Q1

1) $F(x) = (1+\cos x)^{2x}$

$$\boxed{F'(x) = 2\left(\ln(1+\cos x) - \frac{x\sin x}{1+\cos x}\right)F(x)}$$

$$\boxed{F'(0) = 2\ln 2}$$

2) $\int e^{x^2} g(x)\,dx = -e^{x^2} + C$

$$\left(-e^{x^2}+C\right)' = e^{x^2} g(x)$$

$$-2x\,e^{x^2} = e^{x^2} g(x) \rightsquigarrow \boxed{g(x) = -2x}$$

Q2

1) $\int \left(2^x + 2^{-x} + 1\right) dx = \frac{2^x}{\ln 2} - \frac{2^{-x}}{\ln 2} + x + C$

2) $\int \frac{1}{\sqrt{e^{2x}-1}}\,dx = \int \frac{1}{t\sqrt{t^2-1}}\,dt \quad (t = e^x) \quad = \sec^{-1}(e^x) + C.$

3) $\int \frac{1}{\sqrt{x}\sqrt{x+4}}\,dx = 2\int \frac{1}{\sqrt{t^2+4}}\,dt \quad (t=\sqrt{x}) = 2\sinh^{-1}\left(\frac{\sqrt{x}}{2}\right) + C$

Q3

1) $\int \frac{1}{x\sqrt{1-x^8}}\,dx = \frac{1}{4}\int \frac{1}{t\sqrt{1-t^2}}\,dt \quad (x^4 = t)$

$$= -\frac{1}{4}\,\mathrm{sech}^{-1}(x^4) + C.$$

2) $\int \frac{1}{(25-x^2)^{3/2}}\,dx = \frac{1}{25}\int \sec^2\theta\,d\theta = \frac{1}{25}\tan\theta + C \quad (x = 5\sin\theta)$

$$= \frac{1}{25}\,\frac{x}{\sqrt{25-x^2}} + C.$$

$$\int \frac{x^2+12x+3}{x^3-4x}\,dx = \int\left[-\frac{3}{4x}+\frac{31}{8}\frac{1}{(x-2)}-\frac{17}{8}\frac{1}{(x+2)}\right]dx$$

$$= -\frac{3}{4}\ln|x| + \frac{31}{8}\ln|x-2| - \frac{17}{8}\ln|x+2| + C.$$

Q4

1) $\int_1^{\infty} \frac{\ln x}{x}\,dx = \lim_{t\to\infty}\left[\frac{(\ln x)^2}{2}\right]_1^t = \infty.$

2) $A = \int_0^1 \left[(x^2-1) - 2x\right]dx = \frac{1}{3}$

3) $\boxed{V = \pi \int_{-\sqrt{2}/2}^{\sqrt{2}/2} \left((1-x^2)^2 - x^4\right)dx}$

Q5

1) $L = \int_0^4 \sqrt{1+\sinh^2 x}\,dx = \int_0^4 \cosh x\,dx = \sinh 4.$

2) $\theta \in \{0, \pi/2, \pi\}.$

3) $A = 1/2 \int_0^{\pi/4} \tan^2\theta\,d\theta = 1/2\int_0^{\pi/4} (1+\tan^2\theta) - 1\;d\theta$

$$= 1/2\left[\tan\theta - \theta\right]_0^{\pi/4} = \frac{1-\pi/4}{2}$$

4) $\frac{dr}{d\theta} = -\sin\theta$

$S = 2\pi \int_{-\pi/2}^{\pi/2} \cos^2\theta\,d\theta = \pi^2$

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