كلية العلوم
قسم الإحصاء وبحوث العمليات

College of Science.
Department of Statistics & Operations Research

Second Midterm Exam
Academic Year 1443-1444 Hijri-First Semester

Exam Information معلومات الامتحان			
Course name	**Modeling and Simulation**	**النمذجة والمحاكاة**	اسم المقرر
Course Code	**OPER 441**	**441 بحث**	رمز المقرر
Exam Date	**27-10-2021**	**2-4-1444**	تاريخ الامتحان
Exam Time	**1:00 pm**		وقت الامتحان
Exam Duration	**2.5 hours**	ساعتان ونصف	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of ☐ PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
-

تعليمات عامة:

- عدد صفحات الامتحان ☐ صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
-

هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	Understanding the processes and steps for building a simulation model			
2	Implement an inverse cumulative distribution function based random variate generation algorithm			
3	Explain and implement the convolution algorithm for random variate generation			
4	Explain and implement the acceptance rejection algorithm for random variate generation			
5	Compute statistical quantities from simulation output			
6	Generate random numbers from any given distribution discrete or continuous			
7	Building simulation models from basic applications			
8				

EXAM COVER PAGE

Question #1:

The period of time (in months) between rainfalls in Riyadh city is modeled using the following pdf:

$$f(x) = 1.06\, \boldsymbol{e}^{\frac{-x}{2}} \qquad ; \qquad 1 \le x \le 4$$

Where random variable X is time between rainfalls in months.

a) Write the inverse transform for measuring the time between rainfalls.
b) Simulate the rainfalls (in months) in Riyadh city using all number given below (move by rows)
c) From simulated date, compute the average rainfall in Riyadh per year.
d) Assume that the period of time of each rain fall (in hours) is a Binomial distribution that last for a maximum of 3 hours with parameter (p=0.75). Simulate the duration of rainfall using the numbers below (move by columns).

Move by rows →	0.744	0.443	0.820	0.166
	0.256	0.542	0.844	0.936
	0.744	0.444	0.017	0.967

Question #2:

Consider the continuous random Y with the following pdf:

$$f_Y(y) = \begin{cases} 0, & y < 0 \\ 0.2y, & 0 \le y \le 1 \\ 0.1 + 0.1y, & 1 < y \le 2 \\ 0.25 + 0.025y, & 2 < y \le 4 \\ 0, & y > 4. \end{cases}$$

a) Write the cumulative distribution function of $f_Y(y)$ and compute the expected value of Y?

b) Write the Inverse transform for $f_Y(y)$?

c) Write the algorithm for generating 10 random numbers from $f_Y(y)$.

d) Let Y be the time (in hourse) for surgery in an Operations Room (OR) in K.A.N Hospital. The hospital has one Operations Room. Patients are transferred to the OR according to a Poisson Process with average time between arrivals equals to 5 hours. The operations Room work 24 hours per day. Define the random processes for the simulation model for the OR and apply it for 5 patients. Use the following U[0,1] numbers as needed. Starting simulations time is zero.

Move by rows →	0.744	0.443	0.820	0.166
	0.256	0.542	0.844	0.936
	0.744	0.444	0.017	0.967

Question #3:

A car repair workshop manager wants to develop a simulation model. For one particular repair, the times to completion can be represented by the following distribution (x in days):

$$f(x) = \begin{cases} \dfrac{x}{8} - \dfrac{1}{4} & ; \quad 2 \le x \le 4 \\ \\ \dfrac{10}{24} - \dfrac{x}{24} & ; \quad 4 \le x \le 10 \end{cases}$$

(a) Write the inverse transform to generate random numbers for repair time.

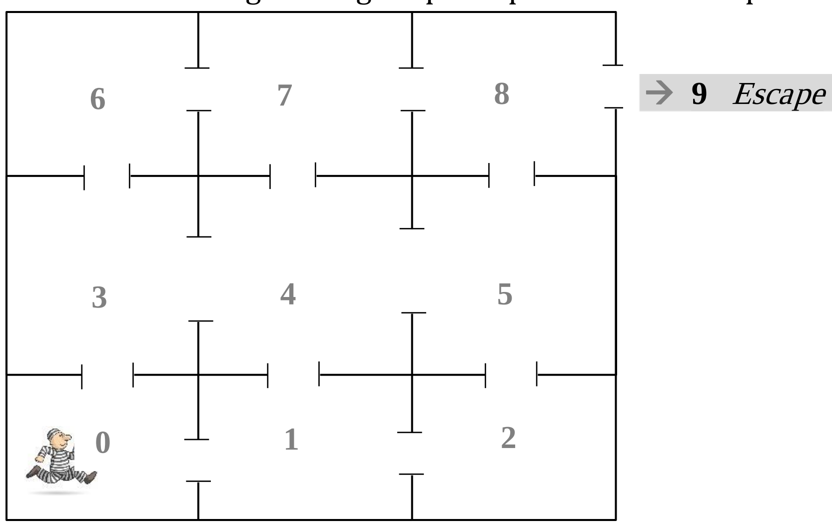
(b) Using U[0,1] random number in the following table, use the inverse transform in part (a) to determine the time of each car repair to compute the average speed of completion for this workshop (number of services completed per day). The workshop works daily from 8:00am to 8:00pm

(c) Write the algorithm for using the acceptance/rejection method to simulate random number from f(x). Use the same random numbers in the table to apply the algorithm.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
U[0,1]	0.138	0.776	0.911	0.259	0.458	0.343	0.105	0.940	0.188	0.343
Repair Time										

Question #4:

Consider an escaped prisoner who entered in a maze. The maze contains 8 chambers. If the prisoner enters any chamber he is equally likely to choose any door in the chamber (including the door he entered through). The prisoner has no time to waste, he has only 5 moves to escape out starting from chamber 0 before he gets caught up and put back into the prison.



a) Define all random processes and write the algorithm for generating the prisoner moves to escape.

b) Using your answer in (a), Simulate the path of the prisoner for 5 attempts in the following table

	Move-1	Move-2	Move-3	Move-4	Move-5	Move-6	Move-7	Move-8	Move-9	Move-10
Attept#1	0.338	0.765	0.976	0.107	0.154	0.684	0.901	0.715	0.013	0.228
Chambers										
Attept#2	0.849	0.145	0.081	0.308	0.543	0.959	0.113	0.381	0.492	0.158
Chambers										
Attept#3	0.731	0.487	0.396	0.144	0.982	0.180	0.631	0.802	0.712	0.507
Chambers										

c) From the simulation data, what is the estimate number of moves to escape.

Question #5:

Airplanes land on a small airport according to time between airplanes follows Erlang distribution with parameters r=3 and rate λ=5 airplanes per day. Also, the airplanes depart at random from the same airport according to Weibull distribution with parameter α=3 and β=0.5 for the time between air planes

$$f(x) = \frac{\alpha}{\beta^{\alpha}}\, x^{\alpha-1}\, e^{-\left(\frac{x}{\beta}\right)^{\alpha}} \qquad ; \qquad x \geq 0$$

Assume that the airport works 18 hours.

1. Give a random number for total number of air planes landed in the airport on one working day using the following U[0,1] numbers ***as needed***. *(Answer on the back of the page)*

n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
$U_n(0,1)$	0.171	0.023	0.879	0.305	0.696	0.415	0.721	0.901	0.344	0.051

2. Give a random generation for the time of ***the last airplane departed*** from the airport on one day using the following U[0,1] numbers ***as needed***.

n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
$U_n(0,1)$	0.815	0.636	0.563	0.923	0.295	0.605	0.971	0.023	0.879	0.305

3. Assume that the percentage of departing airplanes from the airport is 44.5%. Make a discrete event simulation run of the airport for 12 hours. Write the simulation algorithm for this system and use it with the following U[0,1] as needed. *(Answer on the back of the page)*

Event	U[0,1]		U[0,1]		U[0,1]		U[0,1]	
1	0.248		0.817		0.132		0.214	
2	0.968		0.465		0.668		0.482	
4	0.876		0.860		0.694		0.732	
5	0.639		0.002		0.546		0.695	
6	0.035		0.243		0.321		0.328	
7	0.174		0.416		0.923		0.455	
8	0.439		0.280		0.432		0.255	
9	0.815		0.522		0.104		0.377	
10	0.199		0.479		0.963		0.420	