TIME: 3hours **KING SAUD UNIVERSITY** **FULL MARKS :100**

M - 107 **DEPARTMENT OF MATHEMATICS**

(SEMESTER II, 1429-1430) FINAL EXAM

Question: 1 . (a) Use Cramer's Rule to solve the system of equations

$$\begin{aligned} 2x + 2y &= 1 \\ -2x + y + z &= 0 \\ 3x + z &= 1 \end{aligned}$$ **[7]**

(b) Given

$$\begin{aligned} x + 3y + z &= 4 \\ 2x + 2y + z &= -1 \\ 2x + 3y + z &= 3 \end{aligned}$$ **[8]**

i. Write the system of equations in the form AX=B where A, X and B are matrices,

ii. Use elementary row operations to find A^{-1}, and

iii. Use A^{-1} to solve the given system

Question: 2 .(a) Determine whether the points P (3,2,-2), Q (4, 4, -4) and R (2, 0 ,-1) lie in the same line? **[6]**

(b) The volume of the parallelepiped having adjacent sides :

$$a = 3j,\ b = -2r\ i + j \text{ and } c = -i - j + k$$

is 36 unit^3, find value of r. **[8]**

(c) Determine whether the line $x = 3 + 8t$, $y = 4 + 5t$, $z = -3 - t$ and the plane $x - 3y - 5z = 12$ are parallel? If the above line and the plane are not parallel, find their point of intersection. [8]

Question: 3 . (a) Find the equation of the tangent plane and normal line to the graph of the surface $x^3y - y^2 + z^2 = 7$ at the point P (1, 2, 3). **[10]**

(b) Find unit tangent and principal normal vectors of the vector-valued function $r(t) = \cos 2t\ \mathrm{i} + \sin 2t\ \mathrm{j}$ **at** $t = \frac{\pi}{4}$. **[8]**

(c) Show that $\lim\limits_{(x,y)\to(0,0)} \frac{2xy}{x^2 + y^2}$ **does not exist.** **[5]**

Question: 4 . (a) Find the directional derivative of $f(x,y,z) = \sqrt{x^2 - y^2 + z^2}$ **at the point P(1, -4, 8) in the direction of the vector** $a = i + j - 2k$ **[10]**

(b) Use the differential to approximate the change in

$f(x,y,z) = xy + yz + zx$ **[10]**

as (x,y,z) moves from the point (-1, 2 , 3) to the point (-0.98, 1.99, 3.03).

Question: 5 .(a) Show that the function $w = (\sin ax)(\cos by)e^{-\left(\sqrt{a^2+b^2}\right)z}$

satisfies the Laplace equation $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0$. **[10]**

(b) Find the relative extrema and saddle points, if any, of the function

$f(x,y) = 2x^2 - y^3 - 2xy$. **[10]**