TIME: 90 min — KING SAUD UNIVERSITY — FULL MARKS:50

M - 107 — DEPARTMENT OF MATHEMATICS

II MID TERM EXAM ( SEM I ) 1436-1437

---

Question: 1. (a) Find the equation of line passing through point A (1, 1, 1 ) and perpendicular to plane $x + 2y + z = 1$.

[6+6+6] (b) Find the distance between skew lines

$$L_2 : \quad x = 1+s, \; y = -2s, \; z = -1+s, \quad s \in \mathbb{R}$$

$$L_1 : \quad x = 1+2t, \; y = 2-t, \; z = 1, \quad t \in \mathbb{R}$$

( c) The magnitude and direction of constant force are given by $f = 5i + 2j + 6k$.

Find the work done if point of application moves from $P(1,-1,2)$ to $Q(4,3,-1)$.

Question: 2 .

(a) Let $u = < 2,-2,3 >$ and $a = < 4,-1,2 >$. Find $\mathrm{Proj}_a^u$ and show that $\left(u - \mathrm{Proj}_a^u\right)$ is orthogonal to a.

[8+6] (b)Identify the surface $z = \frac{y^2}{9} + \frac{x^2}{4}$, find its traces on the coordinate planes and sketch the surface.

Question: 3. (a) If the velocity of a moving particle is given by $v(t) = 4\cos(2t)i + 2\sin(2t)j + \frac{1}{1+t^2}k$,

[6+6+6] find the position of particle when the initial position of particle is $r(0) = i + 2j + 3k$.

(b) If the position vector of an object is $r(t) = \cos(2t)i + \sin(2t)j + tk$, find the tangential and normal components of acceleration and the curvature of the curve at point $\left(0, 1, \frac{\pi}{4}\right)$

(c) Let C be the curve determined by the vector valued function

$$x(t) = t, \; y(t) = \frac{t^2}{2}, \; z(t) = \frac{t^3}{3}.$$

Find the parametric equation of tangent line to curve C at $t = 1$.

2015/12/1

Q.1 a. A vector perpendicular to plane is $n = \langle 1,2,1\rangle$. **2nd midterm**

[6] A Line passing through point $A(1,1,1)$ and parallel to vector $n = \langle 1,2,1\rangle$ is

$x = 1+t, \quad y = 1+2t, \quad z = 1+t, \quad t \in \mathbb{R}.$

b. Let a point on line $L_2$ is $A_2(1,0,-1)$ and vector parallel to line $L_2$ $u_2 = \langle 1,-2,1\rangle$

[6] and a point on line $L_1$ is $A_1(1,2,1)$ and vector $\parallel$ to $L_1$ $u_1 = \langle 2,-1,0\rangle$

$\underline{n} = u_1 \times u_2 = \langle -1,-2,-3\rangle$ is perpendicular to plane parallel to both $L_1$ and $L_2$

$\overrightarrow{A_1A_2} = \langle 0,-2,-2\rangle$ is vector joining line.

$$d = \text{Comp}_n^{\overrightarrow{A_1A}} = \frac{|0+4+6|}{\sqrt{1+4+9}} = \frac{10}{\sqrt{14}}$$

c.

[6] force $f = 5i + 2j + 6k$ $\quad d = \overrightarrow{PQ} = \langle 3,4,-3\rangle$

work done $= f \cdot d = \langle 5,2,6\rangle \cdot \langle 3,4,-3\rangle = 15+8-18 = 5$

Q.2. a. $\text{Proj}_a^u = \left(\text{Comp}_a^u\right)\frac{a}{\|a\|} = \left(\frac{u\cdot a}{\|a\|}\right)\frac{a}{\|a\|}$

$u\cdot a = 8+2+6 = 16 \qquad \|a\| = \sqrt{16+1+4} = \sqrt{21}$

[8] $-$ $\text{Proj}_a^u = \frac{16}{21}\langle 4,-1,2\rangle = \left\langle \frac{64}{21}, -\frac{16}{21}, \frac{32}{21}\right\rangle$

$-$ $u - \text{Proj}_a^u = \langle 2,-2,3\rangle - \left\langle \frac{64}{21}, \frac{-16}{21}, \frac{32}{21}\right\rangle$

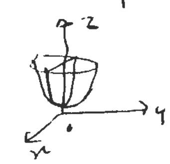
$= \left\langle \frac{-22}{21}, \frac{-26}{21}, \frac{31}{21}\right\rangle$

$-$ $(u - \text{Proj}_a^u)\cdot a = \frac{-88+26+62}{21} = 0 \implies u - \text{Proj}_a^u \perp a.$

b.

[6] 1. surface is Elliptic Paraboloid.

2. xy-Trace is origin $(0,0)$, yz-Trace is parabola, xz-Trace is parabola




Q.3. a. $r(t) = \int \left(4\cos(2t)\,i + 2\sin(2t)\,j + \frac{1}{1+t^2}\,k\right)dt$

$\rightarrow$ $r(t) = 2\sin(2t)\,i - \cos 2t\, j + \tan^{-1} t\, k + c_1 i + c_2 j + c_3 k$

[6] $\rightarrow$ $t=0$ $\quad \langle 1,2,3\rangle = -j + c_1 i + c_2 j + c_3 k \implies c_1 = 1, \ c_2 = 3, \ c_3 = 3$

$\rightarrow$ $r(t) = (2\sin(2t)+1)\,i + (-\cos 2t + 3)\,j + (\tan^{-1} t + 3)\,k.$

b. $r'(t) = -2\sin 2t\, i + 2\cos 2t\, j + k$

$r''(t) = -4\cos 2t\, i - 4\sin 2t\, j$

At $t = \frac{\pi}{4}$ $\quad r'(\frac{\pi}{4}) = \langle -2,0,1\rangle \qquad a_T = 0$

[6] $r''(\frac{\pi}{4}) = \langle 0,-4,0\rangle \qquad a_N = \frac{4\sqrt{5}}{\sqrt{5}} = 4$

$r'\cdot r'' = 0 \qquad \kappa = \frac{4\sqrt{5}}{(5)^{3/2}} = 4/5$

$r' \times r'' = \langle 4,0,8\rangle$

$\|r'(\frac{\pi}{4})\| = \sqrt{4+1} = \sqrt{5}$

$\|r' \times r''\| = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$

(c). Tangent vector is $r'(t) = \langle 1, t, t^2\rangle$, $\quad r'(1) = \langle 1,1,1\rangle$

Point at $t=1$ is $P(1, \frac{1}{2}, \frac{1}{3})$

[6] Equation of Tangent line is $x = 1+t, \ y = \frac{1}{2}+t, \ z = \frac{1}{3}+t, \ t \in \mathbb{R}.$