

Course Title:	Mathematical logic
Course Code:	132 Math
Course Instructor:	Reem Almahmud
Exam:	2 nd MIDTERM
Semester:	1 st term 1445/1446
Date:	15-11-2023
Duration:	90 minutes
Marks:	20

Privileges: Calculator is not Permitted

Student Name:	
Student ID:	
Section No:	54945
Serial No:	

Instructions:

- Cell Phones should be switched off or on silent mode during the exam.
- Write your answers directly on the question sheet.
- There are 4 questions in 5 pages.

Official Use Only		
Question	Students Marks	Question Marks
1		10
2		3
3		2
4		5
Total		20

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

Q1

Q1: **Choose** the correct answer and **write** it in the top table:

1- The power set $\mathcal{P}(A)$, where $A = \{\phi, \{\phi\}\}$ is

- a) $\{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$ b) $\{\phi, \{\phi\}\}$
c) $\{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$ d) $\{\{\phi\}, \{\phi, \{\phi\}\}\}$

2- $\{x|x \text{ is an integer such that } x^2 = 3\} =$

- a) $\{3, -3\}$ b) $\{\sqrt{3}, -\sqrt{3}\}$
c) ϕ d) $\{3\}$

3- One of the following statements is true

- a) $\{0\} \in \{0\}$ b) $\{0\} \subseteq \phi$
c) $0 \in \phi$ d) $\{\phi\} \subseteq \{\phi\}$

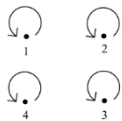
4- The cardinality of the set $\mathcal{P}(\mathcal{P}(\phi))$ is

- a) 2 b) 4
c) 0 d) 1

5- If $A - B = A$, then

- a) $A \subseteq B$ b) $A = B$
c) $B \subseteq A$ d) $A \cap B = \phi$

6- The relation R represented in the diagram



is

- a) Reflexive only b) Reflexive and symmetric
c) Reflexive, symmetric, antisymmetric, and transitive d) Reflexive, symmetric and transitive

7- If $R_1 = \{(a, b) \in \mathbb{R}^2 \mid a \geq b\}$, and $R_2 = \{(a, b) \in \mathbb{R}^2 \mid a \neq b\}$. Then, $R_2 - R_1 =$

a) $\{(a, b) \in \mathbb{R}^2 \mid a = b\}$

b) $\{(a, b) \in \mathbb{R}^2 \mid a \leq b\}$

c) $\{(a, b) \in \mathbb{R}^2 \mid a < b\}$

d) $\{(a, b) \in \mathbb{R}^2 \mid a \neq b\}$

8- If $R_1 = \{(a, b) \in \mathbb{R}^2 \mid a \geq b\}$, and $R_2 = \{(a, b) \in \mathbb{R}^2 \mid a \neq b\}$. Then, $R_2 \cap R_1 =$

a) $\{(a, b) \in \mathbb{R}^2 \mid a \neq b\}$

b) $\{(a, b) \in \mathbb{R}^2 \mid a = b\}$

c) $\{(a, b) \in \mathbb{R}^2 \mid a \leq b\}$

d) $\{(a, b) \in \mathbb{R}^2 \mid a > b\}$

9- The set $\{(1,2), (2,4), (3,8), (4,16)\}$ represents the relation

a) $xRy \Leftrightarrow x + 2 < y$

b) $xRy \Leftrightarrow y = 2^x$

c) $xRy \Leftrightarrow x + y$ is odd

d) $xRy \Leftrightarrow x + y$ is even

10- If $R = \{(x, y) \in \mathbb{R}^2 \mid x > y\}$. Then, $R^{-1} =$

a) $\{(x, y) \in \mathbb{R}^2 \mid x \leq y\}$

b) $\{(x, y) \in \mathbb{R}^2 \mid x = y\}$

c) $\{(x, y) \in \mathbb{R}^2 \mid x < y\}$

d) ϕ

Q2

Q2: I - **Prove** the following statement:

$$\mathcal{P}(A) \subseteq \mathcal{P}(B) \Leftrightarrow A \subseteq B$$

Q3: Let R_1 and R_2 be relations on a set A represented by the matrices:

$$M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

- I- **Find** $M_{R_2 \circ R_1}$
II- **Represent** $R_2 \circ R_1$ by a diagram. (Take $A = \{a, b, c\}$)

Q4: Let R be a relation defined on \mathbb{Z} by $aRb \Leftrightarrow 3|a^2 - b^2$.

- I- **Prove** that R is an equivalence relation.
- II- **Find** the equivalence class $[4]$.