KING SAUD UNIVERSITY
DEPARTMENT OF MATHEMATICS
(SEMESTER II, 1434-1435) FINAL

M - 107
FULL MARKS :80
TIME: 3hours

Question: 1 . (a) Given

[5+ 5+5]

$$\begin{aligned} x - y \quad &= 2 \\ x \quad - z &= 3 \\ -6x + 2y + 3z &= 1 \end{aligned}$$

i. **Write the system of equations in matrix form** $AX = B$,
ii. **Use Elementary matrix method to find** A^{-1}, **and**
iii. **Use** A^{-1} **to solve the given system.**

(b) Find the value of k so that the system of equations

$$\begin{aligned} x + y + 3z &= 0 \\ 4x + 3y + kz &= 0 \\ 2x + y + 2z &= 0 \end{aligned}$$

has a non-trivial solution.

(c) If C is a 4x4 matrix with $\det C = \frac{1}{4}$, **find**

$$i)\det C^{-1}, \quad ii)\det C^{T}, \quad iii)\det 2C.$$

Question: 2.(a) Find the volume of parallelepiped with adjacent edges AB, AC, and AD where
[5+ 5+5] **A (2, 1, 3) , B(1, 4, -2), C(3,-4,1) and D(5 ,3, 1)**

(b) Find a parametric equation of the line of intersection of the planes,
$x+y+z=2$ **and** $x-3y+2z=1$.

(c) Show that the line $x=-1-6t,\ y=1+4t,\ z=6+2t$ **and the plane** $3x+2y+5z=7$ **are parallel. Hence find the distance between the line and the plane.**

Question: 3. (a) A particle is moving along the curve

[6+ 6+6]

$$r(t) = t\cos t\ i + t\sin t\ j + t^2\ k.$$

Find its velocity and acceleration and speed at $t = \frac{\pi}{2}$.

(b) Find the unit tangent vector and principal normal vector for the curve C

$$r(t) = (\cos t + t\sin t)i + (\sin t - t\cos t)j + 3k \quad , t > o$$

(c) Show that $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^4+4y^2}$ **does not exist.**

Question: 4. (a) Find equation of the tangent plane and the normal line to the surface
$x^2+3xy-y^2+z^2=6$ at $(1,-1,3)$

[6+ 6+6] **(b) Show that the function** $u(x,t) = e^{-a^2 t}\sin ax$

satisfies the heat equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ **for every real number *a*.**

(c) If $f(x,y,z) = \begin{vmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$, evaluate the deteminant and find the value of $f_x + f_y + f_z$.

Question: 5. (a) Find the relative extrema and saddle points, if any, of the function
[7+ 7] $f(x,y) = 3x^3 - 5x^2 + xy^2 - 2x^2y - 4x$.

(b) Use Lagrange multipliers to find the extreme value of the function
$f(x,y) = x^2 - 2y$ **subject to constraint** $x^2 + y^2 = 4$.