

Department of Mathematics
King Saud University
Final Examination
M-203(Differential & Integral Calculus)
Summer Term (1433/1434)

Max. Marks: 40

Time: 180 minutes

Marking Scheme: Q.No:1[3+3+3+3], Q.No:2[3+3+3+3], Q.No:3[4+4+4+4]

Q. No: 1 (a) Determine whether the sequence $\{\sqrt{n+1} - \sqrt{n}\}$ converges or diverges and if it converges, find its limit.

(b) Test the convergence of the series $\sum_{n=1}^{\infty} n^2 \tan^2\left(\frac{1}{n}\right)$.

(c) Find the radius of convergence and interval of convergence for the power series

$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n^2} (x-1)^n.$$

(d) Use Maclaurin Series of $\ln(1+x)$ to approximate the integral $\int_1^{1.1} \frac{\ln(x^2+1)}{x} dx$ using first two non-zero terms.

Q. No: 2 (a) Evaluate $\int_0^1 \int_y^1 \frac{\sin x}{x} dx dy$.

(b) Find the surface area of portion of **upper half** of the sphere $x^2 + y^2 + z^2 = 2$ cut off by the cylinder $x^2 + y^2 = 1$.

(c) A solid is bounded by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 2$ where the density at any point $P(x, y, z)$ is $\sqrt{x^2 + y^2}$. Find the mass of the solid.

(d) Use **spherical coordinates** to evaluate the integral

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} \frac{z^2}{x^2 + y^2 + z^2} dz dy dx.$$

Q. No: 3 (a) Show that the line integral is independent of path by finding the potential

function and hence find its value $\int_{(-1,1,1)}^{(0,e,2)} e^{-x} \ln y dx - \frac{e^{-x}}{y} dy + z dz$.

(b) Use **Green's theorem** to evaluate the line integral $\oint_C xy dx + \left(\frac{1}{2}x^2 + xy\right) dy$, where C is

the closed curve consisting of upper half of the ellipse $x^2 + \frac{y^2}{1/4} = 1$ and the interval $[-1,1]$.

(c) Use the **Divergence theorem** to evaluate the integral $\iint_S \vec{F} \cdot \vec{n} dS$ where

$\vec{F} = \sin y \vec{i} + y \vec{j} + e^x y \vec{k}$ and the surface S is bounded by $z = 1 - x^2 - y^2$ and $z = 0$.

(d) Use **Stokes's theorem** to evaluate $\iint_S (\text{curl } \vec{F}) \cdot \vec{n} dS$, if

$\vec{F}(x, y, z) = yz \vec{i} + xz^2 \vec{j} - xyz \vec{k}$ and S is the surface $z = x^2 + y^2$, $0 \leq z \leq 4$.