King Saud University

Faculty of Sciences

Department of Mathematics

Midterm Exam	Math 106	October 2023

Question 1 : (2+3+3)

1. Let $F(x) = \int_{\tan x}^{x^2} \frac{dt}{2+t^4}$. Find $F'(x)$.

2. Use the substitution $u = x^3 + 2$ to compute $\int x^5 \sqrt{x^3+2}dx$.

3. Find the number z in the mean value theorem for $f(x) = |x|$ on $[-1, 1]$.

Question 2 : (2+3+3)

1. Compute $\int (x+1) 7^{x^2+2x} dx$

2. Find the indefinite integral $\int \frac{x}{(x^2+1)\cos^2(\ln(x^2+1))} dx$.

3. If $F(x) = \tan^{-1}(\cosh(x)) + x^{x+1}$, find $F'(x)$.

Question 3 : (3+3+3)

1. Evaluate the integral $\int \frac{\sqrt{x^3}}{1+x^5} dx$

2. Compute $\int \frac{\sinh x}{\sqrt{2^{\cosh x} - 4}} dx$.

3. Find $\int \frac{1-2x^4}{x\sqrt{1-x^4}} dx$.

Solution Midterm Exam	Math 106	October 2023

Question 1 :

1. Let $F'(x) = \dfrac{2x}{2+x^8} - \dfrac{\sec^2 x}{2+\tan^4 x}$. **1+1**

2. $$\begin{aligned} \int x^5\sqrt{x^3+2}\,dx &\overset{u=x^3+2}{=} \frac{1}{3}\int \sqrt{u}(u-2)du \quad \mathbf{1.5} \\ &= \frac{1}{3}\left(\frac{2}{5}(x^3+2)^{\frac{5}{2}} - \frac{4}{3}(x^3+2)^{\frac{3}{2}}\right) + c. \quad \mathbf{1.5} \end{aligned}$$

3. $\int_{-1}^{1} |x|dx = 1,$ **1.5**
   then $|z| = \frac{1}{2}$ and $z = \pm\frac{1}{2}$. **1.5**

Question 2 :

1. $$\begin{aligned} \int (x+1)7^{x^2+2x}dx &\overset{u=x^2+2x}{=} \frac{1}{2}\int 7^u du \quad \mathbf{1} \\ &= \frac{1}{2\ln 7}7^{x^2+2x} + c \quad \mathbf{1} \end{aligned}$$

2. $$\begin{aligned} \int \frac{x}{(x^2+1)\cos^2(\ln(x^2+1))}dx &\overset{u=\ln(x^2+1)}{=} \frac{1}{2}\int \sec^2 u du \quad \mathbf{2} \\ &= \frac{1}{2}\tan u + c = \frac{1}{2}\tan(\ln(x^2+1)) + c \quad \mathbf{1} \end{aligned}$$

3. $F'(x) = \dfrac{\sinh(x)}{1+\cosh^2 x} + (\dfrac{x+1}{x} + \ln x)x^{x+1}$ **1.5+1.5**

Question 3 :

1.

$$\int \frac{\sqrt{x^3}}{1+x^5}dx \overset{u^2=x^5}{=} \frac{2}{5}\int \frac{du}{1+u^2} \qquad \mathbf{2}$$

$$= \frac{2}{5}\tan^{-1}(x^{\frac{5}{2}})+c \qquad \mathbf{1}$$

2.

$$\int \frac{\sinh x}{\sqrt{2^{\cosh x}-4}}dx \overset{u=2^{\frac{1}{2}\cosh x}}{=} \frac{2}{\ln 2}\int \frac{du}{u\sqrt{u^2-4}} \qquad \mathbf{2}$$

$$= \frac{1}{\ln 2}\sec^{-1}\left(\frac{2^{\frac{1}{2}\cosh x}}{2}\right)+c \qquad \mathbf{1}$$

3.

$$\int \frac{1-2x^4}{x\sqrt{1-x^4}}dx = \int \frac{1}{x\sqrt{1-x^4}}dx - \int \frac{2x^3}{\sqrt{1-x^4}}dx \quad \mathbf{0.5}$$

$$= \frac{1}{2}\int \frac{du}{u\sqrt{1-u^2}} + \sqrt{1-x^4}+c \qquad \mathbf{2}$$

$$= \frac{1}{2}\text{sech}^{-1}(x^2)+\sqrt{1-x^4}+c. \qquad \mathbf{0.5}$$