FINAL EXAMINATION, SUMMER, 1430-1431
Department of Mathematics
King Saud University
MATH: 107
Time: 3 H   Full Marks: 50

Question # 1. Marks: 4+4=8

(a) Solve the system of linear equations by Gauss-Jordan method:

$$\begin{cases} x+y+2z &= 8 \\ x+2y-3z &= -1 \\ 3x-7y+4z &= 10. \end{cases}$$

(b) Under what condition on a, b and c, the following system of linear equations would be consistent?

$$\begin{aligned} x+y+2z &= a \\ x+z &= b \\ 2x+y+3z &= c. \end{aligned}$$

Question # 2. Marks: 5+4=9

(a) Find adjoint and inverse of the matrix $A = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{pmatrix}$.

(b) Find a unit vector normal to the plane containing the points $A(1,1,1)$, $B(2,-3,3)$ and $C(5,1,3)$.

Question # 3. Marks: 4+4=8

(a) Find the point of intersection of the following two lines:
$l_1 : x = 1-6t, y = 3+2t, z = 1-2t$
$l_2 : x = 2+2v, y = 6+v, z = 2+v.$

(b) Find the equation of the plane that contains the point $P(4,-3,0)$ and the line $l : x = 5+t, y = -1+2t, z = 7-t$.

Question # 4. Marks: 4+3+4=11

(a) Find the tangential and normal components of acceleration for the position vector given by $\mathbf{r}(t) = 3t\mathbf{i} - t\mathbf{j} + t^2\mathbf{k}$.

(b) If $f(x,y) = \frac{xy}{x^2+y^2}$, does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist ?

(c) The base radius and height of a right circular cone are measured as 10 cm and 25 cm, respectively, with possible error in measurement of as much as 0.1 cm in each. Use differentials to estimate the maximum error in calculated volume of the cone.

Question # 5. Marks: 3+4=7

(a) If $u = x^4y + y^2z^3$, where $x = rse^t, y = rs^2e^{-t}$, and $z = r^2s\sin t$, then use **Chain Rule** to find $\frac{\partial u}{\partial s}$ when $r = 2, s = 1, t = 0$.

(b) Suppose that the temperature at a point $P(x,y,z)$ in space is given by $T(x,y,z) = \frac{80}{(1+x^2+2y^2+3z^2)}$, where T is measured in degrees Celsius and x, y, z in meters. In what direction does the temperature increase fastest at the point $(1,1,-2)$? What is the maximum rate of increase?

Question # 6. Marks: 4+3=7

(a) Identify the surface $z^2 - 2x^2 - 2y^2 = 12$. Find the equations of the tangent plane and normal line at the point $P(1,-1,4)$ to the given surface.

(b) Find the local maximum and local minimum values and saddle points, if any, for the function $f(x,y) = x^4 + y^4 - 4xy + 1$, for real values of x, y.