

Example: Find the elements of the ellipse of equation $4x^2 + 9y^2 - 36y = 0$ and sketch it.

Solution:

① Completing the square:

We have:

$$\begin{aligned} 9y^2 - 36y &= 9(y^2 - 4y) \\ &= 9(y^2 - 2 \times 2y + 2^2 - 2^2) \\ &= 9[(y-2)^2 - 4] \\ &= 9(y-2)^2 - 36 \end{aligned}$$

The equation becomes:

$$4x^2 + 9(y-2)^2 - 36 = 0$$

$$\Leftrightarrow 4x^2 + 9(y-2)^2 = 36$$

$$\Leftrightarrow \frac{x^2}{9} + \frac{(y-2)^2}{4} = 1$$

$$\Leftrightarrow \boxed{\frac{x^2}{3^2} + \frac{(y-2)^2}{2^2} = 1}$$

$$a = 3 ; b = 2 ; c = \sqrt{9-4} = \sqrt{5}$$

$$\begin{aligned} 4 &< 5 < 9 \\ 2 &< \sqrt{5} < 3 \end{aligned}$$

The elements:

① The center $C(0; 2)$

② The foci $F_1(\sqrt{5}, 2)$

$F_2(-\sqrt{5}, 2)$

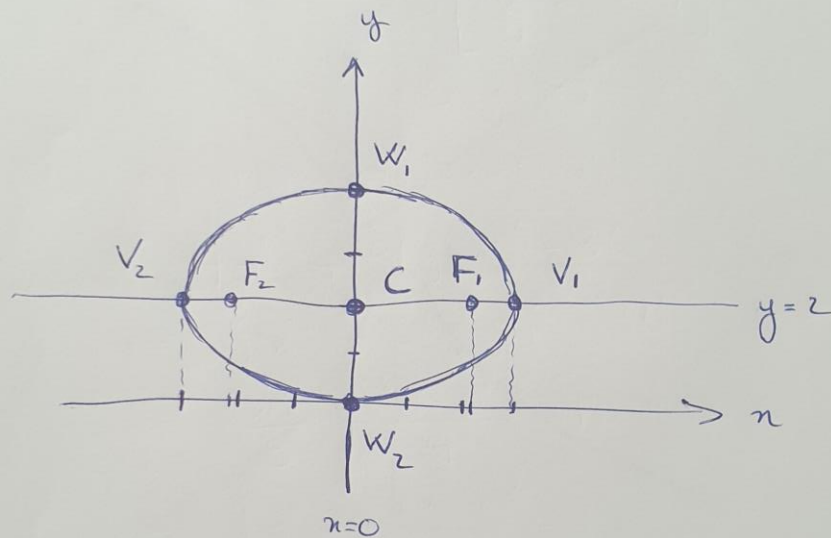
③ The vertices $V_1(3, 2)$ | $W_1(0, 4)$

$V_2(-3, 2)$ | $W_2(0, 0)$

④ The axes :

The major axis : $y = 2$

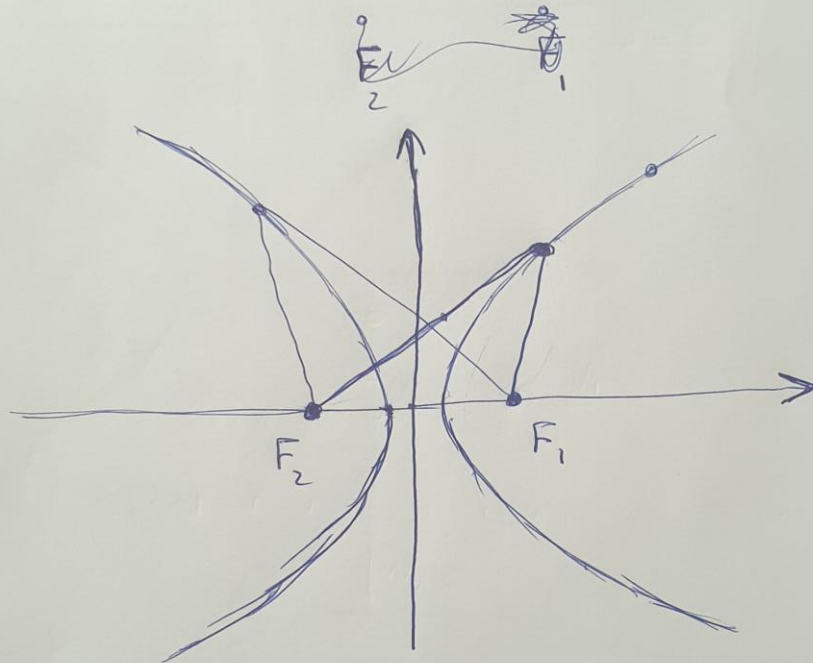
The minor axis : $x = 0$



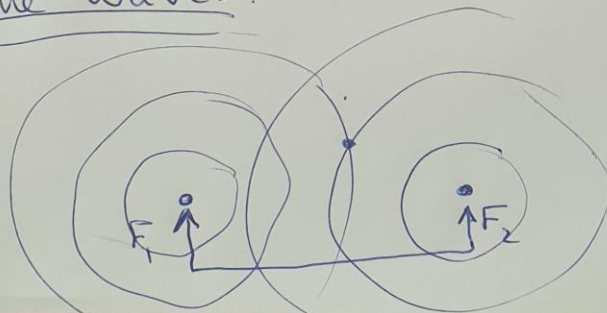
The hyperbola القطع الزائد

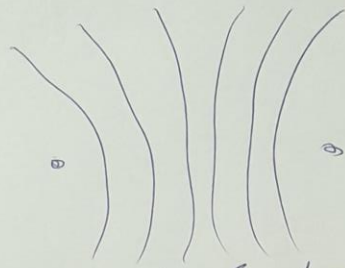
Definition: F_1, F_2 : The foci البؤرتان
 $a > 0$

The hyperbola of foci F_1, F_2 is defined
by the set of points M such that
 $|MF_1 - MF_2| = 2a$

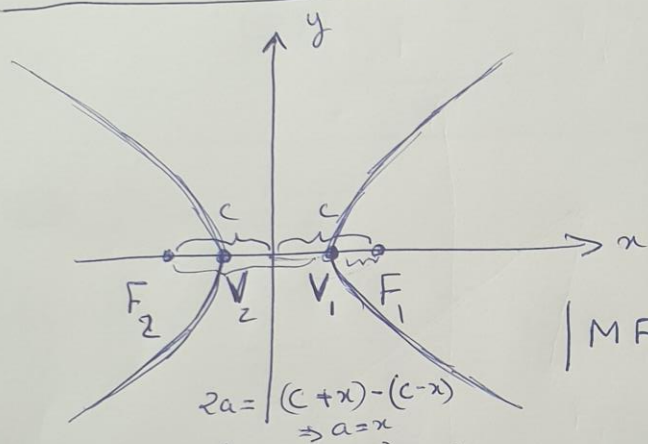


In the waves!





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الهواء



$F_1(c, 0)$
 $F_2(-c, 0)$
 $0 < a < c$

$$|MF_1 - MF_2| = 2a$$

$$2a = (c+x) - (c-x)$$

$$\Rightarrow a = x$$

The equation is :

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

We have the relation: $c^2 = a^2 + b^2$

The vertices: $V_1(a, 0)$
 $V_2(-a, 0)$

Asymptotes:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$

$$\Leftrightarrow \frac{x^2}{a^2} = \frac{y^2}{b^2}$$

$$\Leftrightarrow \frac{y}{b} = \pm \frac{x}{a}$$

$$\Leftrightarrow \boxed{y = \pm \frac{b}{a} x} \leftarrow \text{lines.}$$

How to draw!

The hyperbola: $\frac{x^2}{3^2} - \frac{y^2}{4^2} = 1$

$$a=3; b=4; c=\sqrt{3^2+4^2}=5$$

The foci: $F_1(5, 0); F_2(-5, 0)$

The vertices: $V_1(3, 0); V_2(-3, 0)$

The asymptotes: $y = \frac{4}{3}x$ (1st) ; $y = -\frac{4}{3}x$ (2nd)

The axis: $y = 0$

