Application of Simultaneous and Sequential Transportation Network Equilibrium Models to Riyadh, Saudi Arabia

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The approach we've used in practice to predict short-run transport equilibria involves a sequential process, often with four stages: trip generation, trip distribution, modal split, and traffic assignment. This approach has its inherent weaknesses—its prediction need not be internally consistent. This deficiency has motivated attempts to predict all these stages simultaneously. The (traditional) sequential and simultaneous models are compared by calibrating and applying both models to the same transportation network of Riyadh, Saudi Arabia. The main finding is that the simultaneous model predicts better traffic flow predictions than the prediction of the conventional sequential model. These predictions are much better for the heavy-volume links that are the most important links in the prediction process.

Early procedures, which have been applied hundreds of transport studies throughout the world for the last 40 years and still are in use today, viewed transportation planning as a sequential process, often with four stages: trip generation, trip distribution, modal splits, and route choice (1-4). Unfortunately, the sequential approach has its inherent weaknesses in several respects. This approach is a sequential decision process. For most trips, this is understood to be a highly optimistic representation of travelers’ decision-making. Also, its prediction need not be internally consistent. That is, because each step in this type of sequential planning depends on the others, the performance and demand levels that each yields to assume as input as a step need not agree with those that are determined as outputs from the other steps. Also, the basic forecasting travel choices, as defined in terms of variables and parameters, is inconsistent across the several models, for example, trip assignment often is based on travel times only, whereas modal split is based on a weighted combination of travel time and trip costs. These deficiencies motivated an attempt to predict all four steps simultaneously. Research intended to develop simultaneous models and related computational procedures for predicting short-run transport equilibria has proceeded in three directions. One line of research, equivalent optimization approaches, has significant applications for traffic assignment problems under the assumption of constant travel time and traffic equilibria. Approaches permit richer modeling of user behavior. The focus of such models is the elastic demand and traffic assignment problem, which is the work of Beckman et al. (5). In this problem, the number of trips between each origin-destination (O-D) pair is a function of the traffic time during which that pair's demand is not completely satisfied.

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pair, Beckman's model was cast as an equivalent optimization problem that when solved yields the desired transport equilibrium solution. Evans (6) extended the formulation to include trip distribution, assuming fixed trip generation and an entropy model for trip distribution. Finally, the fact that an entropy distribution model implies a logit mode split model. Flis and Nygren (9) extended the formulation to include modal split.

Safwat and Maggo (10) further enhanced the behavioral features of the equilibrium optimization approach. In their model, the simultaneous transportation-equilibrium model (STEM), trip generation can depend on the system's performance through an accessibility measure that is based on the random utility theory of users' behavior (instead of being fixed), and trip distribution is given by a more flexible logit model based on the random utility theory (instead of being given by a less flexible entropy model). In practice, the STEM model was applied to some real-world transportation systems (11-17).

From a computational point of view, the STEM model may be solved by globally convergent and efficient algorithms such as the logit distribution of trips (LDT) algorithm (18). This algorithm was used in the present research to solve the equilibrium concert program of the STEM model. The LDT algorithm belongs essentially to the class of feasible direction methods and in such iterative schemes, the method involves two main steps. The first step determines a direction for improving the second step determines an optimal step size (λ*) along this direction. The current solution x* then is updated, that is \( x^{*+1} = x^* + \lambda^* \delta^* \), and the process is repeated until a convergence criteria is met. Safwat and Walston (14) showed, in their application of the STEM model, using LDT algorithm, to the Austin transportation network (which consisted of 250 routes, 19,214 O-D pairs, 7,959 link link, and 122 4-link networks) and Nash (15) showed that the computer was 430 s for a typical iteration and about 79 min to arrive at a reasonably accurate solution in no more than 10 iterations. Safwat and Hasen (16) proposed the LDT algorithm as a function of demand, performance, and network parameters.

In 1957, the Riyadh (Saudi Arabia) Development Authority (EDA) (4) developed its transportation model that included trip generation and modal split, trip generation attraction model, trip distribution model, and trip assignment model to forecast the transportation demand. These models follow the traditional sequential approach. Another form of the sequential four-step procedure introduces what is called feedback from the four steps by iterating the four-step procedure until the last flows, their associate generalized costs (impedances), and the corresponding O-D matrix are converged to a
consistent relationship with each other. That is, resolve the four-step procedure by using the travel times yielded by the trip assignment step. Joyce et al. (19) examined how solutions produced by various methods of introducing this kind of feedback procedure compare with the equilibrium solution of a model containing the trip distribution, model split, and trip assignment steps. The comparison was performed on a sketch-planning model for the Chicago region. This comparison showed that the feedback procedure does not produce the desired results. Instead, one must apply an algorithm designed to converge to a well-defined equilibrium of the travel flows and the link times and costs determined by these flows.

This report compares the simultaneous approach, using the STEM model as an example, and the conventional sequential approach used by RDA, by applying both models to the urban transportation network of Riyadh. This comparison is of the accuracy of their predicted solutions for the traffic equilibrium problem.

**STEM Modeling Procedure**

Model Description and Assumption

The STEM model and its assumptions are as follows (15):

\[
G_i = \alpha S_i + E_i \quad \forall i \in L
\]

\[
S_i = \max \left\{ 0, \lambda_i \sum_{j \in A_i} \exp(-B_{ij} + A_{ij}) \right\} \quad \forall i \in L
\]

\[
T_i = \frac{G_i \exp(-B_{i} + \lambda A_{i})}{\sum_{j \in A_i} \exp(-B_{ij} + A_{ij})} \quad \forall j \in A, i \in L
\]

\[
C_i = \begin{cases} 0 & \text{if } b_i > 0 \\ \alpha_i & \text{if } b_i = 0 \end{cases} \quad \forall j \in A, i \in L
\]

\[
C_i = \sum_{j \in A_i} C_i(j) \quad \forall i \in L
\]

where

\( (N,A) = \) a basic network (i.e., any transportation network) consisting of a set \( N \) of nodes and a set \( A \) of links;

\( i = \) the set of origin nodes (\( N \subset N \));

\( j = \) the set of destination nodes (\( N \subset N \));

\( D_s = \) the set of destinations that are accessible from a given origin \( i \) (\( i \subset N \));

\( p = \) a simple path (i.e., no node repeated) in the network \( (N,A) \);

\( s = \) the set of origin parts (\( i \subset N \));

\( f_i = \) the flow on link \( i \);

\( f_{ij} = \) the flow on path \( p \);

\( b_i = \) the average minimum per-trip cost of transportation between origin \( i \) and destination \( j \);

\( E_i = \) the number of trips generated from origin \( i \);

\( S_i = \) the accessibility variable that measures the expected maximum utility of travel on the transport system to period \( i \) from origin \( i \);

\( C_i(j) = \) a composite measure of the effect that the socioeconomic variables, which are exogenous to the transport system, have on trip generation from origin \( i \);

\( A_i = \) a composite measure of the effect that the socioeconomic variables, which are exogenous to the transport system, have on trip attraction at destination \( j \);

\( \alpha_i = \) a parameter that measures the additional number of trips that would be generated from a given origin \( i \) if the expected maximum utility of travel, as perceived by travelers at \( i \), increased by \( \alpha_i \); and

\( 0 = \) a constant that measures the sensitivity of the utility of travel between any given O-D pair \( i,j \) due to changes in the system's performance that give O-D pair \( i,j \).

The basic assumptions of this STEM model may be summarized as follows:

1. Trip generation, \( G_i \), is given by any general function as long as it is linearly dependent on the system's performance through an accessibility measure, \( S_i \), based on the maximum utility theory of travel behavior (i.e., the expected maximum utility of travel).

2. Trip distribution, \( T_i \), is given by a logit model in which each measured utility function includes the average minimum per-trip cost, \( b_i \), as a linear variable.

3. Modal split and trip assignment are simultaneously user-equilibrium. Notice that the STEM framework allows for the model to fit in to be given by a logit model or to be system optimized together with trip assignment (15).

**STEM Model Calibration**

The data needed to calibrate a STEM model for Riyadh were collected mainly from RDA. The data were gathered for 2 years, 1985-1986, and 1987-1988. The 1986 data were used to calibrate the STEM model for the urban transportation network of Riyadh. Then, the 1972 data with observed socioeconomic variables were used to input the calibrated STEM to predict demand and performance levels on that network.

**Calibrating STEM Trip Distribution Model**

The trip distribution model of STEM includes the composite effect \( h_i \) of the socioeconomic variables on trip attraction at destination \( j \) (equation 3). This effect can be expressed by the following function (15):

\( G_i = \frac{1}{T_i} \sum_{j \in D_i} f_{ij} h_i(j) \)

\( h_i(j) = \exp \left( \beta_i + \beta_j + \beta_{ij} \right) \)

\( \beta_i = \) the effect of socioeconomic variable \( i \); and

\( \beta_j = \) the effect of socioeconomic variable \( j \).
\[ A_j = \sum_{i} b_{i,j} \alpha_i \]

where:

- \( \alpha_i \) is the value of the socio-economic variable that influences trip attraction at destination \( j \).
- \( \beta_{i,j} \) is a given factor specifying how the socio-economic variable \( \alpha_i \) influences trip attraction; and
- \( b_{i,j} \) are parameters to be estimated \( \forall i, j \).

Four categories of the gross floor area, GAR (retail), OFAME (medical and educational), GPAAO (offices), and OFAG (government), for zone \( j \) were chosen as \( \alpha_5, \alpha_6, \alpha_7, \) and \( \alpha_8 \), respectively. The trip distribution model takes the form

\[ T_j = \frac{G_j \cdot \sum_{i} \gamma_i \cdot \beta_{i,j} \cdot b_{i,j} \cdot \alpha_i}{\sum_{i} \beta_{i,j} \cdot b_{i,j} \cdot \alpha_i} \]

where

- \( T_j \) is the observed vehicle trips for zone \( j \) for base year 1986;
- \( G_j \) is the observed travel time matrix for base year 1986.

By the above specification for the trip distribution model, the calibration problem is to estimate the values of the parameters \( b_{i,j}, \alpha_i, \) and \( \beta_{i,j} \). The logit-form (2) was used to calibrate this trip logit trip distribution model in which there were 216 regression equations, one model for each traffic analysis zone.

Table 1 shows some of these 216 regression models results, and Table 2 summarizes the results of all 216 regression models. Table 2 shows that the estimated values of \( \beta \) vary from 0.00665 to 0.13230 with a mean of 0.08428. These estimated values of \( \beta \) (along with its negative signs as in Equation 2) reflect the behavioral assumption that, everything else being equal, the observed utility of travel between \( i \) and \( j \) decreases as travel time (cost) increases. The average estimated values of \( \beta_5, \beta_6, \beta_7, \) and \( \beta_8 \) are 4.96 \times 10^{-4}, 4.45 \times 10^{-4}, 1.49 \times 10^{-4}, \) and 8.15 \times 10^{-4}, respectively, which reflect, as expected, that the observed utility of travel between \( i \) and \( j \) increases as the values of the socio-economic variables (retail, medical and educational, offices, or government gross floor area), respectively, increase.

**Calibrating STEM Trip Generation Model**

The outcome of the trip distribution process is a set of estimated parameters \( \beta_{i,j}, b_{i,j}, \alpha_i, \) and \( \beta_{i,j} \). These parameters will be

<table>
<thead>
<tr>
<th>Zone Number</th>
<th>No. of Observations</th>
<th>( \hat{\beta}_{5,5} ) [p-value]</th>
<th>( \hat{\beta}_{6,6} ) [p-value]</th>
<th>( \hat{\beta}_{7,7} ) [p-value]</th>
<th>( \hat{\beta}_{8,8} ) [p-value]</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>186</td>
<td>4.39 [0.0001]</td>
<td>5.91 [0.0001]</td>
<td>12.24 [0.0001]</td>
<td>2.68 [0.0001]</td>
<td>0.55</td>
</tr>
<tr>
<td>3</td>
<td>192</td>
<td>4.20 [0.0001]</td>
<td>5.85 [0.0001]</td>
<td>13.12 [0.0001]</td>
<td>2.57 [0.0001]</td>
<td>0.59</td>
</tr>
<tr>
<td>6</td>
<td>164</td>
<td>4.36 [0.0001]</td>
<td>5.22 [0.0001]</td>
<td>16.11 [0.0001]</td>
<td>2.17 [0.0001]</td>
<td>0.55</td>
</tr>
<tr>
<td>165</td>
<td>212</td>
<td>4.63 [0.0001]</td>
<td>5.17 [0.0001]</td>
<td>27.71 [0.0001]</td>
<td>2.89 [0.0001]</td>
<td>0.42</td>
</tr>
<tr>
<td>166</td>
<td>312</td>
<td>5.17 [0.0001]</td>
<td>5.71 [0.0001]</td>
<td>31.31 [0.0001]</td>
<td>2.65 [0.0001]</td>
<td>0.46</td>
</tr>
<tr>
<td>257</td>
<td>175</td>
<td>5.38 [0.0001]</td>
<td>4.45 [0.0001]</td>
<td>9.41 [0.0001]</td>
<td>2.66 [0.0001]</td>
<td>0.58</td>
</tr>
<tr>
<td>218</td>
<td>202</td>
<td>6.42 [0.0001]</td>
<td>5.98 [0.0001]</td>
<td>13.92 [0.0001]</td>
<td>2.48 [0.0001]</td>
<td>0.53</td>
</tr>
<tr>
<td>239</td>
<td>189</td>
<td>5.15 [0.0001]</td>
<td>5.14 [0.0001]</td>
<td>18.27 [0.0001]</td>
<td>2.31 [0.0001]</td>
<td>0.55</td>
</tr>
</tbody>
</table>
used in the calibration of the STEM's trip generation model through composition of the estimated accessibility measure, \( \hat{S}_i \), which is the natural logarithm of the denominator of the trip distribution model, that is:

\[
\hat{S}_i = \log \left( \sum_{o} \exp \left( -\delta_{ao} \right) \right)
\]

From Equation 1 the trip generation model of STEM includes the composite effect \( E_i \) of the socioeconomic variables on trip generated from origin \( i \). This effect can be expressed by the following functional form:

\[
E_i = \sum_{s} \alpha_s f_i(s_i)
\]

where

- \( E_i \) is the value of the \( i \)th socioeconomic variable that influences trip generation from origin \( i \);  
- \( f_i(s_i) \) is a given function specifying how the \( i \)th socioeconomic variable \( E_i \) influences trip generation; and  
- \( \alpha_s \) is a \( s \) parameter to be estimated.

Three categories of population for zones \( i \): POPE (Saudi) POPA (Arabs) and POPO (Others), were chosen as \( E_{POPE} \), \( E_{POPA} \), and \( E_{POPO} \) variables, respectively. Hence, the trip generation model takes the form:

\[
G_i = \alpha_0 + \alpha_1 POPE + \alpha_2 POPA + \alpha_3 POPO
\]

The calibration results of the STEM's trip generation model are shown in Table 3 where \( \alpha = 522.69 \) means that if the estimated accessibility measure, \( \hat{S}_i \), increases by one unit, and everything else being equal, the number of vehicle trips generated from origin \( i \) will increase by about 523 daily trips. This reflects that the trip generation model is very sensitive to the accessibility variable that means the expected maximum utility of travel on the traffic system perceived by travelers at origin \( i \). The values of other parameters \( \delta_{ao} = 0.21, \delta_{1} = -0.775, \delta_{2} = 0.939 \) are expected. In other words, the effect of increase of Saudi population in a zone has the lowest influence on trips generated from that zone compared to the influence of the increase of Arab and non-Saudi, non-Arab population. However, it might be argued that this could be because of the percentage of Saudi females (they are not allowed to drive in Saudi Arabia) and children compared to the Arabs and others. The same argument can be made for the relative values between Arab and non-Saudi, non-Arab populations. It should be noted that the regression model form for this trip generation model was chosen as the best fit form after several trials of different functional forms.

**ANALYSIS AND COMPARISON RESULTS**

This section presents a behavioral comparison analysis of the application results of the simultaneous approach represented by the STIM model and sequential approach represented by the RDA model.

**Riyadh Transportation Network Description**

To perform the comparative analysis, the calibrated STEM is applied to Riyadh's 1992 roadway network. The network is composed of 5,764 directed links, representing streets, and 2,231 nodes, which generally represent intersections (including 427 zonal or triangle). Each link is described by its beginning node, ending node, length, mode (always auto), link type (a two-digit code relating both to the functional class (freeway, expressway, principal arterial, etc.) and the facility type (one-way, two-way undivided, two-way divided, etc.), number of lanes, volume delay function, and speed.

**Volume Delay Function**

RDA used the Federal Highway Administration (FHWA) value of time function, but calibrated it by using demographic, economic, land use, and transportation (DELTAR) (22) data to determine the value of the first hour speed. A set of factors was calculated for various roadway types in the region from the time-of-day traffic count data obtained from the DELTA study. These factors represent the decile function of daily travel that occurs in the highest peak hour for each type. RDA assumed a set of link capacities based on the capacities typically used in the U.S. Highway Capacity Manual (HCM) and then adjusted it to the levels of capacity determined from the DELTA study, to better represent the Riyadh area. A total of 49 different calibrated and adjusted volumes were finally arrived at by RDA. They were used as link performance functions, \( C_i (Q) \) for STEM application.

**Statistical Performance Measures**

The comparative analysis was based on comparing the predicted daily link flows output from each approach to the observed daily link flows.
flows. These observed link flows were classified into different levels and two statistical measures of performance were used for the comparison: the ratio of total predicted to total observed traffic flows for each observed flow level k (Ratio) that is given by

\[
\text{Ratio}_k = \frac{\sum f'_k}{\sum f_k}
\]

and the root mean square error for each observed flow level k (RMSE), which is given by

\[
\text{RMSE}_k = \sqrt{\frac{\sum (f'_k - f_k)^2}{n-1}}
\]

where

\[ f'_k \] observed traffic flow on link k,
\[ f_k \] predicted traffic flow on link k,
\[ n \] the number of links on observed flow level k; and
\[ A_k \] the set of links in observed flow level k.

### Comparative Results

Figure 1 shows the results of the ratio of total predicted to total observed traffic flows for each observed flow level for STEM and sequential models. For example, for observed flow level 1, which includes all the 13 links with observed traffic counts, STEM's ratio was about 28 percent and sequential's ratio was 35 percent (a difference of 12 percent). This means that, in general, STEM flow predictions are closer to the observed ones than for the sequential flow predictions. The STEM's predictions are always better than those of the sequential for all observed flow levels. As the observed flow level increases, the difference between STEM's and sequential's ratio increases (a difference of 19 percent for observed flow level 4).

![FIGURE 1 Comparison of STEM and sequential models' ratio results for different observed flow levels](image1)

### SUMMARY AND CONCLUSIONS

The sequential approach used in practice to predict short-run traffic equilibrium has several inherent weaknesses and is internally inconsistent in its models structure. To overcome these deficiencies, research intended to develop integrated models and related computational procedures proceeded in three lines: equivalent optimization, variational inequality, and stochastic equilibrium approaches. STEM/ID is one of the most recent models that follow the equivalent optimization line.

The main objective of this research was to perform a behavior comparison between the sequential and simultaneous approaches.
for predicting short-run transportation equilibrium on the transpor-
tation network of Riyadh. To achieve this objective, STEM was
calibrated to the urban transportation network of Riyadh, on the basis of
data available for 1986, then the calibrated STEM was applied to that
network by using 1992 data. The results of the application, which are the predicted link flows, and those obtained by the appli-
cation of the conventional sequential approach currently used by
RDA, were compared with the actual (observed) values, such that a
conclusion of which model yields better forecasts was drawn.

The major conclusions of this study are summarized as follows:

1. Although the number of links with observed traffic flows (123) was small (2.3 percent of the total number of links in the
1992 network), the application of STEM produces better predictions than the conventional sequential approach currently used.

2. STEM's predictions at heavy-volume links, which are the most
important links in the prediction process, is much better than the
sequential's predictions.

3. Both sequential and simultaneous approach results under-
predict the observed flows, and their predictions failed to be good
representatives of the observed flows (both models failed to pass the
chisquare test). This may be because of the small number of socio-
economic variables that were used in the calibration process for trip
generation and trip distribution models for both approaches, and
because of the very small number of observed link flows.

The results of this study are encouraging. Development of an in-
teractive program (software package) able to achieve a comprehensive
application of STEM methodology in a real-world, large-scale trans-
portation network is recommended. If developed, it will be the first
software package for the application of simultaneous models in trans-
portation planning studies and will encourage transportation agencies
to use simultaneous models.

Finally, STEM is behaviorally richer than currently available
equivalent optimization models and it is computationally tractable.
But the question is, Why has it taken so long for so few kinds of
different models with their global convergent algorithms to be
adopted in professional practice? The answer may be summarized as
follows (26):

1. Planning agencies have not faced much pressure in the last
10 years to improve their methods. It is clear they continue to do
what they know from their own experience, which is the traditional
four-step procedure.

2. Even if these planning agencies had wished to adopt these new
combined models, the software was not available. Why not? The
traditional software developer response is "Software developers provide what the market wants."

Now in the time for metropolitan planning agencies, as well as
FHWA and the Federal Transit Administration, to demand software
that yields the equilibrium solution needed to meet the planning
requirements of the 21st century. There should be no further delay by
practitioners and software developers (26).

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