

KING SAUD UNIVERSITY

Math Department

June 2023

Time: 180mn

Final exam Math106

Question 1(2+3+3+3)

a) Set $F(x) = x^3 \int_0^x \sqrt{2 + \sin t} dt$. Find $F'(0)$.

b) Use the substitution $u = x^2 + 1$ to compute $\int \frac{x^3 dx}{(x^2+1)^2}$

c) Compute the integral $\int \frac{dx}{\sqrt{e^{2x}-25}}$

d) Find $\lim_{x \rightarrow 0^+} \sin x \cdot \ln(\sin x)$

Question 2(3+3+3+2+3)

a) Find $\int x (\sec x)^2 dx$

b) Compute the indefinite integral $\int \frac{dx}{(1+x^2)^2}$

c) Evaluate $\int \frac{x+1}{x(x^2+4)} dx$

d) Find $\int \frac{dx}{\sqrt{x^2-4x}}$

e) Evaluate $\int \frac{dx}{x^{3/2}+x^{1/2}}$

Question 3(3+3+3+3+3)

a) Determine whether the integral $\int_0^{\infty} \frac{e^x}{e^{2x+1}} dx$ converges or diverges and find its value if it converges.

b) Sketch the region bounded by the curves $y = x^2 + 5$ and $y = 2x + 8$ and find its area.

c) Find the volume obtained by revolving the region bounded by the curves $y = x^2$ and $y = \sqrt{8x}$ about the y-axis.

d) Find the arc length of the parametric curve given by $x = \frac{t^5}{5}$, $y = \frac{2}{15}t^{15/2}$

$$0 \leq t \leq 1.$$

e) Sketch the polar curve $r = 2 + 2\cos\theta$, $0 \leq \theta \leq 2\pi$, and find the area of the region inside this curve.