TIME: 90 min **KING SAUD UNIVERSITY** **FULL MARKS:50**

M - 107 **DEPARTMENT OF MATHEMATICS**

SECOND MID TERM EXAM (SEM I) 1434-1435

Question: 1. **(a) Find values of a, b, and c such that** $u = ai + bj + ck$,

[6 + 6] **is orthogonal to** $v = i + 2j + k$ and $w = i - j + k$.

(b) Let $a = < 2,3,4 >$ **and** $b = < -1,3,5 >$ **and** $c = 2a + 3b$,

show that $(a \times b).c = 0$.

Question: 2 . (a)Find parametric equations of the line passing through point P $(5,-2,4)$ **that is**

[6 + 6] **normal to the plane containing points A(2,3,1), B(4,3,2) and C (1,4,5).**

(b) Use cross product to find the distance from the point P(3,3,1) to the line passing through the points A(1,1,1) and B(1,2,3)

Question: 3 . Let A(0,1,0), B(-1,1,2) and C(2,1,-1) be three points,

[6 + 6] **a) Find the area of the triangle ABC, and**

b) Find value of y, if $D(2, y, 1)$ **be a point such that the volume of the box having adjacent sides AB, AC, and AD is 3 units^3.**

Question: 4. The motion of a point moving along the curve is give by

[8 + 6] $r(t) = (t-5)i + (2t^3 - 5)j + (t^2 + 4)k$,

(a) Find the velocity, acceleration and speed at time $t = 1$.

(b) Also find the components of velocity and acceleration in direction of vector $b = i - j + 2k$ **at time** $t = 1$.

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(b) Let $a = \langle 2,3,4 \rangle$ **and** $b = \langle -1,3,5 \rangle$ **and** $c = 2a + 3b$,

show that $(a \times b).c = 0$.

Solution (a) u is orthogonal to $v \Rightarrow u.v = 0$

u is orthogonal to $w \Rightarrow u.w = 0$

Note

Q4(a)

$\|v(1)\| = \sqrt{41}$

$u.v = a + 2b + c = 0 \rightarrow E_1$

$u.w = a - b + c = 0 \rightarrow E_2$

$E_1 - E_2 \Rightarrow 3b = 0 \Rightarrow b = 0$

$E_1 \Rightarrow a + c = 0$

$E_2 \Rightarrow a + c = 0 \Rightarrow c = -a$

Let $a = t$, $c = -t$,

Solutions are $a = t$, $b = 0$, $c = -t$, $t \in \mathbb{R}$.

(b) $c = 2a + 3b = 2\langle 2,3,4 \rangle + 3\langle -1,3,5 \rangle = \langle 4-3, 6+9, 8+15 \rangle$

$= \langle 1, 15, 23 \rangle$

$$(a \times b).c = \begin{vmatrix} 2 & 3 & 4 \\ -1 & 3 & 5 \\ 1 & 15 & 23 \end{vmatrix} = 2(69 - 75) - 3(-23 - 5) + 4(-15 - 3)$$

$= -12 + 84 - 72$

$= 0$

Q4(a) $v \times w = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 3i - 0j - 3k$ u is parallel to $v \times w \Rightarrow \frac{a}{3} = \frac{b}{0} = \frac{c}{-3} = t$

$a = 3t$

$b = 0$

$c = -3t \quad t \in \mathbb{R}$

11/24/2013 Q4(a) u is orthogonal to $v \times w \Rightarrow u \times (v \times w) = 0$

$$u \times (v \times w) = \begin{vmatrix} i & j & k \\ a & b & c \\ 3 & 0 & -3 \end{vmatrix} = -3bi - (-3a - 3c)j - 3bk = 0$$

$\Rightarrow b = 0, \quad a = -c$

$a = -t$

$b = 0$

$c = t$

$t \in$

→ Note Q4(a) $\|v(1)\| = \sqrt{41}$