132 Math Midterm Exam

Name:

ID:

QUESTION 1: Fill in the planks in the following and Explain your answer:

a- For any proposition p, The truth value of the proposition $p \leftrightarrow p$ is _______.

b- If $p \vee q$ is true, then the truth value of $\neg p \rightarrow q$ is _______.

c- The negation of the statement $[\forall x \in \mathbb{R}: x^2 \geq 0]$ is ________.

d- The inverse of the contrapositive of the proposition $p \rightarrow q$ is ________.

e- To prove that for any integer n, 2 divides $n^2 + n$ using proof by cases, we need to discuss two cases which are ___________ and __________.

f- The truth value of the statement $\exists x \in \{1,2,3,4\}, 2^x < x$ is _________.

QUESTION 2:

a- without using truth tables, prove that

$$\neg(p \to r) \to \neg q \equiv (p \land q) \to r.$$

b- Show that the statement "For every positive integer n, $n^2 \geq 2n$" is false.

c- Prove that there exists an integer m such that $m^2 > 10^{100}$.

QUESTION 3:

a- Prove that if n is an integer, then n is even if and only if $3n^2 + 2n + 1$ is odd.

b- Prove that $\forall n \in A(3^n < n^2)$ is true, where $A = \{-1, -2, -3\}$.

<u>QUESTION 4:</u> Use mathematical induction to prove that for every positive integer n,

$$2 + 2(2^2) + 3(2^3) + \cdots + n2^n = (n-1)2^{n+1} + 2$$