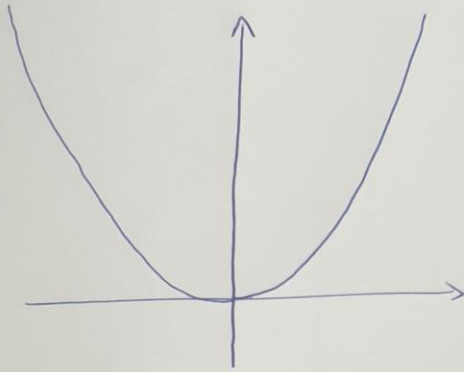
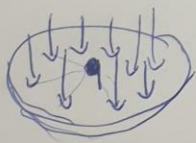
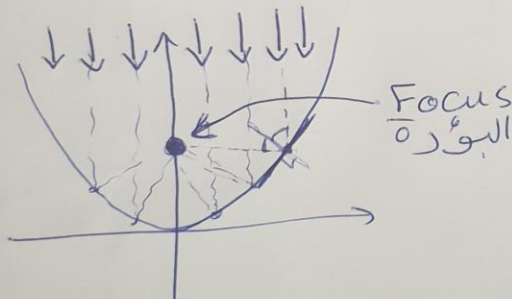


القَطْعُ العِزْوِيَّةُ Conic Sections

1 - Parabola القَطْعُ المِكَافِيءُ

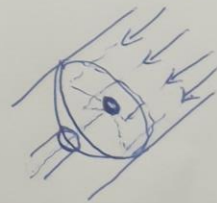


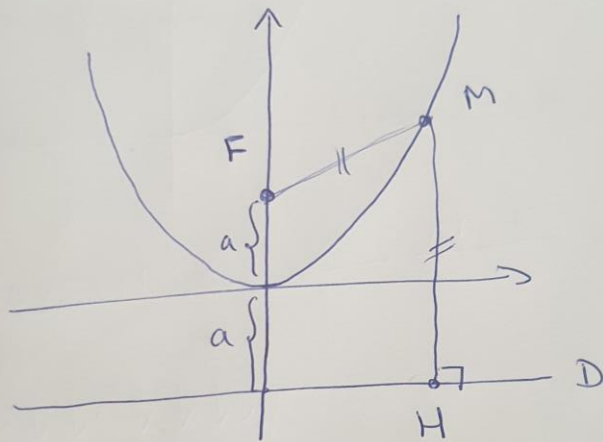
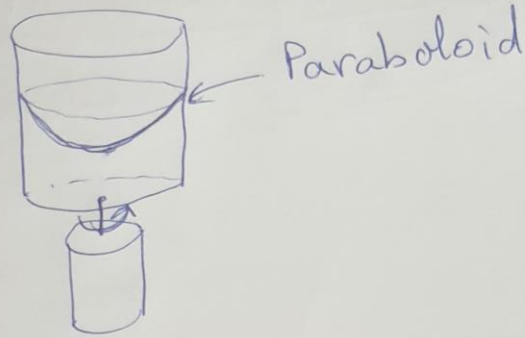
① Geometric properties:



The dish

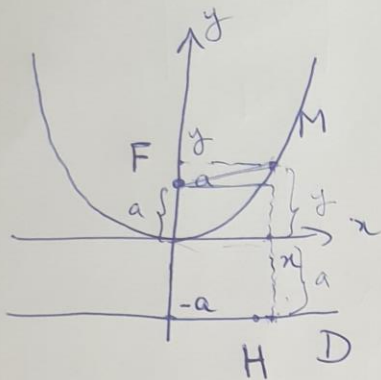
The telescope





The parabola of focus F and القطب F and directrix D is the curve which المنحنى satisfies $MF = MH$.

② Equation of a parabola:



$$F(0, a)$$

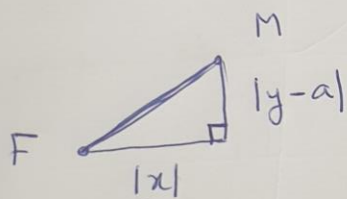
$$D: y = -a$$

$$M(x, y)$$

$$H(x, -a)$$

We have $MH = y + a$

$$MF^2 = x^2 + (y - a)^2$$



We solve:

$$MF = MH$$

$$\Leftrightarrow MF^2 = MH^2 \Leftrightarrow x^2 + (y - a)^2 = (y + a)^2$$

$$\Leftrightarrow x^2 + y^2 - 2ay + a^2 = y^2 + 2ay + a^2$$

$$\Leftrightarrow \boxed{x^2 = 4ay}$$

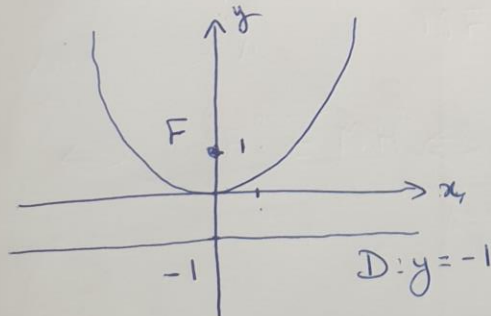
The parabola of focus $F(0, a)$ and directrix $D: y = -a$ has the equation $x^2 = 4ay$

Examples:

Give the equation of the parabola of focus $F(0, 1)$ and directrix $D: y = -1$ and sketch it.

Answer: Here $a = 1$

The equation is $x^2 = 4y$

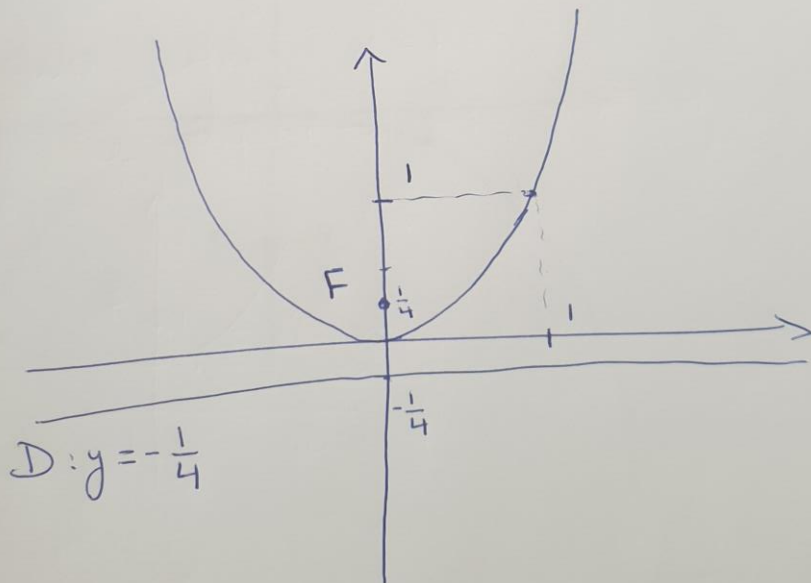


② Find the focus and the directrix
Give
of the parabola of equation

$$x^2 = y$$

and sketch it

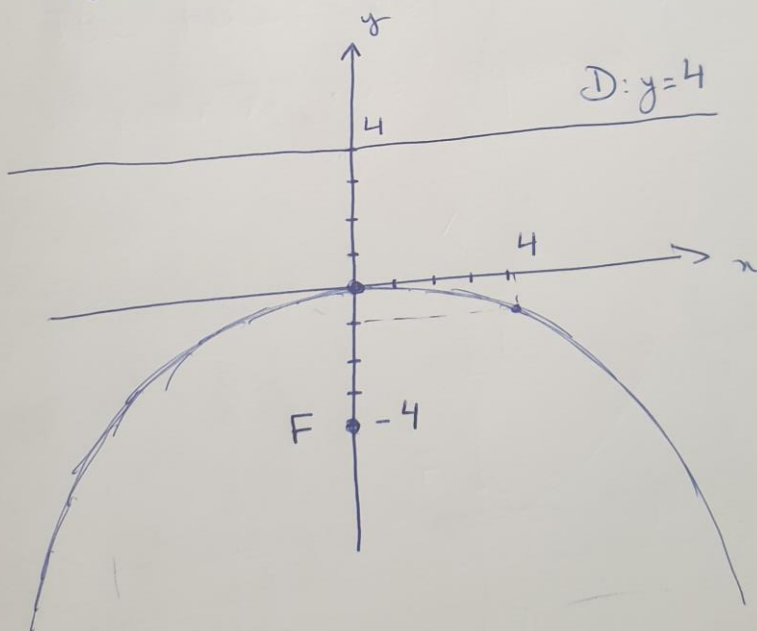
Answer: Here $4a = 1 \Leftrightarrow a = \frac{1}{4}$
The focus is $F(0, \frac{1}{4})$ and the
directrix has equation $D: y = -\frac{1}{4}$



③ The same question for $F(0, -4)$
and $D: y = 4$

- ① The equation.
- ② Sketch the graph.

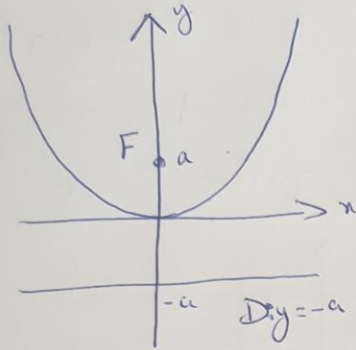
Answer: Here $a = -4$
So the equation is: $x^2 = -16y$
We sketch it:



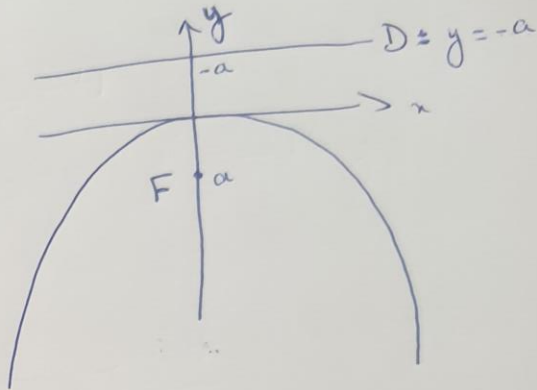
General rule:

$$x^2 = 4ay$$

$$a > 0$$



$$a < 0$$



Examples:

① $x^2 = \frac{1}{2}y$

← $a = \frac{\frac{1}{2}}{4} = \frac{1}{8}$

② $x^2 = -\frac{1}{3}y$

← $a = \frac{-\frac{1}{3}}{4} = -\frac{1}{12}$