MATH 201: Differential and Integral Calculus

Text Book: Calculus, The classic edition by: Swokowski

COURSE DESCRIPTION

In this course we cover chapters 11, 16 and 17, but we start with chapter 16 and 17 then finally chapter 11, also section 14.6 should be read by the students.

Cartesian, cylindrical and spherical coordinate systems. Functions of two and three variables, limits and continuity, partial derivatives, the chain rule, extrema of functions of two variables, Lagrange multipliers. Double integrals, double integrals in polar coordinates, triple integrals, application of triple integrals, triple integrals in cylindrical and spherical coordinates. Sequences, infinite series, convergence tests, representation of functions by power series, Taylor and Maclaurin series.

DISTRIBUTION for MARKS:

20 marks for First Mid Term.

20 marks for Second Mid Term.

10 marks for tow quizzes in the lectures.

10 marks for tutorials.

40 marks for Final Exam.

SYLLABUS and EXERCISES

CHAPTER 16: PARTIAL DIFFERENTIATION

Section 16.1: Functions of Several Variables

Do the following problems from the book

1, 3, 5, 8, 9, 14, 21, 22, 23, 24, 37.

Section 16.2: Limits and Continuity

I- Do the following problems from the book

3, 5, 6, 9, 12, 14, 16, 19, 20, 25, 26, 28, 29, 37, 38, 42.

II- Find the following limits, if they exist:

(1) $\lim_{(x,y)\to(1,2)} \frac{xy-2x-y+2}{x^2+y^2-2x-2y+5}$ (2) $\lim_{(x,y)\to(1,1)} \frac{x-y^4}{x^3-y^4}$ (3) $\lim_{(x,y)\to(0,0)} \frac{3xy}{5x^4+2y^4}$

(4) $\lim_{(x,y)\to(0,0)} \frac{3x^3-2x^2y+3y^2x-2y^3}{x^2+y^2}$ (5) $\lim_{(x,y)\to(0,0)} \frac{10xy}{5x^3+2y^3}$ (6) $\lim_{(x,y)\to(0,1)} \frac{x+(y-1)^3}{\sqrt{x^2+(y-1)^2}}$

(7) $\lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz+zx}{x^2+y^2+z^2}$ (8) $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^3+y^6}$ (9) $\lim_{(x,y,z)\to(0,0,0)} \frac{y^3+x^3 sinz^3}{x^2+y^2+z^2}$

(10) $\lim_{(x,y)\to(2,1)} \frac{(y-1)(x-2)^2}{(y-1)^{3+}(x-2)^3}$ (11) $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^4+y^2}$ (12) $\lim_{(x,y,z)\to(0,0,0)} \frac{xy^2}{x^2+y^2+z^2}$

(13) $\lim_{(x,y)\to(1,1)} \frac{3x^3+xy^2-3xy-y^3}{x^2-y^2}$ (14) $\lim_{(x,y,z)\to(0,0,0)} \begin{cases} \frac{x}{y^2}-\frac{x}{e^{y^2}} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$

III – Show that

(1) $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0$ (2) $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2} = 0$

(3) $\lim_{(x,y,z)\to(0,0,0)} f(x,y,z) = 0$, If $f(x,y,z) = \begin{cases} \frac{3xyz}{x^2+y^2+z^2} & (x,y,z)\neq(0,0,0) \\ 0 & \big((x,y,z)=(0,0,0)\big) \end{cases}$

III- Discuss the continuity of the following functions on their domain:

(1) $f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$

(2) $f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)\neq(0,0) \end{cases}$, at $(x,y)=(0,0)$

(3) $f(x,y,z) = \begin{cases} \frac{x^3+y^3+z^3}{x^2+y^2+z^2} & (x,y,z)\neq(0,0,0) \\ 0 & (x,y,z)=(0,0,0) \end{cases}$, at $(x,y,z)=(0,0,0)$

(4) $f(x,y,z) = \begin{cases} \frac{xz-y^2}{x^2+y^2+z^2} & (x,y,z)\neq(0,0,0) \\ 0 & \big((x,y,z)=(0,0,0)\big) \end{cases}$, at $(x,y,z)=(0,0,0)$

(5) $f(x,y) = \begin{cases} \frac{xy}{|x|+|y|} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$

(6) $f(x,y,z) = \begin{cases} \frac{xzy^2}{x^2+y^2+z^2} & (x,y,z)\neq(0,0,0) \\ 0 & \big((x,y,z)=(0,0,0)\big) \end{cases}$

IV - Discuss the continuity of the following functions:

(1) $h(x,y) = e^{x^2+5xy+y^2}$ (2) $h(x,y) = sin\big(\sqrt{y-4x^2}\big)$ (3) $f(x,y) = e^{xy} sin^{-1}(x^2+y^2)$

(3) $h(x,y,z) = ln(36-4x^2-y^2-9z^2)$ (4) $f(x,y,z) = \frac{xz}{\sqrt{x^2+y^2+z^2-1}}$

Section 16.3: Partial Derivatives

I- Do the following problems from the book

4, 6, 8, 12, 13, 16, 17, 21, 22, 27, 29, 32, 34, 36, 39, 42, 47.

II- Do the following problems

1. Using the definition, find f_x, f_y of the function $f(x,y) = 3x^2 - 2xy + y^2$

II. Let $f(x,y) = e^{x-y}sin(x+y)$, show that $(f_x)^2 + (f_y)^2 = \frac{2(f(x,y))^2}{sin^2(x+y)}$

Section 16.4: Increments and Differentials

Do the following problems from the book

2, 9, 11, 12, 16, 18, 20, 39, 41.

1. Use the differential to approximate the change in the function

$w = f(x,y) = x^2 ln(z^2 + y^2)$, as (x, y, z) changes from (1, 2, 3) to (0.9, 1.9, 3.1).

2. Use the differential to approximate the change in the function

$w = f(x,y) = yx^{\frac{2}{5}} + \sqrt{x-y}$. As (x, y) changes from (52, 16) to (35, 18).

3. $f(x,y,z) = \begin{cases} \frac{xy^2z}{x^4+y^4+z^4} & (x,y,z) \neq (0,0,0) \\ 0 & ((x,y,z) = (0,0,0)) \end{cases}$

(1) Show that $f_x(0,0,0)$, $f_y(0,0,0)$and $f_z(0,0,0)$ exist.

(2) Discuss the differentiability of $f(x,y,z)$ at $(0,0,0)$.

Section 16.5: Chain Rules

Do the following problems from the book

2, 4, 6, 10, 12, 14, 18, 19, 22, 33, 37, 38, 41, 42.

1. If $w = f(x,y,z) = x^2 + y^2 + z^2$, where $x = rcos\theta$, $y = rsin\theta$ and $z = r$. Use the differential to show that $dw = 4rdr$.

2. Let $z = f(x,y)$ be determined implicitly by $x^2 + z^2 + cos(xyz) - 4 = 0$. Find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$. Then show that

$2y\frac{\partial z}{\partial y} - x\frac{\partial z}{\partial x} = \frac{xyzsin(xyz)}{2z - xysin(xyz)}$

Section 16.8: Extrema of Functions of Several Variables

Do the following problems from the book;

1, 9, 11, 15, 20, 21, 23, 24, 26,29,31,32.

Section 16.9: Lagrange Multipliers

Do the following problems from the book; 1, 2, 3, 11.

Chapter 17: **Multiple Integral**

Section 17.1: Double Integral

Do the following problems from the book;

1 - 10, 13, 16, 18, 19, 20, 21, 23, 25, 26, 27, 29, 31, 32, 33, 37, 38, 39, 43, 44, 50.

1. Sketch the region bounded by the graphs of the given equations, and then evaluates the given integral $y = x, y = \sqrt{x}, x = 0;\ \iint_R\ siny^2\, dA$

2. Evaluate the double integral $\int_0^2 \int_{y/2}^1 e^{x^2}\, dx\, dy$

Section 17.2: Area and Volume

Do the following problems from the book;

2, 4, 6, 7, 11, 14, 18, 22, 24, 27, 28, 30, 31, 32.

1. Sketch the region bounded by the graphs of the equation$y = sinx,$ $y = cosx, x = 0, x = \frac{\pi}{4}$. Then use the double integral to find its area.

Section 17.3: Double Integral by Polar Coordinate

Do the following problems from the book;

1 - 12, 13, 15, 17, 18, 19, 21, 23, 24.

1. Use polar coordinate to evaluate the double integral $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} (x^2 + y^2)^{3/2}\, dy\, dx$

Section 17.5: Triple Integral

Do the following problems from the book;

2, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 23, 26, 28.

SYLLABUS and EXERCISES

CHAPTER 11: **INFINITE SERIES**

Section 11. 1 Sequences

Section 11. 2 Convergent or Divergent Series

Section 11. 3 Positive -Term Series

Section 11. 4 The Ratio and Root Tests

Section 11. 5 Alternating Series and Absolute Convergence

Section 11. 6 Power Series

Section 11. 7 Power Series Representations of Functions

Section 11. 8 Maclaurin and Taylor Series

EXERCISES

11. 1: 3, 5, 7, 11, 12, 13, 16, 17, 18, 20, 23, 24, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 38, 41, 42.

11. 2: 2, 4, 5, 6, 8, 10, 14, 15, 18, 20, 25, 28, 30, 34, 37, 38, 39, 40, 42, 43, 45, 46, 50, 57, 58.

11. 3: 2 - 11, 14, 15, 16, 18, 20, 22, 23, 24, 25, 26, 31, 33, 34, 35, 39, 40, 42, 43, 45, 46, 57, 58.

11. 4: 2, 4, 6, 8, 9, 10, 11, 12, 14, 15, 16, 18, 20, 21, 22, 23, 25, 27, 28, 29, 31, 32, 33, 34, 35, 38.

11. 5: 2 - 7, 9, 10, 12, 13, 16, 19, 20, 21, 22, 27, 28, 29, 32, 33, 34, 35, 38, 40, 41, 43, 44, 45, 46.

11. 6: 5, 6, 7, 8, 14, 15, 19, 23, 25, 27, 30, 35, 36, 41, 42, 44, 45, 46.

11. 7: 2, 4, 6, 7, 10, 13, 14, 16, 19, 22, 25, 29, 30, 32, 33, 34, 37.

11. 8: 2, 4, 8, 10, 13, 15, 18, 19, 21, 26, 29, 32, 34, 36, 38, 39, 42.

CHAPTER 16: PARTIAL DIFFERENTIATION

Section 16.1 Functions of Several Variables

Section 16.2 Limits and Continuity

Section 16.3 Partial Derivatives

Section 16.4 Increments and Differentials

Section 16.5 Chain Rules

Section 16.8 Extrema of Functions of Several Variables

Section 16.9 Lagrange Multiplier

EXERCISES

16.1: 1, 3, 5, 8, 9, 14, 21, 22, 23, 24, 37.

16.2 : 3, 5, 6, 9, 12, 14, 16, 19, 20, 25, 26, 28, 29, 37, 38, 42.

16.3 : 4, 6, 8, 12, 13, 16, 17, 21, 22, 27, 29, 32, 34, 36, 39, 42, 47.

16.4 : 2, 9, 11, 12, 16, 18, 20, 39, 41.

16.5 : 2, 4, 6, 10, 12, 14, 18, 19, 22, 33,37, 38, 41, 42.

16.8 : 1, 9, 11, 15, 20, 21, 23, 24, 26,29,31,32.

16.9 : 1, 2, 3, 11.

CHAPTER 17: MULTIPLE INTEGRAL

Section 17.1 Double Integral

Section 17.2 Area and Volume

Section 17.3 Double Integral by Polar Coordinate

Section 17.5 Triple Integral

EXERCISES

17.1: 1 - 10, 13, 16, 18, 19, 20, 21, 23, 25, 26, 27, 29, 31, 32, 33, 37, 38, 39, 43, 44, 50.

17.2: 2, 4, 6, 7, 11, 14, 18, 22, 24, 27, 28, 30, 31, 32.

17.3: 1 - 12, 13, 15, 17, 18, 19, 21, 23, 24.

17.5: 2, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 23, 26, 28.