College of Science.
Department of Statistics & Operations Research

كلية العلوم
قسم الإحصاء وبحوث العمليات

Final Exam
Academic Year 1443 – 1444 Hijri – First Semester

Exam Information معلومات الامتحان			
Course name	Modeling and Simulation	النمذجة والمحاكاة	اسم المقرر
Course Code	OPER 441	441 بحث	رمز المقرر
Exam Date	13/11/2022	19/4/1444	تاريخ الامتحان
Exam Time	Sunday 8:00 am – 11:00 am		وقت الامتحان
Exam Duration	3 hours	ثلاث ساعات	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of ☐ PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان ☐ صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	To know the basics of pseudo random generation and apply different methods of random generation techniques			
2	Chose and fit theoretical distribution on collected data			
3	Generate random variates from different probability functions and directly from collected data			
4	build simple simulation models of real-life problems			
5	Define and compute performance measures from simulation models			
6	Recognize and analyze simple models and its main elements for simulation			
7	Understanding how to use computer software (ECXEL) for simulation models			
8	use appropriate statistical techniques to analyze and evaluate outputs of simulation models			

Q#	Q#1	Q#2	Q#3	Q#4	Q#5	Q#6	Total
Marks							
Score							

Question #1: Consider the following are functions in EXCEL.

A	RANDBETWEEN(a,b)	**E**	GAMMA.DIST(a,b,c,d)	**I**	RAND()
B	NORM.DIST(a,b,c,d)	**F**	DATA TABLE	**J**	BINOM.INV(a,b,c)
C	GAMMA.INV(a,b,c,d)	**G**	BINOM.DIST(a,b,c)	**K**	CONFIDENCE.T(a,b,c)
D	VLOOKUP(a,b,c)	**H**	NORM.INV(a,b,c)		

Read each statement and assign the letter of EXCEL function from the table to the statement.

The Statement	Excel Function
1. Used to compute the PDF function of the Normal distribution with $\mu=a$ and $\sigma=b$ at the value a	
2. Used to generates random values from relative frequency table.	
3. Used generate integer random values with equal distributions between (a) and (b) including the (a) and (b)	
4. Used to generate random values from Exponential distribution	
5. Used to compute the upper limit of the confidence interval from sample	
6. Used generate integer random values with equal distributions between (a) and (b) without the (a) and (b)	
7. Used to generate random values from Erlang distribution	
8. Used to compute the PDF function of the Gamma distribution at the value a	
9. Used to compute the CDF function of the Normal distribution with $\mu=a$ and $\sigma=b$ at the value a	
10. Used to Generates random values from Gamma distribution	
11. Used to generate random values from of integer values (a) to (b) with different probabilities	
12. Used to compute the CDF function of the Gamma distribution at the value a	
13. Used to generate continuous random values between a and b with uniform distribution	
14. Used to compute the PMF function of the Binomial distribution with value a	
15. Used to run the simulation model for many times and record the measures from every run	
16. Used to compute the CDF function of the Binomial distribution with at the value a	
17. Used to compute the CDF function of the Exponential distribution at the value a	
18. Used to compute the CDF function of the Erlang distribution at the value a	
19. Used to Generates random values from normal distribution with mean b and standard deviation c	
20. Used to generate continuous random values between 0 and 1 with uniform distribution	
21. Used to compute $Z_{\alpha/2}$	
22. Used to compute the lower limit of the confidence interval from sample	
23. Used to generate random values from binomial distribution	
24. Used to compute the half width of the confidence interval from sample	

Question #2: Answer the following with *True* or *False*:

	1. To apply Chi-Square Test for a sample of random numbers between 0 and 1 we must sort data from smallest to largest
	2. LCG has full period if and only if number of random numbers equal $(m-1)$.
	3. The LCG ($X_n = (aX_{n-1} + c) \bmod(m)$) it is possible to find values for: X_0 , a , c and m to generate more than m different random numbers.
	4. The sequence of random numbers generated from a given seed is called a random number a Stream.
	5. The K-S test use the empirical distribution which means that $\Pr\{X \le x_{(k)}\} = k/n$, for n is the sample size.
	6. The Geometric distribution has a closed form CDF and can be used for generating random numbers from Geometric.
	7. To apply the inverse transform to the continuous distribution *g(x),* we must find a CDF for the function *G(x)*
	8. We can use the inverse transform for any continuous random variable Y.
	9. If the random variable X is discrete then there is no inverse transform for this variable.
	10. To simulate the arrival time of Event (3) in Poisson process, we generate one random number from Exponential and multiply it by 3.
	11. To simulate the number of events in (3) hours in Poisson process, we generate three independent random numbers from Exponential and sum them together.
	12. If the time between events is exponential distribution with mean α then the time for 1st event to happen is $AT_1 = -\alpha \ln(1-u)$ where $u \sim U[0,1]$
	13. It is always possible to use inverse transform directly to generate random numbers from Normal(μ,σ^2).
	14. For Bernoulli distribution with Success probability p, we get one uniform random numbers from $u \sim U(0,1)$. If $u > p$ then we have Success
	15. The inverse transform for the random variable is obtained taking the pdf *f(x)* for the random variable X and solving the function $f(x) = u$; $u \sim U[0,1]$.
	16. To check for LCG full period, we always must satisfy all three conditions.
	17. Some LCG functions can have FULL period by satisfying any two conditions from three conditions on a, c, m.
	18. In Acceptance/Rejection, if $g(W)/f(W) \ge u_2$ then $W \sim f(x)$
	19. For K-S test the interval [0,1] must be divided in to ***k*** equal subintervals.
	20. Acceptance/Rejection method can be used for any distribution discrete or continuous.
	21. To apply Acceptance/Rejection on pdf $f(x)$, we must choose a majorizing function $g(x)$ that satisfy that $g(x) \ge f(x)\ \forall x$.
	22. In Acceptance/Rejection method for pdf $f(x)$, the majorizing function $g(x)$ is always a *pdf* function.
	23. The inverse transform for triangular distribution, we must get $u_1 \in U[0,1]$ to decide which part of the function to use and get $u_2 \in U[0,1]$ to compute the random variable.

	24. To generate random numbers from U[0,1] in Excel we use the function RANDBETWEEN().
	25. To generate random numbers for $X \in \{1,2,3\}$ with probabilities {0.3,0.3, 0.4} we can use RANDBETWEEN(1,3) directly in EXCEL.
	26. CONFIDENCE in EXCEL is a function used to compute the halfwidth of the mean value given α, standard deviation and the number of simulation runs.
	27. Binomial with parameters (n,p) can be simulated using convolution method using n independent Bernoulli random variables each with parameter p.
	28. It is always possible to simulate Weibull distribution using acceptance/rejection method.
	29. Erlang distribution parameter r = 3 and rate λ = 3.5 could be simulated using convolution method using 3 independent Exponential random variables each with parameter λ.
	30. Gamma distribution parameter α = 3 and rate λ = 3.5 could be simulated using convolution method using 3 independent Erlang random variables each with parameter λ.
	31. The random variable with Exponential distribution is always has mean value equals to the standard deviation.
	32. If the random variable has mean value equals to the variance then it must have a Poisson distribution.
	33. The Erlang distribution is a special case from exponential distribution.
	34. We can always get any Erlang distribution with any parameters using Gamma Distribution
	35. The normal distribution always has the mean equals to the median
	36. In building simulation model, we always have to start data collection after validation.
	37. The beta distribution between (0,1) can be rescaled for any real values
	38. The Erlang distribution always has all parameters positive integer values.
	39. The number of trials until 1st 2 success is a binomial distribution.
	40. The Uniform distribution has a single mode value.

Question # 3:

Consider a random process (T>0) that represents the time intervals between hits on a web-page from remote computers. Let (T) be exponentially distributed, with a mean of 30 seconds. Answer each of the following separately:

1. Find the probability that the first hit occurs between 30 seconds and 60 seconds.
2. Find the expected number of hits during the first 15 minutes.
3. Write the function to generate random numbers for the time intervals between hits (T).
4. Use your answer in (3) and the following table to generate T between the **first 3 hits.**

u~U[0,1]	0.1466	0.7061	0.8585	0.4944
u~U[0,1]	0.5915	0.1998	0.9780	0.6281

5. Assume that the time intervals between hits (T) follows the integer uniform between 5 seconds and 30 seconds. Write the function to generate random numbers for the time intervals between hits (T).
6. Use your answer in (5) and the following table to generate (T) between the **first 3 hits**.

u~U[0,1]	0.176	0.429	0.131	0.830
u~U[0,1]	0.471	0.079	0.221	0.591

Question # 4:

Consider a random process (Y>0) that represents the time intervals between accidents in an intersection. Let (Y) be exponentially distributed, with rate 3 accidents per week. Answer each of the following separately:

1. Find the probability that there will more than 4 accidents in a given week?
2. Given that no accidents happened in first 5 days of the week, what is the probability that this week will end without any accidents.
3. Write the function to generate random numbers for the time intervals between accidents (Y weeks).
4. Use your answer in (3) and the following table to generate Y for the **first 3 accidents**.

u~U[0,1]	0.1466	0.7061	0.8585	0.4944
u~U[0,1]	0.5915	0.1998	0.9780	0.6281

5. Assume that the time intervals between accidents (Y weeks) follows a truncated exponential distribution between 0.25 week and 2 weeks. Write the function to generate random numbers for the time intervals between accidents (Y weeks).
6. Use your answer in (5) and the following table to generate Y between the **first 3 accidents**.

u~U[0,1]	0.176	0.429	0.131	0.830
u~U[0,1]	0.471	0.079	0.221	0.591

Question #5:

Consider an investment with a monthly return on investment. The monthly percentage of return on investment is a random variable X % given by the following probability function:

$$f(x) = \begin{cases} \frac{-x}{10} & ; -2 \le x \le 0 \\ \frac{x}{10} & ; 0 \le x \le 4 \end{cases}$$

Answer the following:

1) Write the inverse algorithm for generating the monthly percentage of return on investment.
2) Using simulation and the following U[0,1] numbers, evaluate the results of the investment for 6 months with initial budget of 100,000 SR.

Mon.	U				
1	0.032				
2	0.138				
3	0.623				
4	0.776				
5	0.301				
6	0.691				

3) From simulation, compute the average and standard deviation of the monthly percentage of return on investment and compute the 95% confidence interval.
4) Compute the From simulation compute the probability that the company will have profit more than 10000 SR per month.
5) From simulation output, estimate the probability of losing.

Question # 6:

Consider a car insurance company that receives claims daily as a random integer value. The company evaluates the total claims in a weekly basis. It is known that the total number of claims (TC) per week is either one of the following values: TC= 100, 200, 300, 400, 500, 600. It is assumed the total number of claims (TC) per week is a Binomial distribution with parameter p=0.65 with TC$\in$\{100, 200, 300, 400, 500, 600\}. It is also assumed that the age (in integer years) of clients (AG) is a random variable that could be in the following age groups: 30 yrs, 35 yrs, 40 yrs, 45 yrs, 50 yrs, 55 yrs, or 60 yrs. The probability distribution that a client's age falls in any of the age groups is also a Binomial distribution with parameter q=0.45

1. Write the inverse transform function and algorithm for generating the weekly number of claims.
2. Write the inverse transform function and algorithm for generating the age group of the client.
3. Assume that the company got 200 claims in a given week. What is the expected number of clients there will be in each age group?
4. Do simulation the results for 4 weeks to give the following table (*use uniform number as needed*):

Week #	number of claims (TC)	number clients in age 30	number clients in age 35	number clients in age 40	number clients in age 45	number clients in age 50	number clients in age 55	number clients in age 60
u~U[0,1]	0.164	0.092	0.588	0.260	0.608	0.094	0.722	0.260
1								
u~U[0,1]	0.938	0.412	0.879	0.138	0.018	0.371	0.247	0.138
2								
u~U[0,1]	0.129	0.874	0.841	0.308	0.165	0.136	0.368	0.308
3								
u~U[0,1]	0.130	0.371	0.220	0.090	0.927	0.680	0.032	0.090
4								

Question #7:

A firm produces LCD screens for the consumer market. Their profit function is:

Profit = (unit price - unit cost) × (quantity sold) – fixed costs

Suppose that the unit price is $200 per LDC screen, and that the other variables have the following probability distributions:

Unit Cost	80	90	100	110
Probability	0.20	0.40	0.30	0.10

Quantity Sold	1000	2000	3000
Probability	0.10	0.60	0.30

Fixed Cost	50000	65000	80000
Probability	0.40	0.30	0.30

(a) Write the flow chart and required random processes for simulation of Profit Function. (***on the back of the page***)

(b) Using the following random U[0,1], do simulation for Profit Function for 10 months.

(c) Estimate the mean profit from your sample and compute a 99% confidence interval for the mean profit

(d) Estimate the probability that the profit will exceed 200,000 $.

mon #	u_1 ~ **U[0,1]**	u_2 ~ **U[0,1]**	u_3 ~ **U[0,1]**	u_4 ~ **U[0,1]**					
1	0.150	0.130	0.176	0.614					
2	0.339	0.180	0.453	0.301					
3	0.220	0.306	0.484	0.139					
4	0.516	0.603	0.949	0.666					
5	0.188	0.213	0.504	0.324					
6	0.804	0.755	0.465	0.237					
7	0.795	0.347	0.548	0.072					
8	0.918	0.355	0.206	0.118					
9	0.742	0.050	0.873	0.463					
10	0.385	0.196	0.517	0.011					

NOTE: Use u_1, u_2, u_3, u_4, as needed for each bus.