Department of Mathematics, College of Science

M-203, Mid-term Examination, Semester-I, 1443 H

Max. Time- 2 Hours　　　　　　　　　　　　　　　　Max. Marks-30

Q.1 Find the interval of convergence and radius of convergence for the power series [5]

$$\sum_{n=0}^{\infty} \frac{n^2}{4^n}(x-2)^{2n}.$$

Q.2 Find the Maclaurin series of the function $f(x) = e^{-x^2}$ and use the first three nonzero terms of the series to approximate

$$\int_0^{0.1} e^{-x^2}\,dx.$$

[5]

Q.3 Evaluate the double integral [5]

$$\int_0^1 \int_y^1 e^{-x^2}\,dx\,dy.$$

Q.4 Evaluate the double integral [5]

$$\int\int_{\mathbf{R}} (2x-y)\,dA,$$

where **R** is the region bounded by the parabola $x = y^2$ and the line $x - y = 2$.

Q.5 Evaluate the integral by changing it to polar coordinates [5]

$$\int_{-1}^1 \int_0^{\sqrt{1-y^2}} \frac{1}{1+x^2+y^2}\,dx\,dy.$$

Q. 6 Find the area of the surface S, where S is the part of the sphere $x^2+y^2+z^2 = 4$ that lies above the plane $z = \sqrt{3}$. [5]

Question 1 :

Find the interval of convergence and radius of convergence for the power series $\sum_{n=0}^{+\infty} \frac{n^2}{4^n}(x-2)^{2n}$.

Solution of the Question 1:

The interval of convergence is $(0,4)$, the radius of convergence is $R=2$.

Question 2 :

Use the non-zeros terms of the power series to approximate $\int_0^{0.1} e^{-x^2}dx$ up to four decimal places.

Solution of the Question 2:

$e^{-x^2} = 1 - x^2 + \frac{x^4}{2} + \varepsilon(x)$.

$$\int_0^{0.1} e^{-x^2}dx \approx \int_0^{0.1} (1 - x^2 + \frac{x^4}{2})dx = 0.100334$$

Question 3 :

Evaluate the integral $\int_0^1 \int_y^1 e^{-x^2}dxdy$.

Solution of the Question 3:

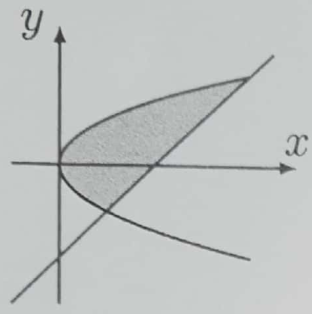
$$\int_0^1 \int_y^1 e^{-x^2}dxdy = \int_0^1 e^{-x^2}\left(\int_0^x dy\right)dx = \int_0^1 xe^{-x^2}dx = \frac{e-1}{2e}.$$

Question 4 :

Evaluate the integral $\iint_R (2x-y)dA$, where R is the region bounded by the parabola $x=y^2$ and $x-y=2$.

Solution of the Question 4:

$$\iint_R (2x-y)\,dA = \int_{-1}^{2}\left(\int_{y^2}^{y+2}(2x-y)\,dx\right)dy$$
$$= \left[4y + y^2 - \frac{1}{5}y^5 + \frac{1}{4}y^4\right]_{-1}^{2} = \frac{243}{20}.$$



Question 5 :

Evaluate the integral by changing to polar coordinates $\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}} \frac{1}{1+x^2+y^2}\,dx\,dy.$

Solution of the Question 5:

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}} \frac{1}{1+x^2+y^2}\,dx\,dy = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{0}^{1}\frac{r}{1+r^2}\,dr\,d\theta = \frac{\pi\ln 2}{2}.$$

Question 6 :

Find the area of the surface S, where S is the part of the sphere $x^2+y^2+z^2=4$ that lies above the plane $z=\sqrt{3}$.

Solution of the Question 6:

$$SA = \int_0^1\int_0^{2\pi}\sqrt{1+\frac{r^2\cos^2\theta}{4-r^2}+\frac{r^2\sin^2\theta}{4-r^2}}\,r\,dr\,d\theta$$
$$= 2\pi\int_0^1 \frac{2r}{\sqrt{4-r^2}}\,dr = 4\pi(2-\sqrt{3}).$$