Exercise 1:

1. Consider the sequence $\{u_n\}_{n=0}^{\infty}$ defined as follows: $\begin{cases} u_0 = 2 \\ u_1 = 4 \\ u_{n+1} = 4u_n - 3u_{n-1}; \quad n \geq 1 \end{cases}$

   Use mathematical induction to prove the following statement:

   $u_n = 3^n + 1$, for each integer n, with $n \geq 0$. (4pts)

2. Consider the set $A := \{1, 2, \{1\}, \{2\}, \{1, 2\}, \{1, \{1\}\}, \{2, \{2\}\}, \emptyset, \{\{\emptyset\}\}\}$.
   Determine whether each of the following five statements is true or false.
   (Justify your answer).

   (a) S_1: "$\{1, 2\} \in A$". (1 pts)

   (b) S_2: "$\{1, 2, \emptyset\} \subseteq A$". (1 pts)

   (c) S_3: "$\{1, \{1\}\} \subseteq A$". (1 pts)

   (d) S_4: "$\{1, \{\emptyset\}\} \subseteq A$". (1 pts)

   (e) S_5: "$A \cap \{1, 2, \{\{1\}, \{2\}\}\} = \{1, 2\}$". (1 pts)

3. Consider the following three sets $C := \{a, b, c, d\}$, $D := \{a, d\}$, and
   $E := \{(a, a), (d, a), (b, d), (c, a), (d, d)\}$. Find the following sets:
   (i) $(C \cap D) \times C$. (ii) $E \setminus (C \times D)$. (iii) $\{\emptyset\} \times E$. (3 pts)

Exercise 2:

1. Let R be the relation from the set $A := \{1, 2, 3\}$ to the set $B := \{2, 3, 4\}$ and T be the relation from the set $B := \{2, 3, 4\}$ to the set $C := \{0, 1, 2\}$, such that, $R := \{(1, 2), (1, 4), (2, 3), (2, 4), (3, 2)\}$ and $T := \{(2, 0), (2, 2), (3, 1), (4, 1), (4, 2)\}$.

   (a) Represent the relation R with a matrix. (1 pts)

   (b) Represent the relation T with a matrix. (1 pts)

   (c) Find $T \circ R$. (2 pts)

2. Let E be the relation defined on the set $\mathbb{Z}$.

   Let $x, y \in \mathbb{Z}$, $(x \; E \; y)$ if and only if $x + y$ is even.

   Prove that E is an equivalence relation on $\mathbb{Z}$. (3 pts)

3. Let P be the relation defined on the set $S := \{2, 16, 8, 64, 32, 4\}$.

   Let $m, n \in S$, $(m \; P \; n)$ if and only if $n | m$.

   (a) Prove that P is partial ordering relation on S. (3 pts)

   (b) Draw the digraph of P. (1 pts)

   (c) Draw the Hasse diagram of P. (1 pts)

   (d) Is P a total ordering? (1 pts)