PHYS 404
1st Midterm Exam
Tuesday 3rd November 2015

Instructor: Dr. V. Lempesis

Student Name: …….

Student ID Number……..

Student Grade: ……./20

Please answer 4 of the following 5 questions

1. If $P_2(x)=\frac{1}{2}\left(3x^2-1\right)$, $P_3(x)=\frac{1}{2}\left(5x^3-3x\right)$, calculate $P_4(x)$, $P_5(x)$. You are given that: $\left(n+1\right)P_{n+1}\left(x\right)=\left(2n+1\right)xP_n(x)-nP_{n-1}(x)$.

2. Expand the function $f(x)=\begin{cases} x & 0<x<1 \\ 0 & -1<x<0 \end{cases}$ in a series of the form $f(x)=\sum_{k=0}^{\infty}A_kP_k(x)$ (calculate only the first four terms of the series). You are given that: $A_k=\frac{2k+1}{2}\int_{-1}^{1}P_k(x)f(x)dx$ and also that $P_0(x)=1$, $P_1(x)=x$, $P_2(x)=\frac{1}{2}\left(3x^2-1\right)$, $P_3(x)=\frac{1}{2}\left(5x^3-3x\right)$.

(5 marks)

3. Given that $j_1(x)=\left(\sin x/x^2\right)-\left(\cos x/x\right)$, $n_1(x)=\left(\cos x/x^2\right)-\left(\sin x/x\right)$, find the functions $j_2(x)$, $n_2(x)$. You are given that both functions satisfy the recurrence relation: $\frac{d}{dx}\left[x^{-n}f_n(x)\right]=-x^nf_{n+1}(x)$.

(5 marks)

4. Show that $e^{(x/2)(t+1/t)}=\sum_{n=-\infty}^{\infty}I_n(x)t^n$, where $I_n(x)=i^{-n}J_n(x)$. You are given the Bessel functions generating function $e^{(x/2)(t-1/t)}=\sum_{n=-\infty}^{\infty}J_n(x)t^n$.

(5 marks)