

(٣٣) مساحة المنطقة داخل المنحنيين  $r = 4 \sin \theta$  و  $r = 4 \cos \theta$  تساوي:

(أ)  $2\pi - 2$  (ب)  $2\pi - 4$  (ج)  $2\pi + 2$  (د)  $2\pi + 4$

(٣٤) طول المنحنى  $r = 4 \sin \theta$  يساوي:

(أ)  $2\pi$  (ب)  $6\pi$  (ج)  $4\pi$  (د)  $8\pi$

(٣٥) مساحة المنطقة داخل  $r = 3 \cos \theta$  وخارج  $r = 1 + \cos \theta$  تساوي:

(أ)  $\theta = \pi/2$  (ب)  $\theta = \pi/2$  (ج)  $\pi$  (د)  $2\pi$

أجوبة التمارين الفردية

### ANSWERS TO ODD EXERCISES

تمارين (١-١)

$$\frac{17}{2} (١٥) \quad 39 (١٣) \quad \frac{1}{3} (١١) \quad \frac{n(n+1)(n+2)(n+3)}{4} (٩) \quad \frac{n(n^2+6n+17)}{3} (٧)$$

$$12.1875 (٢٧) \quad \frac{(1+\sqrt{2})\pi}{4} (٢٥) \quad 46 (٢٢) \quad 50 (٢١) \quad 1 (ب) (١٩) \quad 14 (١٧)$$

$$68.1 (٣٩) \quad 17/2 (٣٧) \quad 75/2 (٣٥) \quad 39/4 (٣٣) \quad 15/4 (٣١) \quad 86 (٢٩)$$

حلول تمارين (٢-١)

$$6 (٧) \quad 0 (٥) \quad 8/3 (٣) \quad 9 (١)$$

تمارين (٣-١)

$$5/2 (١١) \quad 5 (٩) \quad 5/2 (٧) \quad 2/3 (٥) \quad -165/2 (٣) \quad 5\pi (١)$$

$$22 (ج) \quad 17 (ب) \quad -18 (د) \quad 30 (هـ) \quad -1 (و) \quad 7 (ج) \quad -6 (ب) \quad -4 (أ) \quad (١٣)$$

$$c = \sqrt{7} \quad (ب) \quad c = 1 \quad (أ) \quad (٢١)$$

تمارين عامة

$$-1 (٤) \quad (١) \quad (٣) \quad (٥) \quad (٧) \quad (٩) \quad (١١)$$

تمارين (١-٢)

$$29/35(١٥) \quad 28/81(١٣) \quad 1-\sqrt{2}(١١) \quad 53/2(٩) \quad 0(٧) \quad -37/6(٥) \quad -18(٣)$$

$$-55/3(٢٧) \quad -1/3(٢٥) \quad 36(٢٣) \quad 29/6(٢١) \quad 0(١٩) \quad 1(١٧)$$

$$A=A_1+A_2+A_3 = \frac{23}{6} + \frac{343}{6} + \frac{243}{6} + \frac{509}{6} = \frac{(٣٥)}{6} \quad 2(٣٣) \quad \frac{26}{3}(٣١) \quad \frac{107}{10}(٢٩)$$

$$-\sec^2 x \left[ \sin(\tan^4 x) \right] (٤٣) \quad \frac{\cos x}{1+\sin^2 x} (٤١) \quad |x| \sin \sqrt{x} (٣٧)$$

$$\frac{-\sec^2 x}{\sqrt{2+\tan^4 x}} + \frac{2x}{\sqrt{2+x^8}} (٤٧) \quad \frac{-2(2x-1)}{2x+1} + \frac{3(3x-1)}{3x+1} (٤٥)$$

$$3x^2(x^9+1)^{10} - 3(27x^3+1)^{10} (٤٩)$$

$$F''(2) = \frac{6}{\sqrt{13}}, \quad F'(2) = \sqrt{13}, \quad F(2) = 0 (٥١)$$

(٥٣) القيمة الصغرى المطلقة، تزايد على  $[3, \infty)$  و تناقص على  $(-\infty, 3]$ ؛  
تقع لأعلى على  $(-1, 7)$  و لأسفل على كل من  $(-8, -1)$  و  $(7, \infty)$ .

تمارين (٢-٢)

$$I = 2\sin\sqrt{x} + C (٥) \quad I = \frac{\sin 5x}{5} + C (٣) \quad I = \frac{(x^2+1)^{51}}{51} + C (١)$$

$$I = \frac{2(x-1)^{7/2}}{7} + \frac{5}{3} + \frac{4(x-1)^{5/2}}{3} + C (٩) \quad I = \frac{x^2}{2} + \frac{\tan \pi x}{\pi} + C (٧)$$

$$I = \frac{1}{6} \sec^{-1} \left( \frac{\theta^2}{3} \right) + C (١٥) \quad I = \frac{\sin(x^4+2)}{4} + C (١٣) \quad I = \frac{1}{96} (١١)$$

$$I = \frac{2(1+\sin x)^{3/2}}{3} + C (٢١) \quad I = \frac{\tan(x^2)}{2} + C (١٩) \quad I = \sqrt{18-3} (١٧)$$

$$I = \left[ \frac{3}{8}(1+x^{2/3})^4 \right]_1^8 = \frac{1827}{8} (٢٥) \quad I = \frac{x}{2} + \frac{\sin 2x}{4} + C (٢٣)$$

$$I = \frac{-\cot^2 x}{2} + C (٣١) \quad I = \frac{(\sin^{-1} \theta)^2}{2} + C (٢٩) \quad I = \frac{(1+\sqrt{x})^4}{2} + C (٢٧)$$

$$I = \frac{3(2x-3)^{7/3}}{14} + C (٣٧) \quad I = \cos(\sin \theta) + C (٣٥) \quad I = \frac{1}{3(1-3x)} + C (٣٣)$$

$$I = \frac{8(2-x)^{3/2}}{3} + \frac{5}{2(2-x)^{5/2}} + C (٣٩)$$

$$I = (x^{2/3}+1)^{3/2} + C (٤٣) \quad I = \frac{\cos 2\theta}{2} + \frac{\cos^3 2\theta}{6} + C (٤١)$$

$$I = \left[ 2 \tan^{-1} \sqrt{x} \right]_1^3 = \frac{\pi}{6} (٤٧) \quad I = \left[ \frac{2(5+x)^{3/2}}{3} - 10(5+x)^{1/2} \right]_{-1}^4 = \frac{8}{3} (٤٥)$$

$$I = \left[ \frac{1}{2\sqrt{3}} \tan^{-1} \left( \frac{x^2}{\sqrt{3}} \right) \right]_1^{\sqrt{3}} (٥١) \quad I = -\tan^{-1}(\cos \theta) + C (٤٩)$$

$$I = 2 \tan^{-1} \sqrt{x} + C (٥٥) \quad I = -\cos x - \frac{4(\cos x)^{3/2}}{3} - \frac{\cos^2 x}{2} + C (٥٣)$$

$$I = -\tan^{-1}(\cos x) + C (٥٩) \quad I = \frac{1}{3} \tan^3 x + C (٥٧)$$

$$.A = \int_1^2 (x^2+1)^4 dx = \frac{3093}{10} (٦٣) \quad A = \int_0^3 \sqrt{x+1} dx = \frac{14}{3} (٦١)$$

$$\cdot \int_0^3 f(3x) dx = \frac{1}{3} \int_0^9 f(u) du = \frac{5}{3}; u=3x \text{ ضع } (٦٥)$$

$$\cdot \int_{-2}^0 x f(x^2) dx = \frac{1}{2} \int_0^4 f(u) du = -\frac{1}{2}; u=x^2 \text{ ضع } (٦٧)$$

[١, 1] ليست متصلة على  $[-1, 1]$ .

د (٨٥) د (٨٣) ب (٧٩) ج (٧٧)

تمارين (١-٣)

$$f(x) = \frac{1}{x} \ln \ln x \cdot \ln \ln \ln x (٥) \quad y' = \frac{-1-2x}{2-x-x^2} (٣) \quad f'(x) = \frac{1}{x+1} (١)$$

$$\cdot g'(x) = 2\pi \csc(\pi x) (٩) \quad f'(\theta) = \frac{\sec \theta \tan \theta + \sec^2 \theta}{\sec \theta + \tan \theta} (٧)$$

$$h'(t) = \frac{6(t-1)}{t-2} \left[ \ln(t^2-2t) \right]^2 + \left[ \ln(t^2-2t) \right]^3 (١١)$$

$$\frac{e^x(x \ln x - 1)}{x(\ln x)^2} \quad (٧) \quad e^x \cos(e^x) \quad (٥) \quad -10xe^{-5x^2} \quad (٣)$$

$$\frac{2e^{2x}}{\sqrt{e^{4x}+1}} \quad (١٣) \quad \frac{x-1}{e^x-x} \quad (١١) \quad \frac{15x^2}{2\sqrt{1+5x^3}} \exp(\sqrt{1+5x^3}) \quad (٩)$$

$$\frac{e^x(x-1)^2}{(x^2+1)^2} \quad (١٧) \quad e^x \cos(1-\ln x) + \frac{e^x \sin(1-\ln x)}{x} \quad (١٥)$$

$$e^{3x} \left[ \frac{1}{2\sqrt{x}} \sec^2 \sqrt{x} + 3 \tan \sqrt{x} \right] \quad (١٩)$$

$$1 + \frac{e^x(1+x)}{\sin(x-y)} \quad (ج) \quad \frac{e^x \cot y - e^{2y}}{e^x \csc^2 2xe^{xy}} \quad (ب) \quad \frac{3x^2 - ye^{xy}}{xe^{xy} + 6y} \quad (١) \quad (٢١)$$

$$x+2y = 1 + \ln 2 \quad (٢٩) \quad g'(x) \text{ محاسب } \ddot{g}(x) = e^{-x} f(x) \quad (٢٧) \quad \text{ضع}$$

$$e^{-\lambda t} [(wA - \lambda B) \cos wt - (wB + \lambda A) \sin wt] \quad (٣١)$$

$$g^{(n)}(x) = (-1)^n k^n e^{-kx} \quad \text{و} \quad f^{(n)}(x) = k^n e^{kx} \quad (٣٣)$$

$$2e^{\sqrt{x}} + C \quad (٣٩) \quad e/3x^3 - 1/2 \ln 2 \cos x + C \quad (٣٧) \quad 1/3(1+e)^{3/2} + C \quad (٣٥)$$

$$e^x + C \quad (٤٥) \quad (1/2 \ln 3)x^2 - 4\pi e^2 \sin x + C \quad (٤٣) \quad -2e^{-\sqrt{x}} + C \quad (٤١)$$

$$-\frac{1}{8} [(3-4e^x)^2]_{\ln 5}^{\ln 5} = -36 \quad (٤٩) \quad \sin^{-1} e^x + C \quad (٤٧)$$

$$\left[ \ln(e^x + 4) \right]_{-\ln 3}^{\ln 3} = \ln \left( \frac{21}{13} \right) \quad (٥١)$$

تمارين (٣-٣)

$$x^{\sin x} \left[ \frac{\sin x}{x} + (\cos x) \ln x \right] \quad (٧) \quad \pi^{\sin x} \ln \pi (\cos x) \quad (١)$$

$$x^{e^x} \left[ \frac{e^x}{x} + e^x \ln x \right] \quad (٧) \quad (\ln x)^{\tan x} \left[ \frac{\tan x}{x \ln x} + \sec^2 x \ln(\ln x) \right] \quad (٥)$$

$$\frac{4x^3 + 6x}{\ln 10(x^4 + 3x^2 + 1)} \quad (١٣) \quad (\ln a)a^x x^a + a^{x+1} x^{a-1} \quad (١١) \quad x^{x^2+1} (2 \ln x + 1) \quad (٩)$$

$$\frac{30x}{(3x^x + 2) \ln 10} \quad (١٩) \quad \frac{1}{x \ln x \ln 10} \quad (١٧) \quad \frac{-(x^2+1)10^{1/x} \ln 10}{x^2} + 2x 10^{1/x} \quad (١٥)$$

$$f'(x) = \frac{1}{2x\sqrt{\ln x}} \quad (١٧) \quad y' = \frac{x+2x \ln x}{\sqrt{1-x^4 \ln^2 x}} \quad (١٥) \quad g'(x) = \frac{x(1+2 \ln x)}{(1+\ln x)^2} \quad (١٣)$$

$$f'(x) = -\frac{1}{x} \left( \frac{1}{(\ln x)^2} + 1 \right) \quad (٢١) \quad y' = 3(\ln \sin x)^2 \cot x \quad (١٩)$$

$$y' = -\tan x + \frac{3x}{4-3x^2} \quad (٢٥) \quad g'(t) = \frac{2t}{t^4-1} \quad (٢٣)$$

$$f'(x) = \frac{1}{2x} + \frac{1}{3(x+1)} - \cot x - \tan x \quad (٢٧)$$

$$y' = \frac{1}{5} \sqrt{\frac{x-1}{x+1}} \left[ \frac{1}{x-1} - \frac{1}{x+1} \right] \quad (٣١) \quad y' = \frac{-4x}{(2x^2+3) [\ln(2x^2+3)]^2} \quad (٢٩)$$

$$y' = \frac{\sin x \cos x \tan^3 x}{\sqrt{x}} \left[ \cot x - \tan x + \frac{3 \sec^2 x}{\tan x} - \frac{1}{2x} \right] \quad (٣٣)$$

$$\frac{dy}{dx} = \frac{y-x \sin y}{x(\cos y - \ln x)} \quad (٣٩) \quad \frac{dy}{dx} = \frac{(2x^2-1)y}{x(3y+1)} \quad (٣٧) \quad \frac{dy}{dx} = \frac{2x}{x^2+y^2-2y} \quad (٣٥)$$

$$x + 3 \ln |x-3| + C \quad (٤٧) \quad (2, 4 + 4 \ln 2), (1, 1) \quad (٤٣) \quad y = 8x - 15 \quad (٤١)$$

$$\frac{2(\ln x)^{3/2}}{3} + C \quad (٥٣) \quad \frac{x^2 \ln(x^2+1)}{2} + C \quad (٥١) \quad \frac{\ln |x^3+3x+1|}{3} + C \quad (٤٩)$$

$$\frac{(\ln t)^4}{4} + C \quad (٥٩) \quad \ln |1 + \tan x| + C \quad (٥٧) \quad \frac{3 \ln |1 + 2 \cos x|}{2} + C \quad (٥٥)$$

$$\left[ \frac{1}{2} \ln |x^2+5| \right]_{-1}^0 = \frac{1}{2} \ln \frac{5}{6} \quad (٦٣) \quad \frac{(1+\ln x)^5}{5} + C \quad (٦١)$$

$$\frac{3}{2} \ln(1+x^{2/3}) + C \quad (٦٧) \quad \ln \left| \frac{1}{\sin x + \cos x} \right| + C \quad (٦٥)$$

$$\left[ 8\pi \ln(x^2+4) \right]_0^1 = 8\pi \ln \frac{5}{4} \quad (٦٩)$$

تمارين (٢-٣)

$$\frac{1+\sqrt{1+4e}}{2} \quad (٥) \quad \pm 1 \quad (٥) \quad 40 \quad (ج) \quad e^2 \quad (ب) \quad \pi \quad (أ) \quad (١)$$

لا توجد

$$3 \tanh 3x + 2 \tan x \quad (٣٧) \quad x^{\cosh x} \left[ \sinh x \ln x + \frac{\cosh x}{x} \right] \quad (٣٥)$$

$$-3x^2 \cosh(3-x^3) + (\ln x)^{\sinh x} \left[ \frac{\sinh x}{x \ln x} + (\cosh x) \ln \ln x \right] \quad (٣٩)$$

$$2 \sinh(\sqrt{x}) + C \quad (٤٥) \quad (\operatorname{sech}^3 x / 3) + C \quad (٤٣) \quad (\sinh^7 x / 7) + C \quad (٤١)$$

$$\frac{e^{3x}}{3} - e^{-x} + C \quad (٥١) \quad \frac{\ln|1-2 \tanh x|}{2} + C \quad (٤٩) \quad \ln(e^{2x} - 1) + C \quad (٤٧)$$

$$(\ln(2 \pm \sqrt{3}) \pm \sqrt{3}) \quad (٥٧) \quad (e^{3x} / 6) - (e^{-x} / 2) + C \quad (٥٥) \quad 2e^{\sqrt{x}} + C \quad (٥٣)$$

## تمارين (٢-٤)

$$\frac{-1}{(\tanh^{-1} x)^2 (1-x^2)} \quad (٩) \quad \frac{1}{2\sqrt{x}(1-x)} \quad (٧) \quad \frac{1}{\sqrt{9+x^2}} \quad (٥)$$

$$\frac{3x^2 (\sinh^{-1} x)^2}{\sqrt{1+x^2}} + 2x (\sinh^{-1} x)^3 \quad (١٣) \quad \frac{1}{2(1-x^2)} \sqrt{\coth^{-1} x} \quad (١١)$$

$$\frac{x}{|x|(1-x^2)} \quad (١٩) \quad \frac{\tanh^{-1} x}{\sqrt{1+\tanh^2 x}} \quad (١٧) \quad \frac{\operatorname{sech}^2 x}{\sqrt{1+\tanh^2 x}} \quad (١٥)$$

$$\frac{1}{3} \sinh^{-1} \left( \frac{3x}{5} \right) + C \quad (٢٥) \quad \cosh^{-1} \left( \frac{e^x}{4} \right) + C \quad (٢٣) \quad \frac{1}{4} \sinh^{-1} \left( \frac{4x}{9} \right) + C \quad (٢١)$$

$$\frac{1}{2} \coth^{-1} \left( \frac{5}{2} \right) + \frac{1}{2} \coth^{-1} \left( \frac{3}{2} \right) \quad (٢٩) \quad \cosh^{-1}(e^x) + C \quad (٢٧)$$

$$\cdot \coth^{-1} 3 - \cosh^{-1} 2 \quad (٣١)$$

## تمارين عامة (٣-٤)

$$5, 4/3 \quad (١)$$

$$\frac{e^{2x}}{2} + C \quad (ج) \quad \ln(e^{2x} + 1) + C \quad (ب) \quad -\sqrt{4-x^2} - \frac{1}{2} \operatorname{sech}^{-1} \left( \frac{x}{2} \right) + C \quad (د) \quad (٣)$$

$$\frac{(\tanh^{-1} x)^2}{2} + C \quad (ج) \quad \frac{\sinh 2x}{4} + \frac{x}{2} + C \quad (هـ) \quad \frac{\operatorname{sech}^3 x}{3} - \frac{\operatorname{sech}^5 x}{5} + C \quad (د)$$

$$\int (١٩) \quad \int (١٧) \quad \int (١٥) \quad \int (١٣) \quad \int (١١) \quad \int (٩) \quad \int (٧) \quad \int (٥)$$

$$y = (1+2 \ln 2)x + 1 - 2 \ln 2 \quad (ب) \quad y = (10 \ln 10)x + 10(1 - \ln 10) \quad (٢١)$$

$$\frac{10(a^{\sqrt{2}} - a)}{\ln a} \quad (٢٩) \quad \frac{45}{\ln 10} \quad (٢٧) \quad \frac{1}{\ln \pi} \pi^{\sin x} + C \quad (٢٥) \quad \frac{-1}{2 \ln 3} 3^{-x^2} + C \quad (٢٣)$$

## تمارين (٤-٣)

$$\frac{e^x}{\sqrt{1-e^{2x}}} + 2 \tan x + \frac{2}{x} + 1 \quad (١)$$

$$(\cos^{-1} x)^{\sin^{-1} x} \left[ \frac{\ln(\cos^{-1} x)}{\sqrt{1-x^2}} \frac{\sin^{-1} x}{\cos^{-1} x \sqrt{1-x^2}} + e^{\sec^{-1} x} \left( 1 + \frac{1}{\sqrt{x^2-1}} \right) \right] \quad (٣)$$

$$-\sin x e^{\cos x} - e^x \operatorname{sine}^x + \left( \frac{c}{x} \right)^x \left[ \ln \left( \frac{c}{x} \right) - 1 \right] \quad (٥)$$

$$\frac{e^y}{1-xe^y} \quad (٩) \quad (3 \ln 5)^{5^{3x}} + 15(3x)^4 + \frac{1}{2\sqrt{x}} \quad (٧)$$

$$y = -2(1+e)(x-1) + e \quad (ج) \quad y = 2x - e \quad (ب) \quad 3x - 2y + \ln 4 = 0 \quad (١١)$$

$$\cos e^{-x} + C \quad (١٧) \quad \frac{-1}{3 \ln 2} 2^{-x^3} + C \quad (١٥) \quad \frac{[\ln(\cos x)]^2}{2} + C \quad (١٣)$$

$$\frac{1}{\ln 3} [3^{x+1} - \cos 3^x] + C \quad (١٩)$$

$$\int (٣٩) \quad \int (٣٧) \quad \int (٣٥) \quad \int (٣٣) \quad \int (٣١) \quad \int (٢٩) \quad \int (٢٧) \quad \int (٢٥) \quad \int (٢٣) \quad \int (٢١)$$

## تمارين (١-٤)

$$5 \cosh 5x \quad (٢١) \quad 0 \quad (د) \quad \infty \quad (ج) \quad 1/2 \quad (ب) \quad 1/2 \quad (أ) \quad (١٩)$$

$$2 \coth 2x \quad (٢٥) \quad 3x^2 \sinh x^3 + 3 \cosh^2 x \sinh x \quad (٢٣)$$

$$\frac{\cosh x}{1 + \sinh^2 x} \quad (٢٩) \quad \frac{-2x \operatorname{sech} x^2 [(x^2 + 1) \tanh x^2 + 1]}{(x^2 + 1)^2} \quad (٢٧)$$

$$\frac{[-3 \cosh(\cos 3x)] \sin 3x}{x^2} \quad (٣٣)$$

$$\begin{aligned} & \frac{1}{2} [\ln|\csc x - \cot x| + \ln|\sec x + \tan x|] + C \quad (٣١) \quad \frac{\csc^5 x}{5} + \frac{\csc^3 x}{3} + C \quad (٢٩) \\ & \frac{2 \sec^{3/2} x}{3} + C \quad (٣٥) \quad \frac{3}{4} \left[ \cos^3 \left( \frac{x}{3} \right) \sin \left( \frac{x}{3} \right) + \sin \left( \frac{x}{3} \right) \cos \left( \frac{x}{3} \right) \right] + \frac{x}{8} + C \quad (٣٣) \\ & \cos x + \sec x + C \quad (٤١) \quad \frac{5\pi}{16} \quad (٣٩) \quad \frac{2 \cos^{7/2} x}{7} + \frac{2 \cos^{3/2} x}{3} + C \quad (٣٧) \\ & \pi/16 \quad (٤٧) \quad 0 \quad (٤٥) \quad 0 \quad (٤٣) \end{aligned}$$

تمارين (٣-٥)

$$\begin{aligned} & \sqrt{x^2 + 4} + C \quad (٥) \quad \sqrt{x^2 - 25} - 5 \sec^{-1} \left( \frac{x}{5} \right) + C \quad (٣) \quad \frac{-\sqrt{4-x^2}}{4x} + C \quad (١) \\ & \frac{x}{2\sqrt{2-x^2}} + C \quad (١١) \quad \frac{x}{\sqrt{1-x^2}} + C \quad (٩) \quad \frac{-\sqrt{9-x^2}}{x} - \sin^{-1} \left( \frac{x}{3} \right) + C \quad (٧) \\ & \frac{(1-x^2)^{5/2}}{5} + \frac{(1-x^2)^{3/2}}{3} + C \quad (١٥) \quad \frac{x}{2(1+9x^2)} + \frac{\tan^{-1}(3x)}{6} + C \quad (١٣) \\ & -\sin^{-1} \left( \frac{\sin x}{\sqrt{2}} \right) + C \quad (١٩) \quad \frac{x^3 \sqrt{9-x^2}}{4} - \frac{9x \sqrt{9-x^2}}{8} + \frac{81 \sin^{-1} \left( \frac{x}{3} \right)}{8} + C \quad (١٧) \\ & \frac{1}{2} \tan^{-1} (x) + \frac{x}{2(1+x^2)} + C \quad (٢٥) \quad \frac{\sqrt{4x^2-9}}{9x} + C \quad (٢٣) \quad \frac{\sqrt{x^2-16}}{16x} + C \quad (٢١) \\ & \frac{1}{4} \quad (٣٣) \quad \frac{\pi}{4} \quad (٣١) \quad \ln(\sqrt{2}+1) \quad (٢٩) \quad \frac{9 \sin^{-1}(x/3)}{2} + \frac{x \sqrt{9-x^2}}{2} + C \quad (٢٧) \\ & \frac{(x^2+4)^{-3/2}}{3} + C \quad (٣٩) \quad \frac{10\sqrt{3}+18}{243} \quad (٣٧) \quad \frac{8(2-\sqrt{2})}{3} \quad (٣٥) \\ & \frac{(3+2x^2)\sqrt{x^2-3}}{27x^3} + C \quad (٤٣) \quad \frac{1}{2(16-x^2)} + C \quad (٤١) \\ & \frac{\sin^{-1}(x^4)}{4} + C \quad (٤٧) \quad \frac{\ln(x^2 + \sqrt{x^4 - 25})}{2} + C \quad (٤٥) \\ & \sqrt{e^{2x} - 9} - 3 \sec^{-1} \left( \frac{e^x}{3} \right) + C \quad (٤٩) \end{aligned}$$

تمارين (١-٥)

$$\begin{aligned} & -2x \cos \frac{x}{2} + 4 \sin \frac{x}{2} + C \quad (٥) \quad 2 \quad (٣) \quad -(x+1)e^{-x} + C \quad (١) \\ & \frac{16 \ln 4}{3} - \frac{28}{9} \quad (١١) \quad 2 \quad (٩) \quad x^2 \sin x + 2x \cos x - 2 \sin x + C \quad (٧) \\ & \frac{e^{-x} (3 \sin 3x - \cos 3x)}{10} + C \quad (١٥) \quad x \sin^{-1} x + \sqrt{1-x^2} + C \quad (١٣) \\ & x \sec x - \ln|\sec x + \tan x| + C \quad (١٩) \quad \frac{(\sin 2x - 2x \cos 2x)}{8} + C \quad (١٧) \\ & \frac{\csc x \cot x}{2} + \frac{\ln|\csc x - \cot x|}{2} + C \quad (٢١) \\ & x^3 \cosh x - 3x^2 \sinh x + 6x \cosh x - 6 \sinh x + C \quad (٢٣) \\ & x \tan x - (x^2/2) + \ln|\cos x| + C \quad (٢٧) \quad x \cos^{-1} x - \sqrt{1-x^2} + C \quad (٢٥) \\ & \frac{\pi}{4\sqrt{2}} + 1 - \frac{\sqrt{16-\pi^2}}{4} \quad (٣١) \quad x \ln(x^2+4) - 2x + 4 \tan^{-1}(x/2) + C \quad (٢٩) \\ & \frac{2x}{\sqrt{1-x^2}} - 2 \sin^{-1} x + C \quad (٣٥) \quad \frac{3\sqrt{e-4}}{2e} \quad (٣٣) \\ & (\pi^2/2) - 2 \quad (٤١) \quad (2x+1)e^x + C \quad (٣٩) \quad \frac{1}{3} x \sinh 3x - \frac{1}{9} \cosh 3x + C \quad (٣٧) \\ & \text{تمارين (٢-٥)} \\ & 0 \quad (٧) \quad (\sin^5 x/5) - (\sin^7 x/7) + C \quad (٥) \quad 2/3 \quad (٣) \quad (\sin^4 x/4) + C \quad (١) \\ & \frac{\tan^2 x}{2} + C \quad (١١) \quad \frac{1}{8} \left[ \frac{5x}{2} - 2 \sin 2x + \frac{3 \sin 4x}{8} + \frac{\sin^3 2x}{6} \right] + C \quad (٩) \\ & \frac{\cos^5 x}{5} + \frac{2 \cos^3 x}{3} - \cos x + C \quad (١٥) \quad \frac{\tan^5 x}{5} + \frac{\tan^3 x}{3} + C \quad (١٣) \\ & -\frac{\cos^3 2x}{6} + \frac{\cos^5 2x}{10} + C \quad (١٩) \quad \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C \quad (١٧) \\ & -\ln|2-\sin x| + C \quad (٢٧) \quad \frac{-1}{(1+\tan x)} + C \quad (٢٥) \quad \frac{\cot^3 x}{3} - \cot x + C \quad (٢١) \end{aligned}$$

$$\frac{2}{3}x^3 + 7x + \frac{1}{4}\ln|x| - \frac{57}{8}\ln|x+2| + \frac{55}{8}\ln|x-2| + C \quad (٣٧)$$

$$\frac{5}{2}\ln(x^2+1) - 3\tan^{-1}x - \frac{1}{x^2+1} + C \quad (٤١)$$

$$\ln|x| + \frac{1}{2}\ln(x^2+4) - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C \quad (٤٣)$$

$$\ln|x| - \frac{1}{2}\ln|x^2+1| - 2\tan^{-1}x - \frac{2x+1}{2(x^2+1)} + C \quad (٤٥)$$

$$-\frac{7}{15}\ln|3x-1| + \frac{2}{5}\ln(x^2+1) + \frac{3}{5}\tan^{-1}x + C \quad (٤٧)$$

$$\ln|x-1| - 2\ln|x+2| - 3\ln|x+3| + C \quad (٤٩)$$

$$\frac{x-3}{7(x^2+x+2)} + \frac{2\sqrt{7}}{49}\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right) + C \quad (٥١)$$

$$\frac{1}{3}\ln|x-1| + \frac{\sqrt{3}}{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) - \frac{1}{6}(x^2+x+1) + C \quad (٥٣)$$

$$\frac{1}{3}x^3 + \frac{1}{2}\ln(x^2+6x+10) - 3\tan^{-1}(x+3) + C \quad (٥٥)$$

$$\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right) + \frac{1}{x^2+2x+3} + C \quad (٥٩)$$

$$\frac{1}{5}\ln|4x-1| - \frac{1}{10}\ln(4x^2+1) - \frac{1}{10}\tan^{-1}(2x) + C \quad (٦١)$$

$$[\ln x + 3\ln x + 3] + 2\ln(x-1) \Big|_2^3 = 4\ln 6 - 3\ln 5 \quad (٦٣)$$

$$\left[ \frac{1}{2}\ln(x^2+2x+2) - 2\tan^{-1}(x+1) \right]_0^1 = \frac{1}{2}\ln\left(\frac{5}{2}\right) - 2\tan^{-1}2 + \frac{\pi}{2} \quad (٦٥)$$

$$2\ln|x+1| + \ln|x-2| + \frac{3}{x-2} + C \quad (٦٩)$$

$$\cot\theta + \ln|\cot\theta| - \cot\theta + C \quad (٦٧)$$

## تمارين (٦-٥)

$$\frac{1}{6}(2x-1)^{3/2} + \frac{1}{2}(2x-1)^{1/2} + C \quad (٣)$$

$$\frac{1-10x}{80(2x-1)^5} + C \quad (١)$$

## تمارين (٤-٥)

$$\frac{1}{2}\tanh^{-1}\left(\frac{x+1}{2}\right) + C \quad (٣)$$

$$\frac{\ln[(x-2)^2+4]}{2} + \tan^{-1}\left(\frac{x-2}{2}\right) + C \quad (١)$$

$$\sqrt{x^2+2x+2} + 2\sinh^{-1}(x+1) + C \quad (٧)$$

$$-\tanh^{-1}(x-1) + C \quad (٥)$$

$$\frac{1}{4}\tan^{-1}(4x+2) + C \quad (١١)$$

$$-2\sqrt{9-8x-x^2} - 5\sin^{-1}\left(\frac{x+4}{5}\right) + C \quad (٩)$$

$$1 + \frac{\pi}{4} \quad (١٥)$$

$$\frac{1}{4}\ln(x^2+x+\frac{5}{4}) + \frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + C \quad (١٣)$$

$$3\sin^{-1}\left(\frac{x+5}{6}\right) + C \quad (١٩)$$

$$\sin^{-1}(x-1) + C \quad (١٧)$$

$$\frac{1}{6}\ln|x+1| - \frac{1}{2}\ln|x+7| + C \quad (٣)$$

$$\frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| + C \quad (١)$$

$$\ln|x-3| + \frac{2}{3}\ln|3x+1| + C \quad (٧)$$

$$\frac{1}{3}[\ln|x-2| - \ln|x+1|] \Big|_3^7 = \frac{1}{3}\ln\left(\frac{5}{2}\right) \quad (٥)$$

$$3x + 12\ln|x-2| - \frac{2}{x-2} + C \quad (١١)$$

$$\frac{1}{2}x^2 - x + \ln|x+1| + C \quad (٩)$$

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\ln|x-1| + C \quad (١٥)$$

$$x - \ln|x| + 2\ln|x-1| + C \quad (١٣)$$

$$\frac{x}{2(x^2+1)} + \frac{1}{2}\tan^{-1}x + C \quad (١٩)$$

$$4\ln|x| - 4\ln|2x+1| + \frac{2}{x} + \frac{1}{2x^2} + C \quad (١٧)$$

$$x - \frac{1}{3}\ln|x+1| + \frac{1}{6}\ln(x^2-x+1) - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C \quad (٢١)$$

$$\frac{1}{3(x^3+3x)} + C \quad (٢٥)$$

$$3\ln|x| + \frac{2}{x} - \frac{1}{2x^2} - 3\ln|x+1| + \frac{1}{x+1} + C \quad (٢٣)$$

$$-\frac{1}{4}\ln|x-1| - \frac{1}{2(x-1)} + \frac{1}{4}\ln|x+1| + C \quad (٢٩)$$

$$\frac{1}{x} + 3\ln|x-1| + C \quad (٢٧)$$

$$\frac{x^2}{2} - \ln|x-\sqrt{2}| + 2\ln|x+\sqrt{2}| + C \quad (٣٥)$$

$$\frac{3}{2} + 4\ln\left(\frac{3}{4}\right) \quad (٣٣)$$

$$\frac{19}{5} - \frac{9}{5}\ln 5 \quad (٣١)$$

$$(1+\cos x = 2\cos^2(x/2)) \text{ استخدم } 4\sqrt{2} \quad (٢٥)$$

$$\frac{1}{6(x+1)^6} + \frac{3}{7(x+1)^7} + \frac{3}{8(x+1)^8} + \frac{1}{9(x+1)^9} + C \quad (٢٧)$$

$$\frac{1}{2}(x^2+1)(\tan^{-1}x)^2 - x \tan^{-1}x + \frac{1}{2}\ln(x^2+1) + C \quad (٢٩)$$

$$\ln|\sec(e^x) + \tan(e^x)| + C \quad (٣٣) \quad \frac{1}{5}(x^2-25)^{5/2} + \frac{25}{3}(x^2-25)^{3/2} + C \quad (٣١)$$

$$x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6 \cos x + \sin x + C \quad (٣٥)$$

$$\frac{1}{2}\ln(x^2+6x+10) - \frac{1}{3}\tan^{-1}(x+3) + C \quad (٣٧)$$

$$\ln\left(\frac{(x+3)^2(x^2+9)^2}{|x-3|^5}\right) + \frac{1}{3}\tan^{-1}\frac{1}{x} + C \quad (٣٩)$$

$$1(٥٩) \quad 1(٥٧) \quad 1(٥٥) \quad 2(٥٣) \quad 3(٥١) \quad 4(٤٩) \quad 5(٤٧) \quad 6(٤٥) \quad 7(٤٣) \quad 8(٤١)$$

## تمارين (١-٦)

$$3/2(١٥) \quad 1/3(١٣) \quad \ln 2(١١) \quad 0(٩) \quad +\infty(٧) \quad 0(٥) \quad 4(٣) \quad 4(١)$$

$$0(٢٩) \quad 1(٢٧) \quad 1/2(٢٥) \quad -2(٢٣) \quad 0(٢١) \quad 2/3(١٩) \quad 1/2(١٧)$$

$$1/6(٤٣) \quad 1(٤١) \quad 1/9(٣٩) \quad -1/12(٣٧) \quad 2(٣٥) \quad \ln 3(٣٣) \quad -1/2\pi(٣١)$$

$$1(٥٧) \quad e^{-3}(٥٥) \quad 2(٥٣) \quad \pi(٥١) \quad -1/3(٤٩) \quad \frac{1}{3c^{2/3}}(٤٧) \quad -1(٤٥)$$

$$+\infty(٧١) \quad 0(٦٩) \quad e^2(٦٧) \quad e(٦٥) \quad 1(٦٣) \quad e^{2/\pi}(٦١) \quad +\infty(٥٩)$$

$$5(٧٩) \quad -1/2(٧٧) \quad -\infty(٧٥) \quad 1/2(٧٣)$$

## تمارين (٧-٦)

$$\frac{1}{4}e^2(١٣) \quad \ln\left(\frac{3}{2}\right)(١١) \quad \text{تباعدي}(٩) \quad \text{تباعدي}(٧) \quad -1/4(٥) \quad 1/2(٣) \quad 1(١)$$

$$\text{تباعدي}(٢٧) \quad 2(٢٥) \quad \ln 2(٢٣) \quad \pi(٢١) \quad \pi(١٩) \quad 1/\ln 2(١٧) \quad 1(١٥)$$

$$-4(٣٩) \quad \text{تباعدي}(٣٧) \quad 2^{3/4}(٣٥) \quad \text{تباعدي}(٣٣) \quad \text{تباعدي}(٣١) \quad \frac{\pi}{2}(٢٩)$$

$$2\sqrt{x} - 2\ln(1+\sqrt{x}) + C \quad (٧) \quad x - 2\sqrt{x} + 2\ln(\sqrt{x}+1) + C \quad (٥)$$

$$\frac{2(x^3+1)^{5/2}}{2(x^3+1)^{3/2}} + C \quad (١١) \quad \frac{3x^{2/3}}{2} - \frac{3\ln(x^{2/3}+1)}{2} + C \quad (٩)$$

$$\frac{3x^{4/3}}{4} - x + \frac{3x^{2/3}}{2} - 3x^{1/3} + 3\ln|x^{1/3}-1| + C \quad (١٥) \quad \sqrt{x} - \ln|1+\sqrt{x}| + C \quad (١٣)$$

$$\frac{2}{3}\left[2\ln|\sqrt{x}+1-2| + \ln(\sqrt{x}+2+1)\right] + C \quad (١٩) \quad \frac{5}{2}\ln|x^{2/5}-1| + C \quad (١٧)$$

$$2\tan^{-1}(\sqrt{x})^{1/3} = \pi/3 \quad (٢٣) \quad \frac{3}{10}(x^2+1)^{5/3} - \frac{3}{4}(x^2+1)^{2/3} + C \quad (٢١)$$

$$\frac{1}{5}\ln|\cos x + 3| - \frac{1}{5}\ln|\cos x - 2| + C \quad (٢٧) \quad \frac{3}{4}\ln|x^{4/3}-1| + C \quad (٢٥)$$

$$\frac{1}{4}\ln\left|\tan\left(\frac{x}{2}\right) + \tan^2\left(\frac{x}{2}\right)\right| + C \quad (٣١) \quad e^x + \frac{1}{2}\ln|x^2-1| - \frac{1}{2}\ln|x^2+1| + C \quad (٢٩)$$

$$\frac{1}{2}\ln\left|\frac{1+\tan(x/2)}{1-\tan(x/2)}\right| + \frac{1}{1+\tan(x/2)} - \frac{1}{[1+\tan(x/2)]^2} + C \quad (٣٣)$$

$$2\sqrt{1-e^x} + \ln(1-\sqrt{1-e^x}) - \ln(1+\sqrt{1-e^x}) + C \quad (٣٥)$$

$$\frac{1}{\sqrt{2}}\ln\left|\frac{\tan(x/2)-1-\sqrt{2}}{\tan(x/2)-1+\sqrt{2}}\right| = \sqrt{2}\ln(\sqrt{2}+1) \quad (٣٧)$$

$$\frac{1}{4}\ln|\tan(x/2)-3| - \frac{1}{4}3\tan(x/2) - 1 + C \quad (٣٩)$$

## تمارين عامة (٧-٥)

$$\frac{1}{5}x^5 \ln x - \frac{1}{25}x^5 + C \quad (٥) \quad + \frac{1}{3}\ln\left|\frac{x-2}{2x-1}\right| + C \quad (٣) \quad \frac{(\tan^{-1}x)^6}{6} + C \quad (١)$$

$$\ln|x| - \ln\sqrt{x^2+1} + C \quad (٩) \quad \sqrt{1+2x} - 3\ln(\sqrt{1+2x}+3) + C \quad (٧)$$

$$(x+1)\ln(x^2+2x+2) - 2(x+1) + 2\tan^{-1}(x+1) + C \quad (١١)$$

$$\frac{1}{2}x^2 + x + 2\ln|x-1| + C \quad (١٧) \quad e^x + C \quad (١٥) \quad \frac{1}{2}\cos^2 x - \ln|\cos x| + C \quad (١٣)$$

$$\frac{1}{11}\tan^{11}x + \frac{1}{9}\tan^9x + C \quad (٢٣) \quad \sin^{-1}\left(\frac{x-1}{3}\right) + C \quad (٢١) \quad \frac{\sqrt{1+x^2}}{x} + C \quad (١٩)$$





$$V = \int_1^9 \pi[(y-1)^{2/3} - 1^2] dy = \frac{58\pi}{5} \quad (٣٣٧) \quad V = \int_0^{\pi/2} \pi \cos y dy = \pi \quad (٣٣٨)$$

$$V = \int_0^1 \pi[1^2 - (1-x^2)^2] dx = \frac{7\pi}{15} \quad (٣٣٩)$$

$$V = \int_0^1 \pi[1^2 - (1-x^3)^2] dx = \frac{5\pi}{14} \quad (٣٤٠)$$

$$V = \int_1^2 \pi\left(\frac{1}{x^3}\right)^2 dx = \frac{31\pi}{160} \quad (٣٤١) \quad V = \int_0^1 2\pi(1-x)(x-x^2) dx = \frac{\pi}{6} \quad (٣٤٢)$$

$$V = \int_0^{\sqrt{\pi}} 2\pi x \sin x^2 dx = 2\pi \quad (٣٤٣) \quad V = \int_0^1 2\pi(1-x)(x-x^2) dx = \frac{\pi}{6} \quad (٣٤٤)$$

$$V = \int_0^3 2\pi x (2\sqrt{1-(x-2)^2}) dx \quad (٣٤٥)$$

$$= \int_{-1}^1 \pi[(2+\sqrt{1-y^2})^2 - (2-\sqrt{1-y^2})^2] dy = \pi$$

$$V = \int_0^{\pi/4} \pi \tan^2 x dx = \pi \left(1 - \frac{\pi}{4}\right) \quad (٣٤٦) \quad V = \int_0^{\pi} 2\pi \sin x dx = 2\pi^2 \quad (٣٤٧)$$

$$V = \int_0^4 \pi(\sqrt{x})^2 dx + \int_4^6 \pi(6-x)^2 dx = \frac{32\pi}{3} \quad (٣٤٨)$$

$$V = \int_{1/e}^1 I^2 dy + \int_{1/e}^1 \pi(\sqrt{-\ln y})^2 dy = \pi \left(1 - \frac{1}{e}\right) \quad (٣٤٩)$$

$$V = \int_0^1 \pi[(e+1)^2 - (e^y+1)^2] dy = \frac{(e^2+5)\pi}{2} \quad (٣٥٠)$$

$$V = \int_0^3 \frac{2\pi x}{1+x^2} dx = \pi \ln 10 \quad (٣٥١) \quad V = \int_{3\pi/4}^{5\pi/4} 2\pi x \sec^2 x dx = 4\pi^2 \quad (٣٥٢)$$

$$V = \int_2^3 2\pi x [\sqrt{x-2} - (x-2)] dx = \frac{4\pi}{5} \quad (٣٥٣)$$

$$V = \int_0^1 2\pi x (e^x - e^{-x}) dx = \frac{4\pi}{e} \quad (٣٥٤)$$

$$V = \int_0^1 2\pi y [(2-y) - y^2] dy = \frac{5\pi}{6} \quad (٣٥٥)$$

$$V = \int_0^1 2\pi x (x \sin x^3) dx = \frac{2\pi}{3} [1 - \cos 1] \quad (٣٥٦)$$

$$A = \int_0^{\pi/2} [x - x \sin x] dx = \frac{\pi^2}{8} - 1 \quad (٣٥٧) \quad A = \int_0^{\pi/2} [1 - \sin y] dy = \frac{\pi}{2} - 1 \quad (٣٥٨)$$

$$A = \int_e^{\infty} \frac{\ln x - 1}{x^2} dx = \frac{1}{e} \quad (٣٥٩) \quad A = \int_0^{\infty} e^{-3x} dx = \frac{1}{3} \quad (٣٦٠)$$

$$A = \int_{-1}^0 [2^x - 5^x] dx + \int_0^1 [5^x - 2^x] dx = \frac{16}{5 \ln 5} - \frac{1}{2 \ln 2} \quad (٣٦١)$$

$$A = 2 \int_2^3 \frac{3\sqrt{x^2-4}}{2} dx = \frac{9\sqrt{5}}{2} - 6 \ln \left(\frac{3+\sqrt{5}}{2}\right) \quad (٣٦٢) \quad A = \int_0^{\pi/2} \tan x dx = \infty \quad (٣٦٣)$$

$$A = \int_0^1 \left[ \frac{1}{2-\sqrt{x}} - \frac{1}{2+\sqrt{x}} \right] dx = 4 \ln 3 - 4 \quad (٣٦٤) \quad A = \int_0^5 x e^{-x} dx = 1 - 6e^{-5} \quad (٣٦٥)$$

$$A = 2 \int_{-r}^r \frac{b}{a} \sqrt{a^2 - x^2} dx = \pi ab \quad (٣٦٦)$$

## تمارين (٣-٧)

$$V = \int_0^4 \pi(\sqrt{y})^2 dy = 8\pi \quad (٣٦٧) \quad V = \int_0^1 \pi(x^2)^2 dx = \frac{\pi}{5} \quad (٣٦٨)$$

$$V = \int_0^2 \pi[(2y)^2 - (y^2)^2] dy = \frac{64}{15} \pi \quad (٣٦٩)$$

$$V = \int_0^4 \pi(x^2 - 4x)^2 dx = \frac{512\pi}{15} \quad (٣٧٠)$$

$$V = \int_{-1}^2 \pi[(y+2)^2 - (y^2)^2] dy = \frac{72}{5} \quad (٣٧١)$$

$$V = \int_0^{\pi/4} \pi(\cos^2 x - \sin^2 x) dx = \frac{\pi}{2} \quad (٣٧٢)$$

$$V = \int_1^2 \pi(1+x)^{3/2} dx = \frac{373\pi}{14} \quad (٣٧٣)$$

$$V = \int_{-3}^3 \pi[(9)^2 - (x^2)^2] dx = \frac{1944\pi}{5} \quad (٣٧٤)$$

$$V = \int_{-1}^1 \pi(1-y^2)^2 dy = \frac{16\pi}{15} \quad (٣٧٥) \quad V = \int_0^1 \pi[(\sqrt{x})^2 - x^2] dx = \frac{\pi}{6} \quad (٣٧٦)$$

$$S = 2\pi \int_0^2 y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = 8\pi\sqrt{5} \quad (٢١)$$

$$S = 2\pi \int_1^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \frac{16\pi}{9} \quad (٢٢)$$

$$S = 2\pi \int_{-2}^2 x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy = 24\pi \quad (٢٥)$$

$$S = 2\pi \int_1^2 x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy = \frac{16911\pi}{1024} \quad (٢٧)$$

$$S = 2\pi \int_1^3 y \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy = \frac{5813\pi}{30} \quad (٢٩)$$

$$S = 2\pi \int_0^{\sqrt{2}} x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy = \frac{\pi}{4} \left[ 2e\sqrt{1+4e^2} - 2\sqrt{5} + \ln \left[ \frac{2e + \sqrt{1+4e^2}}{2 + \sqrt{5}} \right] \right] \quad (٣١)$$

$$S = 2\pi \int_1^2 x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy = \frac{\pi}{8} [21 - 8\ln 2 - (\ln 2)^2] \quad (٣٣)$$

تمارين (٤-٧)

$$v(t) = 2\sin 2t - 1 \quad (٥) \quad s(t) = t - \cos t - 2 \quad (٦) \quad s(t) = t^2 - 3t + 7 \quad (١)$$

$$s(t) = 3t^3 + 36000t + s_0 \quad (١) \quad v_0 = 24ft / \sec \quad (٩) \quad s(t) = \frac{1}{3}t^3 - \frac{3}{2}t^2 + 4t \quad (٧)$$

$$v(7) = -112 \quad (ج) \quad s(7/2) = 196 \quad (ب) \quad v(3) = 16 \quad (د) \quad (١٣)$$

$$v = -73.1, \quad t = 12.46 \quad (٥) \quad v = -49, \quad t = 10 \quad (ج) \quad s = 272.5 \quad (ب) \quad t = 5 \quad (د) \quad (١٥)$$

تمارين (٥-٧)

$$W = \int_0^{10} (5x^2 + 1) dx = \frac{5030}{3} ft - lb \quad (١)$$

$$W = \int_0^6 \frac{62.4}{4} \pi (8x^2 - x^3) dx \approx 12350.23 ft - lb \quad (٣)$$

$$V = \int_1^2 2\pi(y-1)\ln y \, dy = \frac{\pi}{2} \quad (١٣) \quad V = \int_1^4 2\pi y \left[ 5 - y - \frac{4}{y} \right] dy = 9\pi \quad (١١)$$

$$V = \int_{-1}^2 2\pi(3-x)(x+2-x^2) dx = \frac{45}{2} \pi \quad (١٥)$$

$$V = \int_{-1}^0 2\pi(1-x)e^{-x} dx = 2\pi e \quad (١٧)$$

$$V = \frac{\sqrt{3}\pi r^3}{2} \quad (٧١) \quad V = \int_0^3 2\pi y [(y+1) - (y-1)^2] dy = \frac{27\pi}{2} \quad (١٩)$$

$$V = \int_0^1 \frac{1}{8} \pi x dx = \pi \quad (٧٥) \quad V = \int_0^1 (x-x^2)^2 dx = \frac{1}{30} \quad (٧٣)$$

$$V = \int_0^h \pi \left( R - \frac{R-r}{h} y \right)^2 dy = \frac{1}{3} \pi h (R^2 + Rr + r^2) \quad (٧٧)$$

$$V = \int_{-r}^r \pi [(R + \sqrt{r^2 - y^2})^2 - (R - \sqrt{r^2 - y^2})^2] dy = 2\pi^2 Rr^2 \quad (٧٩)$$

تمارين (٣-٧)

$$L = \int_1^4 \sqrt{1+144y} \, dy = \frac{1}{216} (577^{3/2} - 145^{3/2}) \quad (١)$$

$$L = \int_2^4 \left( x + \frac{1}{4}x^{-1} \right) dx = 6 + \frac{\ln 2}{4} \quad (٥) \quad L = \int_0^4 \left[ \frac{1}{2} \sqrt{x} + \frac{1}{2}x^{-1/2} \right] dx = \frac{14}{3} \quad (٣)$$

$$L = \int_2^4 \left( \frac{1}{8}y^2 + 2y^{-2} \right) dy = \frac{17}{6} \quad (٩) \quad L = \int_0^2 \sqrt{1+4x} \, dx = \frac{13}{3} \quad (٧)$$

$$L = \int_0^1 \sqrt{1+\frac{9y}{4}} dy + \int_0^4 \sqrt{1+\frac{9y}{4}} dy = \frac{13\sqrt{13} + 80\sqrt{10} - 16}{27} \quad (١١)$$

$$L = \int_0^{1/2} \frac{1+x^2}{1-x^2} dx = \ln 3 - \frac{1}{2} \quad (١٥) \quad L = \int_0^1 (x^2 + 1) dx = \frac{4}{3} \quad (١٣)$$

$$S = 2\pi \int_0^1 x \sqrt{1+(y')^2} dx = 2\pi(2^{3/2} - 1)/3 \quad (١٧)$$

$$S = 2\pi \int_0^3 y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = 30\pi \quad (١٩)$$

## تمارين عامة (٨-٧)

$$A = \int_0^{\pi/4} (\cos x - \sin x) dx = \sqrt{2} - 1 \quad (\text{ج}) \quad A = 2 \int_0^1 (x^{1/3} - x^3) dx = 1 \quad (\text{د})$$

$$V = \pi \int_0^{\pi/4} (\cos^2 x - \sin^2 x) dx = \frac{\pi}{2} \quad (\text{ب})$$

$$A = \int_1^2 \ln x dx = [x \ln x - x]_1^2 = 2 \ln 2 - 1 \quad (\text{ج}) \quad (\text{د})$$

$$V = \pi \int_1^2 (\ln x)^2 dx = \pi [2(\ln 2)^2 - 4 \ln 2 + 2] \quad (\text{ب})$$

$$V = \pi \int_0^2 \frac{x^4}{(9-x^2)^2} dx = \pi \left( \frac{19}{5} - \frac{9 \ln 5}{4} \right) \quad (\text{ج})$$

$$A = \int_0^{\pi^2} \sin \sqrt{x} dx = 2[-\sqrt{x} \cos \sqrt{x} + \sin \sqrt{x}]_0^{\pi^2} = 2\pi \quad (\text{أ})$$

$$V = 2\pi \int_1^e x \ln x dx = 2\pi \left[ \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 \right]_1^e = \frac{\pi}{2} (e^2 + 1) \quad (\text{د})$$

$$S(t) = -\frac{1}{3} \cos t + \frac{1}{9} \cos^3 t + 10t \quad (\text{أ}) \quad S(t) = -\frac{1}{2} t e^{-2t} - \frac{1}{4} e^{-2t} + \frac{1}{4} \quad (\text{ب})$$

$$V = 2\pi \int_0^{\pi^2} \frac{x^2}{\sqrt{x^2+25}} dx = 25\pi \left[ \sqrt{2} - \ln(\sqrt{2}+1) \right] \quad (\text{د})$$

$$V = \pi \int_0^3 x^2 \sqrt{9-x^2} dx = \frac{81\pi^2}{16} \quad (\text{أ}) \quad A = \int_0^2 \frac{x}{x^2-2x-3} dx = \frac{1}{2} \ln 3 \quad (\text{أ})$$

$$A = \int_0^{\pi/2} [\sin x - \sin^2 x] dx = \frac{1}{3} \quad (\text{أ}) \quad A = \int_4^9 \frac{1}{\sqrt{x}+4} dx = 2 + 8 \ln \left( \frac{6}{7} \right) \quad (\text{ب})$$

$$V = 2\pi \int_0^{3\sqrt{5}/2} \frac{x^3}{(4x^2+9)^{3/2}} dx = 3\pi/16 \quad (\text{ب})$$

$$V = 2\pi \int_0^2 \frac{x}{x^2+2x+10} dx = \pi \ln \left( \frac{9}{5} \right) + \frac{2\pi}{3} \left[ \tan^{-1} \frac{1}{3} - \frac{\pi}{4} \right] \quad (\text{أ})$$

$$|E_T| < 0.0067, 1.090608 \quad (\text{ب}) \quad A = -\int_0^1 \ln x dx = -\lim_{t \rightarrow 0^+} \int_t^1 \ln x dx = 1 \quad (\text{ب})$$

$$V = \pi \int_{-1}^1 [(3-x^2)^2 - (x^2+1)^2] dx = 32\pi/3 \quad (\text{ب}) \quad 17.74 \quad (\text{ب})$$

$$W = \int_0^{1.8} 5x dx = 8.1 J \quad (\text{أ}) \quad W = \int_{1.2}^{12} 777x^{-1.4} dx = 1083 \text{ in-lb} \quad (\text{ب})$$

$$W = \int_1^3 \frac{9x}{2} dx = 18 \text{ in-lb} \quad (\text{ب}) \quad W = \int_0^4 \frac{9x}{2} dx = 36 \text{ in-lb} \quad (\text{ج}) \quad (\text{د})$$

$$W = \int_0^2 62.5\pi \left( \frac{y^2}{16} \right) (20-y) dy = 163625 \text{ ft-lb} \quad (\text{د}) \quad (\text{هـ})$$

$$W = \int_0^{3000} \frac{x}{2} dx = 625 \text{ ft-lb} \quad (\text{د}) \quad (\text{هـ})$$

$$W = \int_0^{3000} \left( 43 - \frac{x}{500} \right) dx = 120000 \text{ ft-ton} \quad (\text{د}) \quad (\text{هـ})$$

## تمارين (٦-٧)

$$\bar{y} = 1, \bar{x} = 19/15, M_y = 29, M_x = 15, M = 15 \quad (\text{أ})$$

$$\bar{y} = -43/17, \bar{x} = 39/17, M_y = 39, M_x = -43, M = 17 \quad (\text{ب})$$

$$\bar{y} = 2/5, \bar{x} = 2/5, M_y = 4, M_x = 4, M = 10 \quad (\text{ج})$$

$$\left( \frac{1}{2}, \frac{3}{5} \right) \quad (\text{د}) \quad \left( \frac{4}{5}, \frac{2}{7} \right) \quad (\text{هـ}) \quad \left( \frac{4}{5}, 0 \right) \quad (\text{و}) \quad \left( \frac{5}{8}, \frac{17}{20} \right) \quad (\text{ز})$$

$$\left[ \frac{4\sqrt{2}-2}{3} (\sqrt{2} + \ln(1+\sqrt{2}))^4, \frac{4}{3} (\sqrt{2} + \ln(1+\sqrt{2})) \right] \quad (\text{أ}) \quad (\text{ب})$$

$$\left( \frac{1}{8}, \frac{\pi}{8} \right) \quad (\text{أ}) \quad \left( \frac{1}{e-1}, \frac{e+1}{4} \right) \quad (\text{ب}) \quad (\text{ج})$$

## تمارين (٧-٧)

$$|E_S| < 0.000115, |E_T| < 0.014 \quad (\text{ج}) \quad 1.462681 \quad (\text{ب}) \quad 1.467175 \quad (\text{د}) \quad (\text{هـ})$$

$$|E_S| < 0.00114, |E_T| < 0.03125 \quad (\text{ج}) \quad 0.904501 \quad (\text{ب}) \quad 0.895759 \quad (\text{د}) \quad (\text{هـ})$$

$$|E_S| < 0.000044, |E_T| < 0.00088 \quad (\text{ج}) \quad 0.983819 \quad (\text{ب}) \quad 0.984120 \quad (\text{د}) \quad (\text{هـ})$$

$$1.536963 \quad (\text{ب}) \quad 1.526348 \quad (\text{د}) \quad (\text{هـ}) \quad 1.137631 \quad (\text{ب}) \quad 1.133926 \quad (\text{د}) \quad (\text{هـ})$$

$$n=6 \quad (\text{ب}) \quad n=22 \quad (\text{د}) \quad (\text{هـ}) \quad n=8 \quad (\text{ب}) \quad n=41 \quad (\text{د}) \quad (\text{هـ})$$

$$(-4,0) \quad (2\sqrt{2}, 2\sqrt{2}) \quad (7) \quad (3\sqrt{3}, 3) \quad (5) \quad \left( \frac{7\sqrt{3}}{2}, \frac{7}{2} \right) \quad (3) \quad (3\sqrt{3}, 3) \quad (1)$$

$$(r, \theta) = \left( \sqrt{2}, \frac{3\pi}{4} + 2n\pi \right) = \left( -\sqrt{2}, \frac{3\pi}{4} + (n+1)\pi \right) \quad (11)$$

$$(r, \theta) = \left( 4, \frac{11\pi}{6} + 2n\pi \right) = \left( -4, \frac{11\pi}{6} + (n+1)\pi \right) \quad (13)$$

$$(r, \theta) = \left( \sqrt{2}, \frac{7\pi}{4} + 2n\pi \right) = \left( -\sqrt{2}, \frac{7\pi}{4} + (n+1)\pi \right) \quad (15)$$

$$(r, \theta) = \left( 8\sqrt{2}, \frac{3\pi}{4} + 2n\pi \right) = \left( -8\sqrt{2}, \frac{3\pi}{4} + (n+1)\pi \right) \quad (17)$$

$$(r, \theta) = \left( 6, \frac{2\pi}{3} + 2n\pi \right) = \left( -6, \frac{2\pi}{3} + (n+1)\pi \right) \quad (19)$$

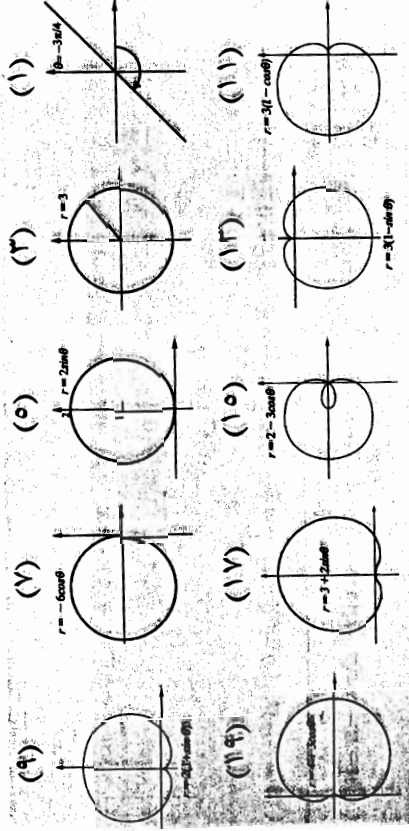
$$r = -4\cos\theta \quad (27) \quad r^2 = \csc 2\theta \quad (25) \quad r = 4\tan\theta \sec\theta \quad (23) \quad r = 5\sec\theta \quad (21)$$

$$x^2 - y^2 = 9 \quad (35) \quad (x-2)^2 - \frac{y^2}{3} = 1 \quad (33) \quad 2x + 3y = 6 \quad (31) \quad r^2 = \sin 2\theta \quad (29)$$

$$y = 2 \quad (37) \quad x \tan(x^2 + y^2) = y \quad (39) \quad x^2 + y^2 = -3x \quad (43)$$

$$(3x^2 + 4y^2 + 12x = 36 \text{ (دائرة)}) \quad (45) \quad (3x^2 - x^2 - 8y = -4 \text{ (قطع زائد)}) \quad (49) \quad (3x^2 + 4y^2 + 12x = 36 \text{ (قطع زائد)}) \quad (47)$$

## تمارين (٣-٨)



$$L = \int_1^2 \sqrt{1 + \frac{9}{4}(x-1)} dx = \frac{13\sqrt{13}-8}{27} \quad (39)$$

$$L = \int_1^2 \frac{\cosh x}{\sinh x} dx = \ln \left( \frac{\sinh 2}{\sinh 1} \right) \quad (41)$$

$$S = 2\pi \int_{1/2}^{3/2} \sqrt{1+x^{-2/3}} dx = \frac{3\pi}{5} (50\sqrt{5} - 2\sqrt{2}) \quad (43)$$

$$S = 2\pi \int_0^{\pi/4} \tan x \sqrt{1+\sec^4 x} dx \approx 3.84 \quad (45)$$

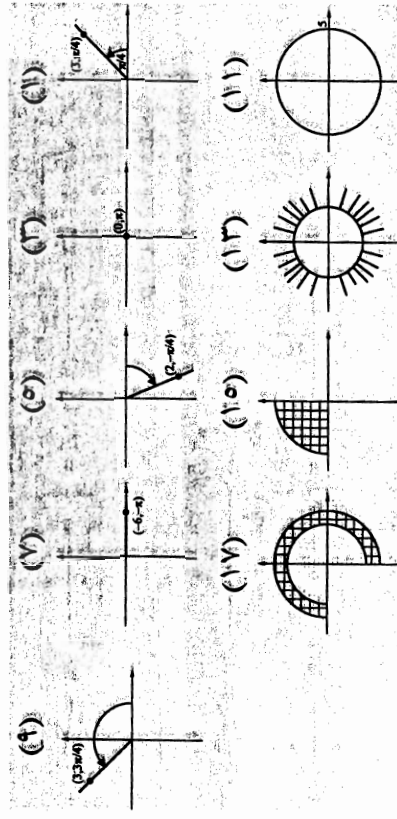
$$S = S_1 + S_2 = \int_{-2}^2 2\pi [2 - \sqrt{4-x^2}] \sqrt{\frac{1+x^2}{4-x^2}} dx \quad (47)$$

$$+ \int_{-2}^2 2\pi [2 + \sqrt{4-x^2}] \sqrt{\frac{1+x^2}{4-x^2}} dx = 16\pi^2$$

$$W = \int_0^4 \pi (2\sqrt{y})^2 62.5(4-y) dy = \frac{8000h}{3} ft-lb \quad (49)$$

$$\text{ب(67) ب(65) ب(63) د(59) د(57) د(55) د(53) د(51) د(49) د(47) د(45) د(43) د(41) د(39) د(37) د(35) د(33) د(31) د(29) د(27) د(25) د(23) د(21) د(19) د(17) د(15) د(13) د(11) د(9)}$$

## تمارين (٢-٨)



## تمارين (٢-٨)

$$A = \int_{-\pi/2}^{\pi/2} (4 - \sin\theta)^2 d\theta = \frac{33\pi}{2} \quad (١١) \quad A = \int_{-\pi/2}^{\pi/2} (1 + \sin\theta)^2 d\theta = \frac{3\pi}{2} \quad (٩)$$

$$A = \int_{2\pi/3}^{\pi} (1 + 2\cos\theta)^2 d\theta = \pi - \frac{3\sqrt{3}}{2} \quad (١٥) \quad A = \frac{1}{2} \int_{\pi/6}^{\pi/3} \theta^2 d\theta = \frac{7\pi^3}{1296} \quad (١٣)$$

$$A = \frac{1}{2} \int_{\pi/4}^{\pi/2} (2\csc\theta)^2 d\theta = 2 \quad (١٩) \quad A = 3 \int_0^{\pi/6} 16\cos^2 3\theta d\theta = 4\pi \quad (١٧)$$

$$A = \frac{1}{2} \int_0^{\pi} (1 + \sin\theta)^2 - \frac{1}{2} \int_0^{\pi} 1^2 d\theta = 2 + \frac{\pi}{4} \quad (٢١)$$

$$A = \frac{1}{2} \int_{-3\pi/4}^{\pi/2} (1 + \sqrt{2}\cos\theta)^2 d\theta = \frac{1}{2} \int_{-3\pi/4}^{\pi/2} (1 + \sqrt{2}\cos\theta)^2 d\theta = \pi + 3 \quad (٢٢)$$

$$A = \frac{1}{2} \int_{-\pi/3}^{\pi/3} [(4 + 4\cos\theta)^2 - 6^2] d\theta = 18\sqrt{3} - 4\pi \quad (٢٥)$$

$$A = \int_{\pi/6}^{\pi/2} [25\sin^2\theta - (2 + \sin\theta)^2] d\theta = \frac{8\pi}{3} + \sqrt{3} \quad (٢٧)$$

$$A = \frac{1}{2} \int_0^{\pi/4} [16\cos^2\theta - 16\sin^2\theta] d\theta = 4 \quad (٢٩)$$

$$A = \int_0^{\cos^{-1}(3/5)} [100 - 36\sec^2\theta] d\theta = 100\cos^{-1}\left(\frac{3}{5}\right) - 48 \quad (٣١)$$

$$A = \frac{1}{2} \int_{-\pi/3}^{\pi/3} [4 - \sec^2\theta] d\theta = \frac{4\pi}{3} - \sqrt{3} \quad (٣٣)$$

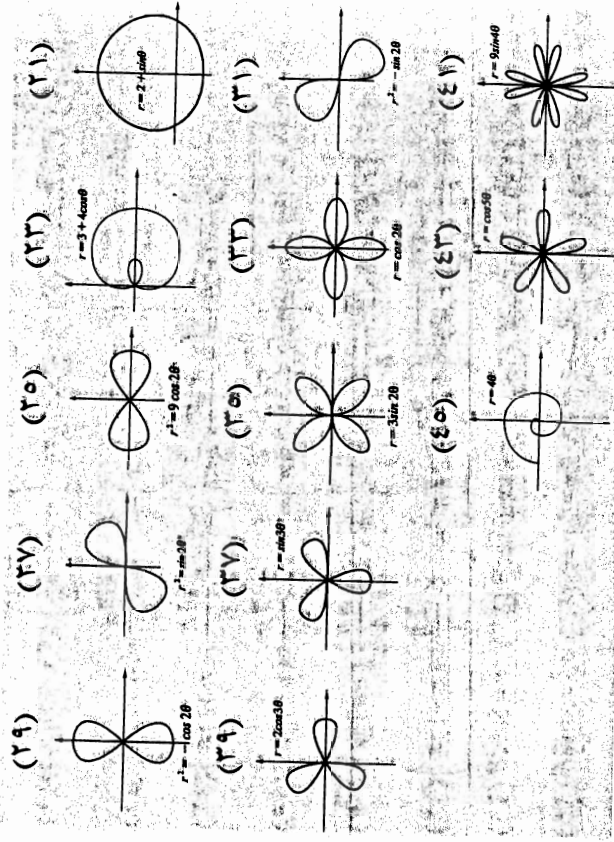
$$A = \frac{1}{2} \int_{-\pi/2}^{\pi/2} [(2\cos\theta)^2 - (\cos\theta)^2] d\theta = \frac{3\pi}{4} \quad (٣٥)$$

$$A = \int_{-\pi/6}^{\pi/6} [2\cos 2\theta - 1^2] d\theta = \sqrt{3} - \frac{\pi}{3} \quad (٣٧)$$

$$A = \int_0^{\pi/6} (\sqrt{2}\sin\theta)^2 d\theta + \int_{\pi/6}^{\pi/2} \sin\theta d\theta = \frac{\pi}{6} + \frac{\sqrt{3}}{4} \quad (٣٩)$$

تمارين (٦-٨)

$$\frac{(x-3)^2}{9} + \frac{(y-2)^2}{4} = 1 \quad \text{القطع الناقص (٣) (3,3) إلى (-1,1) من } x-2y+3=0 \quad (١)$$



تمارين (٤-٨)

$$r = \frac{6}{1+2\sin\theta} \quad (٧) \quad r = \frac{6}{1-\sin\theta} \quad (٥) \quad r = \frac{6}{5+3\sin\theta} \quad (٣) \quad r = \frac{12}{3-4\cos\theta} \quad (١)$$

$$y = -6, e = 1/2, \text{ قطع ناقص, (١١)} \quad r = \frac{10}{3-2\sin\theta} \quad (٩)$$

$$y = 10/3, e = 3/2, \text{ قطع زائد, (١٥)} \quad y = -1, e = 3, \text{ قطع زائد, (١٣)}$$

$$y = 2, e = 3/2, \text{ قطع زائد, (١٩)} \quad x = -15/4, \text{ قطع مكافئ, (١٧)}$$

تمارين (٥-٨)

$$A = \frac{1}{2} \int_{-3\pi/4}^{3\pi/4} (1 + \sqrt{2}\cos\theta)^2 d\theta = \frac{3\pi}{2} + \frac{3}{2} \quad (٣) \quad A = \frac{1}{2} \int_0^{\pi} (2\sin\theta)^2 d\theta = \pi \quad (١)$$

$$A = 4 \left( \frac{1}{2} \right) \int_0^{\pi/2} \sin^2 2\theta d\theta = \frac{\pi}{2} \quad (٧) \quad A = \frac{1}{2} \int_0^{\pi/2} (1 - \cos\theta)^2 d\theta = \frac{3}{8}\pi - 1 \quad (٥)$$

$$L = 2 \int_{-\pi/4}^{\pi/2} \sqrt{2+2\sin\theta} d\theta = 8 \quad (٥١) \quad L = \int_0^{\pi/2} \sqrt{13}\sin 2t dt = \sqrt{13} \quad (٤٩)$$

$$L = \int_0^{2\pi} 2^\theta \sqrt{1+(\ln 2)^2} d\theta = \sqrt{\frac{1+(\ln 2)^2}{\ln 2}} (2^{2\pi} - 1) \quad (٥٣)$$

$$L = 2 \int_0^\pi 2 \cos\left(\frac{\theta}{2}\right) d\theta = 8 \quad (٥٧) \quad L = 2 \int_0^{2\pi} \left| \cos^3\left(\frac{\theta}{4}\right) \right| d\theta = \frac{16}{3} \quad (٥٥)$$

$$L = \int_{\pi/6}^{\pi/2} \tan\theta \sec^2\theta \sqrt{9\sec^2\theta - 5} d\theta = \frac{1}{27} (7^{3/2} - 8) \quad (٥٩)$$

$$S = \int_0^2 2\pi t \sqrt{4t^2 + 1} dt = \frac{\pi}{6} (17^{3/2} - 1) \quad (٦١)$$

$$S = 2\pi \int_0^\pi 2\sin t \sqrt{4\sin^2 t + 4\cos^2 t} dt = 16\pi \quad (٦٣)$$

$$S = 2\pi \int_0^{\pi/2} \sqrt{2} e^{2t} \sin t dt = \frac{2\sqrt{2}}{5} \pi (2e^\pi + 1) \quad (٦٥)$$

$$S = 2\pi \int_0^{2\pi} \sqrt{2} (1 - \cos t)^{3/2} dt = \frac{64\pi}{3} \quad (٦٧)$$

$$S = 2\pi \int_0^1 t^2 \sqrt{9t^4 + 4t^2} dt = \frac{2\pi(247\sqrt{13} + 64)}{1215} \quad (٦٩)$$

$$S = 4\pi \int_0^{\pi/2} \sin\theta d\theta = 4\pi \quad (٧٣) \quad S = 4\pi \int_0^{\pi/4} \sin\theta d\theta = 2\pi(2 - \sqrt{2}) \quad (٧١)$$

$$S = 2\pi \int_{-\pi/2}^{\pi/2} 4\cos^2\theta d\theta = 4\pi^2 \quad (٧٥)$$

$$S = 2\pi \int_0^\pi (1 + \cos\theta) \sin\theta \sqrt{2(1 + \cos\theta)} d\theta = \frac{32\pi}{5} \quad (٧٧)$$

$$S = 2\pi \int_0^{\pi/2} 2\sqrt{5} e^\theta \cos\theta d\theta = 2\pi\sqrt{5}(e^{\pi/2} - 1) \quad (٧٩)$$

تأريخ عامة (٧-٨)

$$x^2 + y^2 = 4x \quad (٥) \quad x = 1 \quad (ج) \quad x^2 = 4(y+1) \quad (ب) \quad \frac{(x-2)^2}{1} - \frac{y^2}{3} = 1 \quad (د) \quad (١)$$

$$1/t \quad (١١) \quad \pi(y + \pi) = x \quad (٩) \quad y - e = 2ex \quad (٧) \quad y - 16 = 2(x - 8) \quad (٥)$$

$$\frac{2}{(\cos t - \sin t)^2} \quad (١٧) \quad \frac{4(\cos t - \sin 2t + \sin t \sin 2t)}{(1 - 2\sin t)^2} \quad (١٥) \quad \frac{6}{t(3t+2)^3} \quad (١٣)$$

$$(±2, 5) \quad (١٩) \quad \text{عند } \left(\frac{65}{8}, \frac{-29}{4}\right) \text{ ورأسه عند } (±2, 5)$$

$$(٢٣) \quad \text{لا يوجد مماس أفقي؛ رأسه عند } (0, 0) \text{ و } (-3, 1)$$

$$(٢٥) \quad \text{أفقي عند } (0, -9) \text{ ورأسه عند } (±2, -6).$$

$$(٢٧) \quad \text{معادلتا المماسين: } y = ±\sqrt{3}(x - 3) \quad (ب) \quad \text{المماس أفقي عند } (1, ±2)$$

(ج) المماس رأسي عند (1, 2)

$$(٢٩) \quad \text{معادلتا المماسين: } y = 5x - 14 \quad \text{و } y = 6x - 17$$

$$(٣١) \quad \text{ميل المماس هو } -1, \text{ المماس رأسي عند } \left(0, \frac{3\pi}{2}\right) \text{ و } \left(\frac{3}{2}, \frac{\pi}{6}\right)$$

$$\text{المماس أفقي عند } \left(\frac{1}{2}, \frac{11\pi}{6}\right) \text{ و } \left(\frac{1}{2}, \frac{7\pi}{6}\right), \left(2, \frac{\pi}{2}\right), \left(\frac{3\pi}{8}, \frac{3\pi}{8}\right)$$

(٣٣) ميل المماس يساوي -1.

$$\text{المماس رأسي عند } \left(\cos \frac{5\pi}{8} + \sin \frac{5\pi}{8}, \frac{5\pi}{8}\right), \left(\cos \frac{\pi}{8} + \sin \frac{\pi}{8}, \frac{\pi}{8}\right)$$

$$\text{عند } \left(\cos \frac{7\pi}{8} + \sin \frac{7\pi}{8}, \frac{7\pi}{8}\right), \left(\cos \frac{3\pi}{8} + \sin \frac{3\pi}{8}, \frac{3\pi}{8}\right)$$

(٣٥) ميل المماس هو -1.

$$\theta = 0, \frac{5\pi}{6}, \frac{7\pi}{6} \quad \text{عندما } \theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}. \quad \text{المماس أفقي عندما } \theta = 0, \frac{5\pi}{6}, \frac{7\pi}{6}$$

$$L = \int_0^2 t \sqrt{9t^2 + 4} dt = \frac{1}{27} (40^{3/2} - 8) \quad (٣٧)$$

$$L = \int_0^1 (1+t) \sqrt{4+9(1+t)^2} dt = \frac{80\sqrt{10} - 13\sqrt{13}}{27} \quad (٣٩)$$

$$L = \int_0^{2\pi} t \sqrt{t^2 + 4} dt = \frac{8[(\pi^2 + 1)^{3/2} - 1]}{3} \quad (٤٣) \quad L = \int_0^{\pi/2} 2 dt = \pi \quad (٤١)$$

$$L = \int_0^\pi \theta d\theta = \frac{\pi^2}{2} \quad (٤٧) \quad L = \int_1^4 (1-t^2) dt = \frac{15}{4} \quad (٤٥)$$

## مخارج

### وئبته المصطالحات

Polar Coordinates	٢٧٥	الاحداثيات القطبية
Archimedes	١	ارشميدس
Right Cylinder	٢٢٢	اسطوانة قائمة
Logarithmic Differentiation	٦٠	الاشتقاق اللوغاريتمي
Partition	(٦)	تجزئ
Uniform Partition	٦	تجزئ منتظم
Trigonometric Substitutions	١٢٨	التعويضات المثلثية
Miscellaneous Substitutions	١٥٧	تعويضات منفردة
Numerical Integration	٢٥٨	التكامل العددي
Definite Integral	١	التكامل المحدد
Integration By Parts	١١٦	التكامل بالتجزئ
Integration By Substitution	٤٣	التكامل بالتعويض
Divergent Integral	١٨٠	تكامل تباعدي
Convergent Integral	١٨٠	تكامل تقاربي
Indefinite Integral	٣٨	التكامل غير المحدد
Integrals of Quadratic Forms	١٤٥	تكاملات الصيغ التربيعية
Improper Integrals	١٧٩	التكاملات المعتلة
Integrals of Powers of Trigonometric Functions	١٢٨	تكاملات قوى الدوال المثلثية

$$A = 5 \int_{-\pi/10}^{\pi/10} \frac{9 \cos 5\theta}{2} d\theta = 9 \text{ (أ) (٣)}$$

$$A = 2 \int_{\sin^{-1}(1/3)}^{\pi/2} \frac{1}{2} (1 - 3 \sin \theta)^2 d\theta = \frac{11\pi}{4} - \frac{11\pi}{2} \sin^{-1} \left( \frac{1}{3} \right) - 3\sqrt{2} \text{ (ب)}$$

$$A = \int_0^{\pi/4} \frac{1}{2} (2 \sin \theta)^2 d\theta + \int_{\pi/4}^{3\pi/4} \frac{1}{2} (\sin \theta + \cos \theta)^2 d\theta = \frac{\pi}{2} - \frac{1}{2} \text{ (ج)}$$

$$A = 2 \int_{-\pi/2}^{\pi/2} \frac{1}{2} [(2 - \cos 2\theta)^2 - (2 + \cos 2\theta)] d\theta = \frac{51\sqrt{3}}{16} \text{ (د)}$$

$$-1 \text{ (هـ) } \frac{\sin e + e \cos e}{\cos e - e \sin e} \text{ (و) } -\sqrt{3} \text{ (هـ) } \frac{4 + 3\sqrt{3}}{11} \text{ (ج) } 0 \text{ (ح) } \frac{1}{2} \text{ (ب) } \frac{1}{2} \text{ (أ) (٥)}$$

$$S = 2\pi \int_1^4 \left[ \frac{t^3}{3} + \frac{1}{2t^2} \right] \sqrt{\left( \frac{2}{\sqrt{t}} \right)^2 + (t^2 - t^{-3})} dt = \frac{471295\pi}{1024} \text{ (أ) (٧)}$$

$$4\pi^2 \text{ (د) } \frac{128}{5} \pi \text{ (ج) } S = 2\pi \int_{\pi/2}^{3\pi/4} \sin t (-\cos t) dt = \pi(2 - \sqrt{2}) \text{ (ب)}$$

$$\text{د (٢٥) د (٢٣) ب (٢١) ج (١٧) ج (١٥) ج (١٣) أ (١١) أ (٩) ج (٣٥) ب (٣٣) أ (٢٩) د (٢٧)}$$