

Find the orthogonal Trajectories of the family of curves

$$12. y^2 = c(1-x^2)$$

مضي

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12. $y^2 = c(1-x^2)$

$$c = \frac{y^2}{1-x^2}$$

$$0 = \frac{2yy'(1-x^2) - y^2(-2x)}{(1-x^2)^2}$$

$$2yy'(1-x^2) = -2y^2x$$

$$y' = \frac{-2y^2x}{2y(1-x^2)} = \frac{-xy}{1-x^2}$$

$$y' = \frac{-1}{f(x,y)} = \frac{1-x^2}{xy}$$

$$xy \, dy = (1-x^2) \, dx$$

$$y \, dy = \left(\frac{1-x^2}{x}\right) \, dx$$

$$\frac{y^2}{2} = \ln|x| - \frac{x^2}{2} + c$$

ROCO

21. Find the member of the orthogonal trajectories for $x^2 + 3y^2 = cy$ that passes through (1, 2).

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21. $x^2 + 3y^2 = cy$, (1, 2)

$$c = \frac{x^2 + 3y^2}{y}$$

$$0 = \frac{y(2x + 6yy') - y'(x^2 + 3y^2)}{y^2}$$

$$0 = 2xy + 6y^2y' - y'(x^2 + 3y^2)$$

$$-2xy = y'(6y^2 - x^2 - 3y^2)$$

$$y' = \frac{-2xy}{3y^2 - x^2}$$

$$y' = \frac{-1}{f(x,y)} = \frac{3y^2 - x^2}{2xy}$$

$$2xy dy = (3y^2 - x^2) dx$$

$$u = \frac{y}{x} \Rightarrow y = ux \quad dy = u dx + x du$$

$$2ux^2(u dx + x du) = (3u^2x^2 - x^2) dx$$

$$2u^2x^2 dx + 2ux^3 du = (3u^2x^2 - x^2) dx$$

$$2ux^3 du = (3u^2x^2 - x^2 - 2u^2x^2) dx$$

$$2ux^3 du = (u^2x^2 - x^2) dx \quad (\div x^2)$$

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$$2u \times du = (u^2 - 1) dx$$

$$\frac{2u du}{u^2 - 1} = \frac{dx}{x}$$

$$\ln|u^2 - 1| = \ln|x| + C$$

$$\ln\left|\frac{y^2}{x^2} - 1\right| - \ln|x| = C$$

at (1, 2)

$$\ln|3| - 0 = C$$

$$C = \ln 3$$

$$\ln\left|\frac{y^2}{x^2} - 1\right| - \ln|x| - \ln 3 = 0$$

10) The population of a town grows at a rate proportional to the population at time t . The initial population of 500 increases by 15% in 10 years. What will be the population in 30 years?.

10. $P(0) = 500$ $P(10) = 575$
 $P(30) = ?$

$$\frac{dP(t)}{dt} = kP(t)$$

$$\frac{dP(t)}{P(t)} = k dt$$

$$\ln|P(t)| = kt + C$$

$$P(t) = C_1 e^{kt}$$

$$P(0) = 500 = C_1$$

$$P(10) = 575 = 500 e^{10k}$$

$$1.15 = e^{10k}$$

$$\ln(1.15) = 10k$$

$$k = 0.014$$

$$P(t) = 500 e^{0.014t}$$

$$P(30) = 760.98$$

11) The population of a town grows at a rate proportional to the population at time. The initial population P_0 becomes double after 50 years. When the population becomes $4P_0$?

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11. $P(0) = P_0$ $P(50) = 2P_0$
 $P(?) = 4P_0$

$$P(t) = C_1 e^{kt}$$
$$P(0) = P_0 = C_1$$
$$P(50) = 2P_0 = P_0 e^{k \cdot 50}$$
$$\ln(2) = 50k$$
$$k = 0.014$$
$$P(t) = P_0 e^{0.014t}$$
$$4P_0 = P_0 e^{0.014t}$$
$$\ln(4) = 0.014t$$
$$t = 99.02 \text{ year}$$

روبو

13) Initially there were 10 mg. of radioactive material present. After 2 months the mass decreased by 5%. If the rate of decay is proportional to the amount present at any time, then determine the half-life of this material.

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13. $P(0) = 10$ $P(2) = 9.5$
half-life?

$$P(t) = ce^{kt}$$
$$P(0) = 10 = c$$
$$P(2) = 0.95 = 10 e^{2k}$$
$$\ln(0.95) = 2k$$
$$k = -0.026$$
$$P(t) = 10 e^{-0.026t}$$
$$5 = 10 e^{-0.026t}$$
$$\ln(0.5) = -0.026t$$
$$t = 26.7 \text{ m}$$

16) If 0.5% of radium disappears in 12 years. Find what percentage will disappear in 1000 years? What is the half-life of radium?

If 0.5% of radium disappears in 12 years, what percentage will disappear in 1000 years? half-life?

$$P(t) = P_0 e^{-kt}$$

$$P_0 = C$$

$$P(12) = P_0 - 0.005 P_0 = C e^{-k(12)}$$

$$0.995 P_0 = P_0 e^{-12k}$$

$$0.995 = e^{-12k}$$

$$\ln(0.995) = -12k$$

$$-4.17 \times 10^{-4} = k$$

$$P(1000) = P_0 e^{-0.4177}$$

$$= 0.6585 P_0$$

$$\text{Percentage} = 65.85\%$$

$$P(t) = 0.5 P_0 = P_0 e^{-kt}$$

$$0.5 P_0 = P_0 e^{-4.177 \times 10^{-4} t}$$

$$-0.69314 = (-4.177 \times 10^{-4}) t \Rightarrow t = 1659.43 \text{ yrs}$$

$$16. \quad P(0) = 100 \quad P(12) = 99.5$$

$$P(1000) = ??$$

$$P(t) = C_1 e^{kt}$$

$$P(0) = 100 = C$$

$$P(12) = 99.5 = 100 e^{12k}$$

$$\ln(0.995) = 12k$$

$$k = -0.00042$$

$$P(t) = 100 e^{-0.00042t}$$

$$P(1000) = 65.7$$

$$\textcircled{1} \quad 100 - 65.7 = \boxed{34.3\%}$$

$$\textcircled{2} \quad 50 = 100 e^{-0.00042t}$$

$$\ln(0.5) = -0.00042t$$

$$= 1650.4 \text{ years}$$

18) A hot iron rod was left in a room where the temperature was 20°C . After one minute the temperature of the rod was recorded 35°C , and after two minutes it was 27.5°C . What was the initial temperature of the rod?

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18. $T_s = 20$ $T(1) = 35$
 $T(2) = 27.5$

$$T(t) = 20 + c_1 e^{kt}$$

$$T(1) = 35 = 20 + c_1 e^k$$

$$15 = c_1 e^k \Rightarrow \frac{15}{c_1} = e^k$$

$$T(2) = 27.5 = 20 + c_1 e^{2k}$$

$$7.5 = c_1 e^{2k} \Rightarrow \frac{7.5}{c_1} = e^{2k}$$

$$\frac{15^2}{c_1^2} = \frac{7.5}{c_1}$$

$$\frac{225}{7.5} = c_1 = 30$$

$$k = \ln(0.5) = -0.69$$

$$T(t) = 20 + 30 e^{-0.69t}$$

$$T(0) = 50^{\circ}$$

←

22) A body initially at 50°C is put into a 375°C oven. After 75 minutes it is found that the temperature of the body is 125°C . How long the body will take to attain the temperature 150°C ?

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22. $P_0 = 50$ $T_s = 375$ $P(75) = 125$
 $P(?) = 150$

$$P(t) = 50 = 375 + C$$

$$C = -325$$

$$P(75) = 125 = 375 - 325 e^{k \cdot 75}$$

$$\ln(0.77) = 75k$$

$$k = -0.0035$$

$$P(t) = 375 - 325 e^{-0.0035t}$$

$$150 = 375 - 325 e^{-0.0035t}$$

$$\ln(0.69) = -0.0035t$$

$$t = 106.02 \text{ m}$$

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