

Determinants المحددات

For any square matrix we associate a number called the determinant.

We denote: $\det A$; $|A|$

How to compute this number:

For a 2×2 matrix:

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} ; \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Examples:

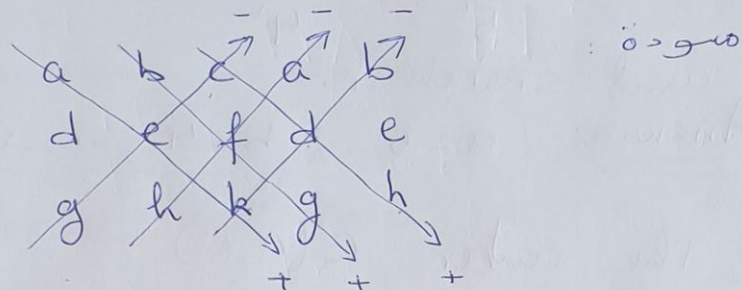
$$\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 1 \times 1 - 3 \times 2 = -5$$

$$\begin{vmatrix} 4 & 3 \\ 3 & 2 \end{vmatrix} = 4 \times 2 - 3 \times 3 = -1$$

$$\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = 0 \times 0 - (-1) \times 1 = 1$$

For a 3×3 matrix:

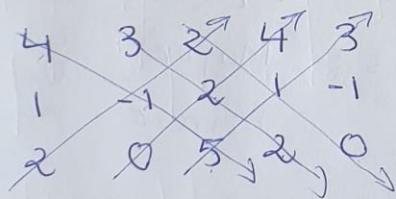
$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} = aek + bfg + cdh - gec - hfa - kdb$$



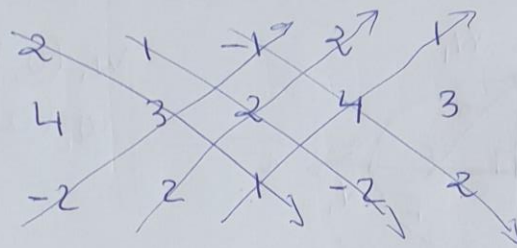
Example :

$$\begin{vmatrix} 4 & 3 & 2 \\ 1 & -1 & 2 \\ 2 & 0 & 5 \end{vmatrix} = -20 + 12 + 0 - (-4) = 0 - 15 = -19$$

في السودا



$$\begin{vmatrix} 2 & 1 & -1 \\ 4 & 3 & 2 \\ -2 & 2 & 1 \end{vmatrix} = 6 + (-4) + (-8) - 6 - 8 - 4 = -24$$



القيمة 0

Another method:

We choose one row or one column
 We put signs in a special way
 and then we distribute with
 respect to the row/column

$$\begin{vmatrix} a^+ & b^- & c^+ \\ d^- & e^+ & f^- \\ g^+ & h^- & k^+ \end{vmatrix} = +g \begin{vmatrix} b & c \\ e & f \end{vmatrix} - h \begin{vmatrix} a & c \\ d & f \end{vmatrix} + k \begin{vmatrix} a & b \\ d & e \end{vmatrix}$$

$$= g(bf - ce) - h(af - cd) + k(ae - bd)$$

$$= gbf + cdh + aek - gce - haf - kbd$$

Example:

$$\begin{vmatrix} 1^+ & 2^- & -1^+ \\ 2^- & 0^+ & 1^- \\ 4 & 3 & 1 \end{vmatrix} = -2 \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} + 0 \begin{vmatrix} 1 & -1 \\ 4 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}$$

$$= -2 \times 5 + 0 - (-5) = -5$$

$$\begin{vmatrix} 1^+ & 2^- & -1 \\ 2 & 0^+ & 1 \\ 4 & 3^- & 1 \end{vmatrix} = -2 \begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} + 0 \begin{vmatrix} 1 & -1 \\ 4 & 1 \end{vmatrix} - 3 \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= (-2) \times (-2) + 0 - 3 \times 3$$

$$= 4 - 9 = -5$$

For a 4x4 matrix:



We cannot use the method of adding columns

The result will be wrong

We use the second method

Example:

$$\begin{vmatrix} 1^+ & 2 & 1 & 3 \\ 4^- & 0 & 5 & 3 \\ 2^+ & -1^- & 0^+ & 0 \\ 3 & 0 & 2 & 1 \end{vmatrix} = +2 \begin{vmatrix} 2^+ & 1 & 3 \\ 0^- & 5 & 3 \\ 0^+ & 2 & 1 \end{vmatrix} - (-1) \begin{vmatrix} 1^+ & 1^- & 3^+ \\ 4 & 5 & 3 \\ 3 & 2 & 1 \end{vmatrix} + 0 \begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 3 \\ 3 & 0 & 1 \end{vmatrix} - 0 \begin{vmatrix} 1 & 2 & 1 \\ 4 & 0 & 5 \\ 3 & 0 & 2 \end{vmatrix}$$

$$= 2 \left(2 \begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix} - 0 + 0 \right) + \left(\begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix} - \begin{vmatrix} 4 & 3 \\ 3 & 1 \end{vmatrix} + 3 \begin{vmatrix} 4 & 5 \\ 3 & 2 \end{vmatrix} \right)$$

→ 0

1	1	3	4	1
4	5	3	4	5
3	2	1	3	2

$$= -4 + (-1 + 5 - 3)$$

$$= \boxed{-21}$$

$$5 + 9 + 24 - 45 - 6 - 4 = -17$$

طريقة أخرى

$$\begin{vmatrix} 1^+ & 2^- & 1 & 3 \\ 4 & 0^+ & 5 & 3 \\ 2 & -1^- & 0 & 0 \\ 3 & 0^+ & 2 & 1 \end{vmatrix}$$

$$= -2 \begin{vmatrix} 4^+ & 5 & 3 \\ 2^- & 0 & 0 \\ 3 & 2 & 1 \end{vmatrix} + 0 + 1 \begin{vmatrix} 1^+ & 1 & 3 \\ 4^- & 5^+ & 3 \\ 3 & 2 & 1 \end{vmatrix}$$

+ 0

$$= -2 \left(-2 \begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix} \right) + \left(-4 \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} + 5 \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} - 3 \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} \right)$$

$$= -4 + 20 - 40 + 3 = \boxed{-21}$$