the table of U(0,1) random numbers above.

d) **(4 points)** Consider a car repair workshop. Assume that the variable X is the time in hours until a car is repaired and Y is the time in hours between. Build a flow chart and use your results in (a) and (b) to simulate the system for 8 arriving customers.

(a) Inverse transform

1. Get the CDF of $f(x)$

$$f(x) = \int_1^x \frac{4}{7}y^3 \; dy = \frac{4}{7}\left[\frac{y^4}{4}\right]_1^x = \frac{1}{7}(x^4 - 1)$$

2. Let u = F(x) and solve for x

$$u = \frac{1}{7}(x^4 - 1) \iff 7u = (x^4 - 1) \iff 7u + 1 = x^4$$

$$Then \;\; x = \sqrt[4]{7u + 1}$$

Question #6: (12 points)

High temperature (ᵒF) in a city on July 21, denoted by the random variable X, has the following probability density function, where X is in degrees F.

$$f(x) = \begin{cases} \dfrac{2(x-85)}{119}, & 85 \le x \le 92 \\ \dfrac{2(102-x)}{170}, & 92 < x \le 102 \\ 0, & \text{otherwise} \end{cases}$$

(a) **(3 points)** What is the mean of the temperature E(X)?
(b) **(3 points)** What is the variance of the temperature V(X)?
(c) **(3 points)** What is the probability that the temperature will be between 90ᵒ and 100ᵒ.
(d) **(3 points)** Write the inverse transform for X.

$$E(X^2) = \int_{85}^{92} \frac{x^2(2)(x-85)}{119}dx + \int_{95}^{102} \frac{x^2(2)(102-x)}{170}dx = 3311.75 + 5349.41 = 8661.16$$

$$E(X) = (a+b+c)/3 = (85+92+102)/3 = 93$$

$$V(X) = E(X^2) - [E(X)]^2 = 8661.16 - (93)^2 = 12.16°F^2$$

P {that the temperature will be between 90ᵒ and 100ᵒ} = P{90 <= X <= 100} = 0.7664

Question #6: (15 points)

Busses arrive to a station at random. It is estimated that the time between busses is an exponential distribution with mean 15 minutes. The number of passengers on the bus is also random follows a binomial distribution with parameter 10 and probability 0.75.

(1) **(3 points)**If you arrive at 10:00 am, What is the probability you will wait more than 30 min for your bus?

(2) **(3 points)**What is the expected number of busses that will arrive to the station between 10:00 to 11:00 am?

(3) **(3 points)**What is the expected number of passengers that will drop off to the station between 10:00 to 11:00 am?

(4) **(3 points)**Given that 15 passengers arrived between 10:00 to 11:00 am what is the probability that the next bus will have 3 passengers on board?

(5) **(3 points)**Draw the flowchart that will simulate the bus arrival and passenger's drop-off to this station? (**Use the command *Generate RV from Dist.*--- to complete your flowchart**)

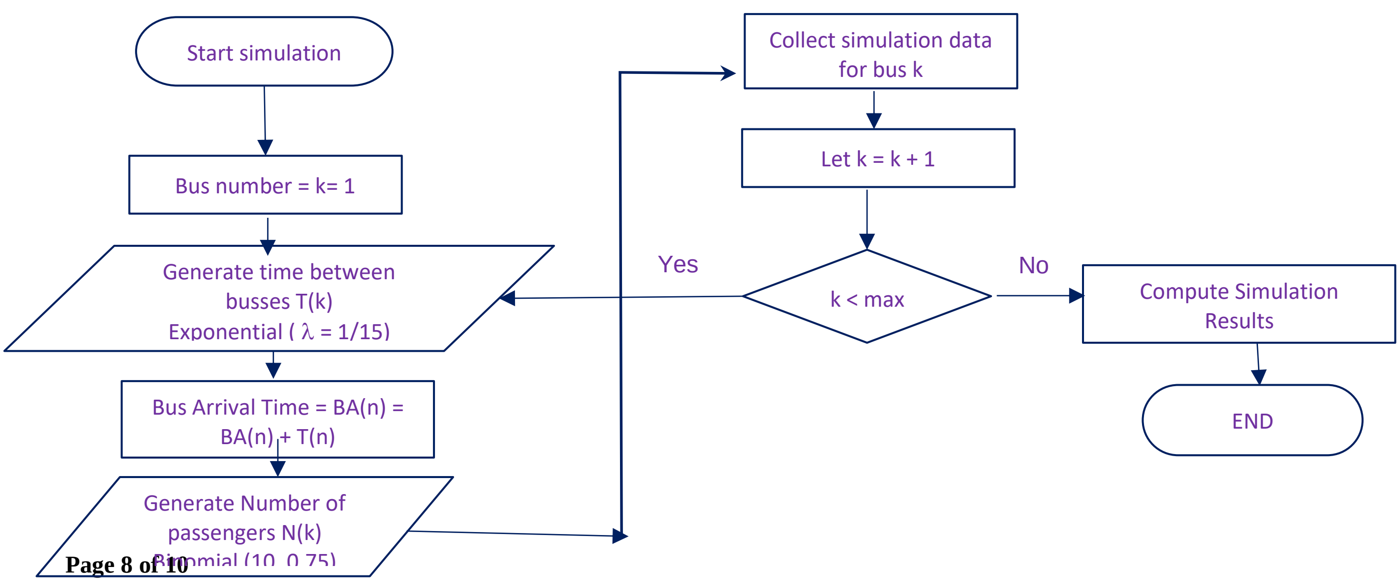
Let N: be number of busses arrived , X : be number of passengers on board the bus, T: time between busses

(1) P{wait more than 30 min for your bus| arrive at 10:00 am} = P{T>30} = e^{-2} = 0.13534

(2) E[busses that will arrive to the station between 10:00 to 11:00 am] = 1 hr (60/15) = 4 busses

(3) E[passengers that will drop off to the station between 10:00 to 11:00 am]
= E[number of busses arrive 10 to 11] * E[number of passengers on each bus]
= 4 (10 * 0.75) = 4 * 7.5 = 30 pasenger

(4) P{ next bus will have 3 passengers on board | 15 passengers arrived 10 to 11}
Busses are independent
P{ X = 3 } = C^{10}_{3} $(0.75)^3$ $(0.25)^7$ = 0.0031



Question #6: (12 points)

A supermarket sells fresh milk daily. Customers arrive to supermarket buy milk. Customers may buy 1 bottle, 2 bottle or 3 bottles at random (let B(j): number of bottles bought by customer j). The number of customers demanding the milk varies between 5 to 15 customers daily (let N(k): number customers arrived in day k asking for milk). The supermarket stores the milk in a refrigerator that can hold up to 20 bottles of milk. By the end of each day, the owner will decide wither to order more milk for next day or not. If he finds 10 or less in that the refrigerator then he will refill the refrigerator for next day.

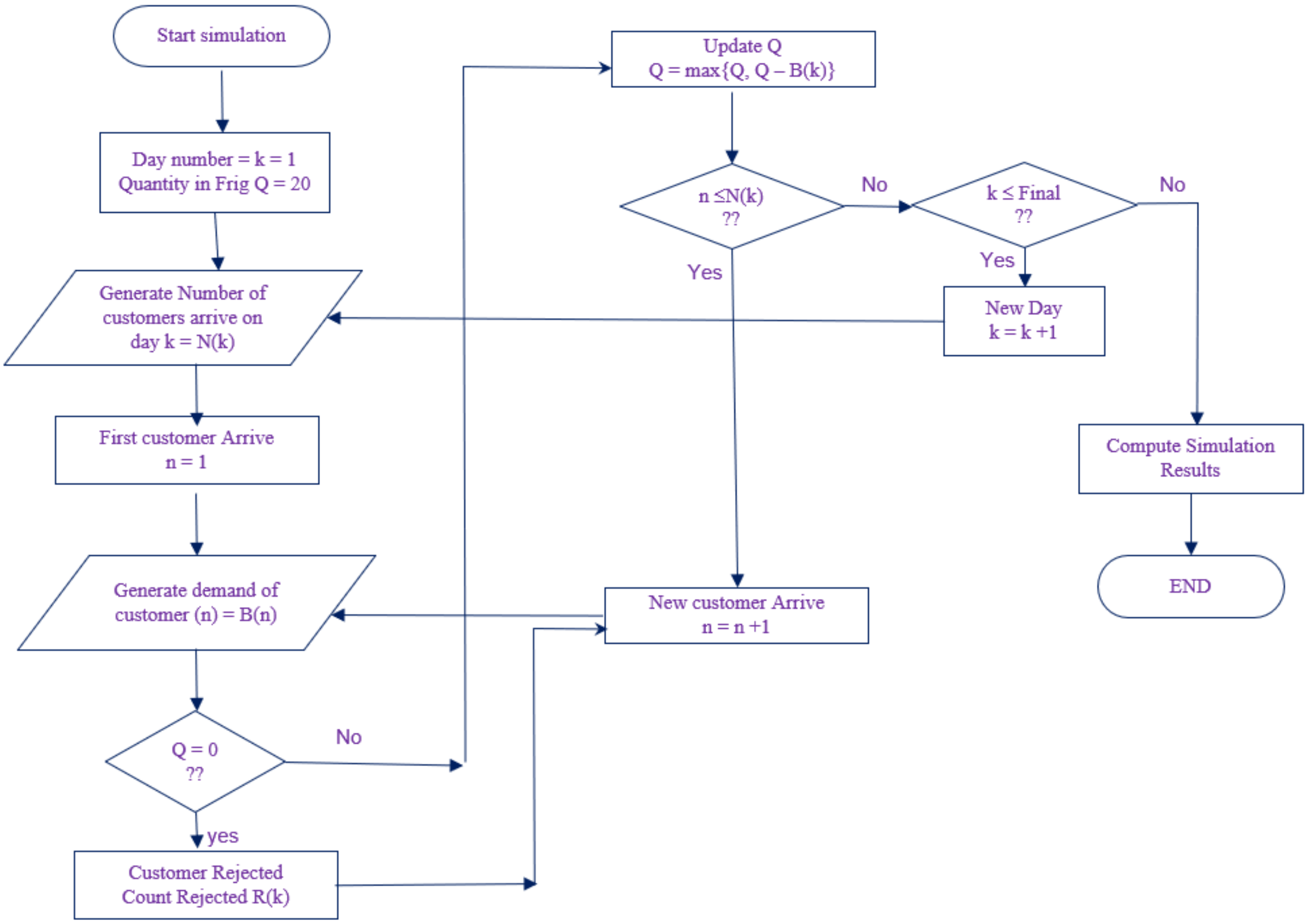
Day	Cust-1	Cust-2	Cust-3	Cust-4	Cust-5	Cust-6	Cust-7	Cust-8	Cust-9	Cust-10	Cust-11	Cust-12	Cust-13	Cust-14	Cust-15
1	1	1	2	1	3	1	1	3	3	1	2				
2	1	3	1	3	3	1	3	2	1	3	1	2	1	3	3
3	3	1	3	2	1	2	1	1	1						
4	2	3	1	2	3	3									
5	1	2	3	1	3	1	2	3							

(1) Draw the flowchart that will simulate number of sold bottles. (4 points)

(2) Do the manual simulation to determine the status of customer (accepted, rejected) and amount of milk in the frig. (4 points)

(3) From the manual simulation, estimate the number of sold units and the number of lost customers per day. (4 points)

	Day	Cust-1	Cust-2	Cust-3	Cust-4	Cust-5	Cust-6	Cust-7	Cust-8	Cust-9	Cust-10	Cust-11	Cust-12	Cust-13	Cust-14	Cust-15
	1	1	1	2	1	3	1	1	3	3	1	2				
	Q1	19	18	16	15	12	11	10	7	4	3	1				
	A/R	A	A	A	A	A	A	A	A	A	A	A				
	2	1	3	1	3	3	1	3	2	1	3	1	2	1	3	3
order	**Q1**	19	16	15	12	9	8	5	3	2	A	0	0	0	0	0
NQ=19	**A/R**	A	A	A	A	A	A	A	A	A	A	A	R	R	R	R
	3	3	1	3	2	1	2	1	1	1						
order	**Q1**	17	16	13	11	10	8	7	6	5						
NQ=20	**A/R**	A	A	A	A	A	A	A	A	A						
	4	2	3	1	2	3	3									
order	**Q1**	18	15	14	12	9	6									
NQ=15	**A/R**	A	A	A	A	A	A									
	5	1	2	3	1	3	1	2	3							
order	**Q1**	19	17	14	13	10	9	7	4							
NQ=14	**A/R**	A	A	A	A	A	A	A	A							


Start simulation
Day number = k = 1
Quantity in Frig Q = 20
Generate Number of customers arrive on day k = N(k)
First customer Arrive
n = 1
Generate demand of customer (n) = B(n)
Q = 0
??
No
yes
Customer Rejected
Count Rejected R(k)
Update Q
Q = max{Q, Q – B(k)}
n ≤N(k)
??
Yes
No
k ≤ Final
??
Yes
No
New Day
k = k +1
New customer Arrive
n = n +1
Compute Simulation Results
END