King Saud University
College of Science
Department of Mathematics

Course Title:	Mathematical logic
Course Code:	132 Math
Course Instructor:	Reem Almahmud
Exam:	Final Exam
Semester:	1st term 1444/1445
Date:	18-10-2023
Duration:	3 Hours
Marks:	40

Privileges: Calculator is not Permitted

Student Name:	
Student ID:	
Section No:	
Serial No:	

Instructions:

- Cell Phones should be switched off or on silent mode during the exam.
- Write your answers directly on the question sheet.
- There are 5 questions in 6 pages.

Official Use Only		
Question	**Students Marks**	**Question Marks**
1		15
2		6
3		6
4		7
5		6
Total		40

Q1: a) without using the truth table, prove the following:

$$A \to (B \to C) \equiv (A \to B) \to (A \to C).$$ 3 points

b) Decide whether each of the following statements is true of false, justify your answer:

i) $\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}, \quad \forall x, y \in \mathbb{R}$. 2points

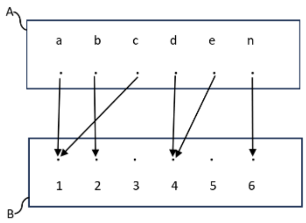
ii) If $x < 1$, then $x^n \leq 1, \quad \forall x \in \mathbb{R}, n \in \mathbb{N}$. 2points

iii) $\overline{\bar{A} \cap (A \cup B)} = A \cup \bar{B}$. 2points

iv) If A and B are sets, A is uncountable and $A \subseteq B$, then B is uncountable. 2points

c) Use mathematical induction to show that $n < 2^n, \quad \forall n \in \mathbb{Z}^+$. 4points

Q2: If $A = \{a, b, c, d, e, n\}, B = \{1,2,3,4,5,6\}$, and the function $f: A \longrightarrow B$ defined by the given graph. Let $A_1, A_2 \subseteq A,\ B_1, B_2 \subseteq B$ where $A_1 = \{a, b, d\}$, $A_2 = \{c, d, e\}, B_1 = \{1,2,3\}, B_2 = \{2,4,5\}$.



a) Find $f^{-1}(B_1 \cup B_2)$. Point

b) Find $f^{-1}(B_1) \cup f^{-1}(B_2)$. Point

c) Is $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$. 2points

d) Determine whether the given function is one-to-one, onto. 2points

Q3: If $\mathbb{R} \xrightarrow{f} \mathbb{R} \xrightarrow{g} \mathbb{R} \xrightarrow{h} \mathbb{R}$, such that $f(x) = x + 1, g(x) = 2x, h(x) = \sin x$.
a) Prove that f is a one-to-one correspondence. 2points

b) Find h^{-1} of the function h. Point

c) Find $h^{-1} \circ h$. Point

d) Show that $(h \circ g) \circ f = h \circ (g \circ f)$. 2points

Q4: a) Let R be the relation from the set of integers $\mathbb{Z}$ defined as follows:

$$let\ n \in \mathbb{N}, aRb \leftrightarrow \exists q \in \mathbb{Z}\ s.t\ a - b = qn$$

i) Show that R is an equivalence relation on $\mathbb{Z}$. 3points

ii) Find [2]. Point

b) Given $S_1 = \{1\}, S_2 = \{-2, -1\}, S_3 = \{0,2,3\}$ and $S_4 = \{-3,4\}$ be the equivalence classes of the relation T on the set $S = \{-3, -2, -1, 0, 1, 2, 3, 4\}$.

i) Draw the digraph of T. Point

ii) List all the ordered pairs in the relation T. 2points

Q5: Let P be the relation defined on the set $\mathbb{Z}^+$ of positive integer by:

$$\forall m, n \in \mathbb{Z}^+, mPn \leftrightarrow \frac{n}{m} \text{ is odd}$$

i) Show that P is a partially ordering relation on $\mathbb{Z}^+$. 3points

ii) Is P a total ordering relation on $\mathbb{Z}^+$? point

iii) Let $C = \{1, 2, 3, 4, 6, 8, 9\}$. Draw the hasse diagram of P on the set C. 2points

The End of Exam