Question 1.[4,4] a) Consider the initial value problem

$$\begin{cases} (x^2 - 1)y'' + (\tan x)\, y = e^x \\ \quad y(0) = 1, \ y'(0) = 0. \end{cases} \qquad (*)$$

Find the largest interval for which the initial value problem (*) has a unique

solution.

b) Solve the initial value problem

$$\begin{cases} 5y'' + y' = -6x, \\ y(0) = 0, \ y'(0) = -10 \end{cases}$$

Question 2 [4,4]. a) if $y_1 = \frac{\sin x}{\sqrt{x}}$ is a solution of the differential equation

$$4x^2 y'' + 4xy' + (4x^2 - 1)y = 0, \quad x \in (0, \pi),$$

then find the second solution.

b) Determine whether the set of functions

$$f_1(x) = e^{x+1}, \quad f_2(x) = e^{x+2}, \quad f_3(x) = e^{x+3},$$

are linearly dependent or linearly independent on $\mathbb{R}$.

Question 3. [4] Find the general solution of differential equation

$$x^2 y'' - xy' + 2y = x \ ; \quad x > 0.$$

Question 4. [5] Solve the following linear system of differential equations

$$\begin{cases} x' - x - 8y = e^t \\ y' - y - 2x = e^{-t}. \end{cases}$$