

Question 1(2+2+3)

a) Find the number c in the mean value theorem for $f(x) = -x^2 + 4x$ on $[0, 3]$

b) Compute the integral $\int \frac{dx}{\sqrt{5^x - 16}}$

c) Evaluate $\int \frac{\cot x dx}{\sqrt{9 - (\sin x)^4}}$

Question 2(3+3+3)

a) Compute $\lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{1}{\ln(x-2)} \right)$

b) Find $\int x^2 \tan^{-1}(x) dx$

c) Evaluate the integral $\int (\tan x)^4 (\sec x)^6 dx$

Question 3(3+3+3)

a) Compute the following integral $\int \frac{x^2 dx}{(x^2 + 9)^{3/2}}$

b) Find the integral $\int \frac{(3x-2)dx}{(x^2+4)(x+2)}$

c) Evaluate the integral $\int \frac{dx}{3 - \sin x + \cos x}$

Question 4(3+2+1)

- a) Sketch the region bounded by the curves: $y = 4 - x^2$, $y = x + 2$
 $x = -3$, $x = 0$ and find its area.
- b) Find the volume obtained by revolving the region bounded by the
curves $y = -x^2 + 2$, $y = 1$ about the line of equation $x = 3$
- c) Set up an integral for the volume obtained by revolving the region in
part b) about the line of equation $y = 4$.

Question 5(3+3+3)

- a) Find the length of the curve given by $r = (\cos(\frac{\theta}{2}))^2$, $0 \leq \theta \leq \pi$.
- b) Sketch the region R that lies inside the curve $r = 1 - \sin\theta$ and
outside the curve $r = 1$ and find its area.
- c) Find the area of the surface obtained by revolving the curve
 $r = 2\cos\theta$ $0 \leq \theta \leq \pi/4$ about the y-axis.