الفيزياء الرياضية -١-

Mathematical Physics -1-

PHYS 201

First Term 2020-2021

Series of Applications in Physics

Application I: **Syst. Of. Lin. Eqs**

Systems of linear equations arise in a wide variety of applications and are one of the central themes in linear algebra.
One of the good applications in Physics is related to Kirchhoff's Laws for electricity.

Kirchhoff's laws: Reminder

1st law: also called *Kirchhoff's point rule,* or *Kirchhoff's junction rule.*

The algebraic sum of currents in a network of conductors meeting at a point is zero.

$$\sum_{k=1}^{n} I_k = 0$$

2nd law: also called *Kirchhoff's loop* or *(mesh) rule*

The directed sum of the potential differences (voltages) around any closed loop is zero.

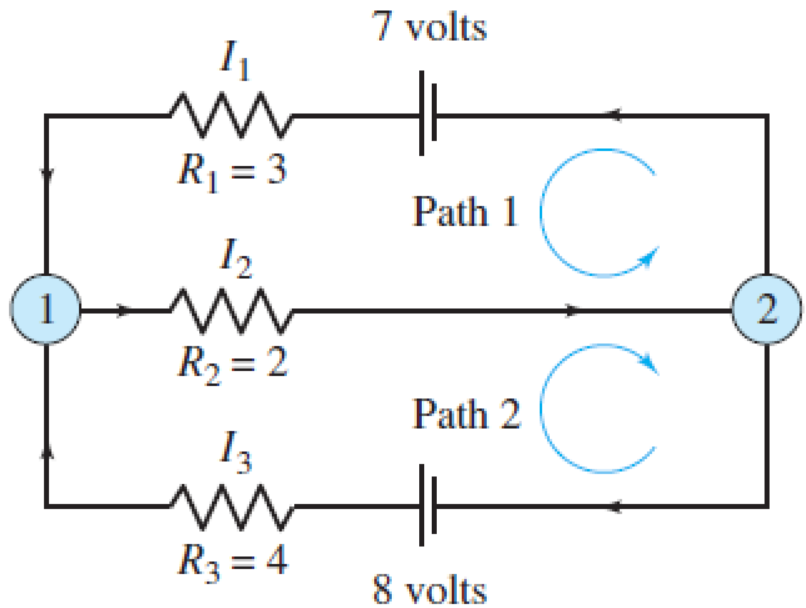
$$\sum_{k=1}^{n} V_k = 0$$

Notes: A *closed* path is a sequence of branches such that the beginning point of the first branch coincides with the end point of the last branch.

In an electrical network, current is measured in amps, resistance in ohms, and the product of current and resistance in volts. Batteries are represented by the symbol  . The larger vertical bar denotes where the current flows out of the terminal. Resistance is denoted by the symbol  . The direction of the current is indicated by an arrow in the branch.

Application:

Determine the currents I_1, I_2, and I_3 for the electrical network shown in Figure



Solutions:

Applying Kirchhoff's first law to either junction produces

$$I_1 + I_3 = I_2 \qquad \text{Junction 1 or Junction 2}$$

and applying Kirchhoff's second law to the two paths produces

$$R_1I_1 + R_2I_2 = 3I_1 + 2I_2 = 7 \qquad \text{Path 1}$$
$$R_2I_2 + R_3I_3 = 2I_2 + 4I_3 = 8. \qquad \text{Path 2}$$

So, you have the following system of three linear equations in the variables I_1, I_2, and I_3.

$$\begin{aligned} I_1 - I_2 + I_3 &= 0 \\ 3I_1 + 2I_2 &= 7 \\ 2I_2 + 4I_3 &= 8 \end{aligned}$$

Now, apply *Gauss-Jordan* elimination method to the associated augmented matrix

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 3 & 2 & 0 & 7 \\ 0 & 2 & 4 & 8 \end{bmatrix}$$

Solutions:

Now, apply *Gauss-Jordan* elimination method to the associated augmented matrix

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 3 & 2 & 0 & 7 \\ 0 & 2 & 4 & 8 \end{bmatrix}$$

Which will produce, Check!, the following R.R.E.F (reduced row-echelon form):

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Hence the solutions are:

$$I_1 = 1 \text{ amp}, I_2 = 2 \text{ amps, and } I_3 = 1 \text{ amp.}$$