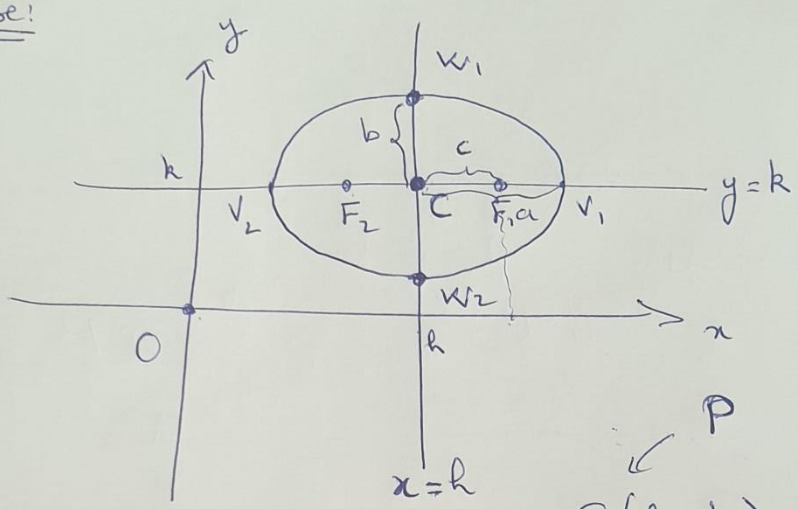
When the center is not the origin

1st case:



When the center is $C(h, k)$

The equation becomes:

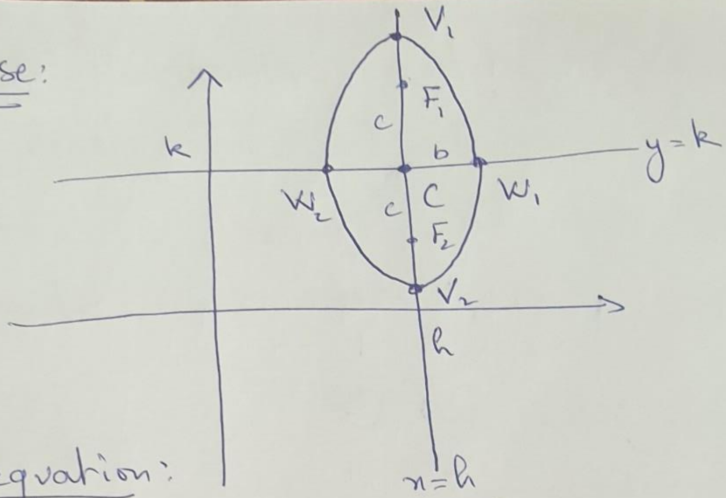
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \qquad c^2 = a^2 - b^2$$

1. The focci $F_1(h+c, k)$
   $F_2(h-c, k)$

2. The vertices $V_1(h+a, k)$ | $W_1(h, k+b)$
   $V_2(h-a, k)$ | $W_2(h, k-b)$

3. The axes: The major axis: $y=k$
   The minor axis: $x=h$

2^{nd} case:



The equation:

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \qquad a>b>0$$

$$c^2 = a^2 - b^2$$

The elements:

1. The center $C(h, k)$
2. The focci $F_1(h, k+c)$
   $F_2(h, k-c)$
3. The vertices $V_1(h, k+a)$ | $W_1(h+b, k)$
   $V_2(h, k-a)$ | $W_2(h-b, k)$

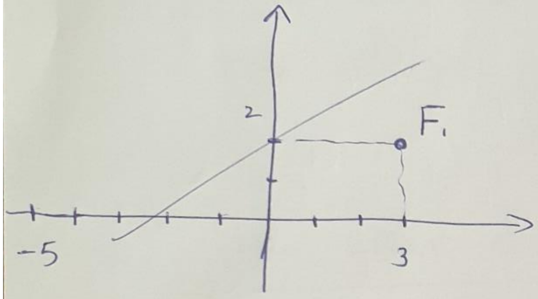
2. The axes: The major axis : $x=h$
   The minor axis: $y=k$

Example: Give the equation of the ellipse of focci $F_1(3,2)$; $F_2(-5;2)$ and vertex $V_1(4,2)$, all the other elements and sketch it.

Solution:

① The center $C(-1,2)$

$\frac{3+(-5)}{2} = -1$



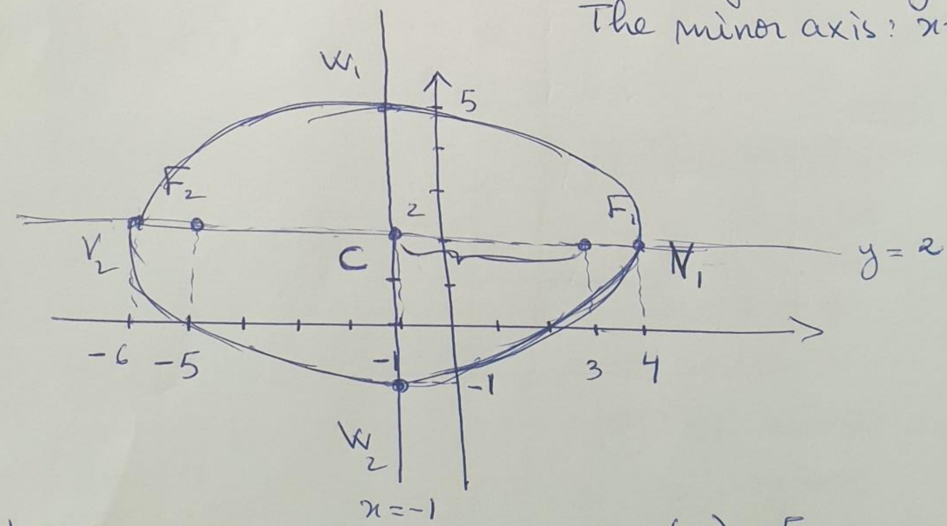
② The vertices

$V_1(4,2)$	$W_1(-1,5)$
$V_2(-6,2)$	$W_2(-1,-1)$

③ The axes:

The major axis: $y=2$

The minor axis: $x=-1$



b? $c = 3-(-1) = 4$; $a = 4-(-1) = 5$

$\Rightarrow b = \sqrt{5^2 - 4^2} = 3$

The equation is:

$$\frac{(x+1)^2}{5^2} + \frac{(y-2)^2}{3^2} = 1$$

Example: Give the equation of the ellipse of vertices $V_1(-1;6)$; $V_2(-1;-2)$; $F_1(-1;4)$; the other elements and sketch it

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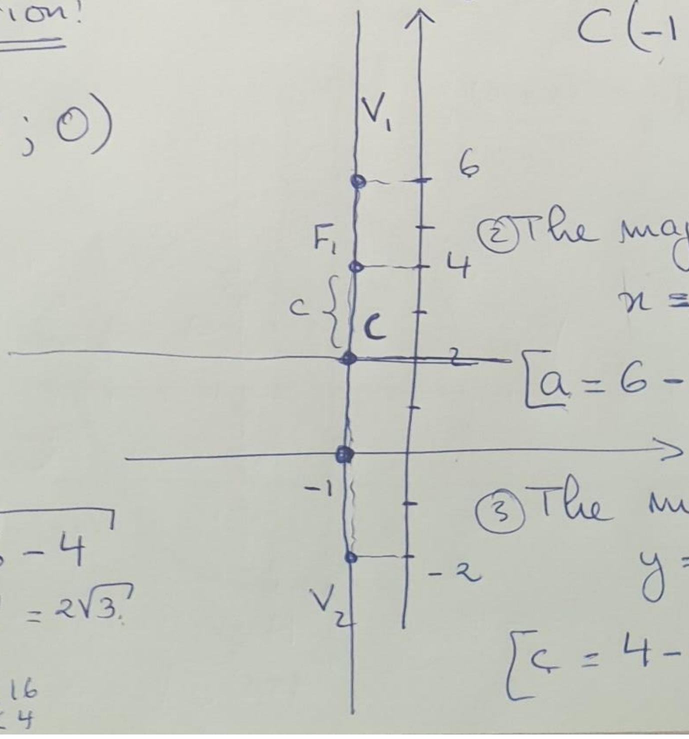
Solution:

① The center: $C(-1;2)$

$$\uparrow \quad \frac{6+(-2)}{2}$$

$F_2(-1;0)$



② The major axis: $x = -1$

$y = 2$

$\boxed{a = 6-2 = 4}$

③ The minor axis: $y = 2$

$b = \sqrt{16-4}$

$= \sqrt{12} = 2\sqrt{3}$

4×3

$[c = 4-2 = 2]$

$9 < 12 < 16$

$3 < \sqrt{12} < 4$

The elements:

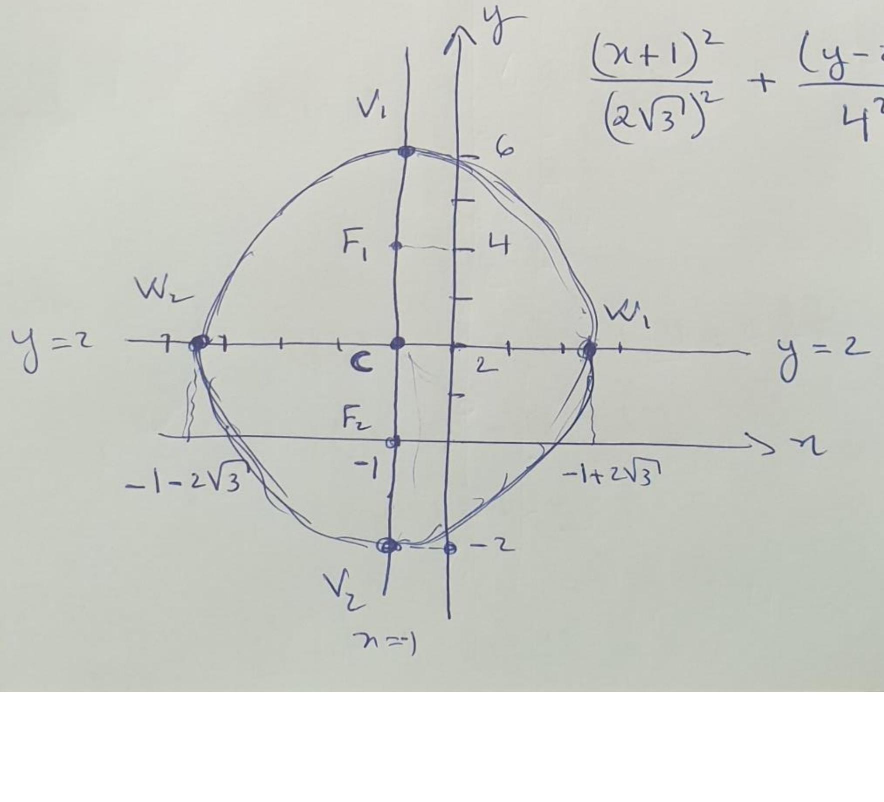
① The center $C(-1, 2)$

② The focci $F_1(-1, 4)$
$F_2(-1, 0)$

③ The vertices $V_1(-1, 6)$ | $W_1(-1+2\sqrt{3}, 2)$
$V_2(-1; -2)$ | $W_2(-1-2\sqrt{3}, 2)$

④ The axes:
The major axis: $x = -1$
The minor axis: $y = 2$

$$\frac{(x+1)^2}{(2\sqrt{3})^2} + \frac{(y-2)^2}{4^2} = 1$$



Example : Find the elements of the ellipse of equation

$$4x^2 + y^2 + 8x + 4y - 28 = 0$$

and sketch it.

Solution :

By completing the square :

$\to$ $$4x^2 + 8x = 4(x^2 + 2x)$$
$$= 4(x^2 + 2\times 1\, x + 1^2 - 1^2)$$
$$= 4\left[(x+1)^2 - 1\right]$$
$$= 4(x+1)^2 - 4 .$$

$\to$ $$y^2 + 4y = y^2 + 2\times 2y + 2^2 - 2^2$$
$$= (y+2)^2 - 4$$

The equation becomes :

$$0 = 4x^2 + y^2 + 8x + 4y - 28$$
$$\Leftrightarrow \quad 0 = 4(x+1)^2 - 4 + (y+2)^2 - 4 - 28$$
$$\Leftrightarrow \quad 0 = 4(x+1)^2 + (y+2)^2 - 36$$
$$\Leftrightarrow \quad 4(x+1)^2 + (y+2)^2 = 36$$
$$\Leftrightarrow \quad \frac{(x+1)^2}{9} + \frac{(y+2)^2}{36} = 1$$

$$\Leftrightarrow \boxed{\frac{(x+1)^2}{3^2} + \frac{(y+2)^2}{6^2} = 1}$$

$a = 6$; $b = 3$; $c = \sqrt{36-9} = \sqrt{27}$

$3 \times 3 \times 3$

$= 3\sqrt{3}$

$25 < 27 < 36$

$5 < \sqrt{27} < 6$

The elements:

1. The center

$C(-1 ; -2)$

2. The focci

$F_1(-1 ; -2 + 3\sqrt{3})$

$F_2(-1 ; -2 - 3\sqrt{3})$

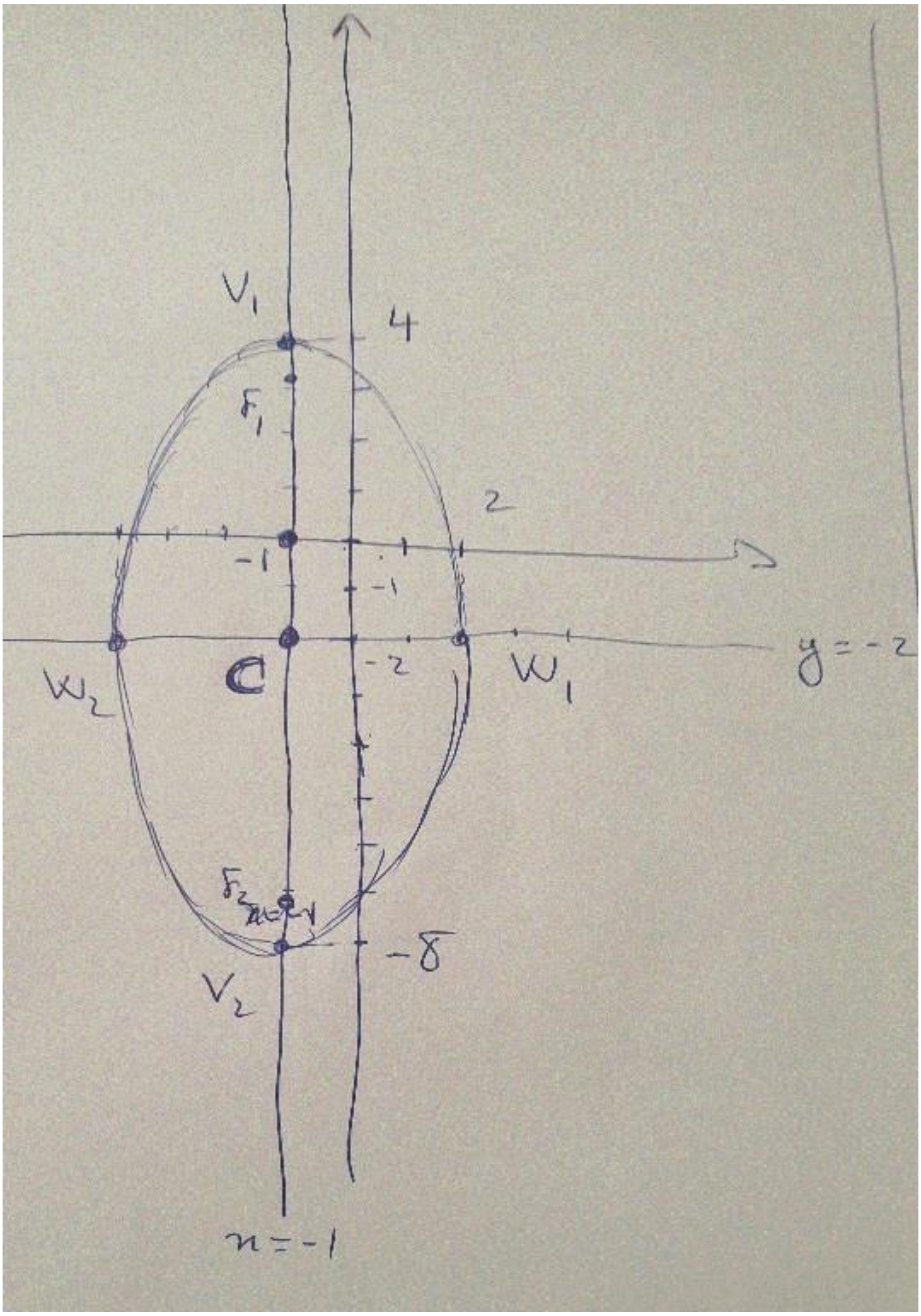
3. The vertices:

$V_1(-1, 4)$	$W_1(2, -2)$
$V_2(-1, -8)$	$W_2(-4, -2)$

4. The axes:

The major axis: $x = -1$

The minor axis: $y = -2$


V1
4
F1
2
-1
-1
W2
C
-2
W1
y = -2
F2
-8
V2
x = -1