

King Saud University,
College of Sciences
Mathematical Department.

Mid-Term 2/S1/2016
Full Mark: 25. Time 1H30mn
28/12/2016

Question 1 [4,4] a) Find the largest interval for which the following initial value problem has a unique solution

$$\begin{cases} (x^2 - 4)y'' + xy' + 2y = \ln x \\ y(3) = 1, y'(3) = 2. \end{cases}$$

b) Solve the nonhomogeneous differential equation

$$y'' + y = \csc x, \quad x \in (0, \frac{\pi}{2})$$

Question 2 [4,3]. a) Show that $y_1 = \sin x$ is a solution of the differential equation

$$y'' + (3 \tan x)y' - 2y = 0, \quad x \in (0, \frac{\pi}{2}).$$

Find the second solution, then obtain the general solution.

b) Show whether the functions

$$f_1(x) = x, \quad f_2(x) = x \ln x,$$

are linearly independent or linearly dependent on $(0, \infty)$.

Question 3 [5] Find the general solution of the differential equation

$$x^2 y'' - 2xy' + 2y = x^3 \ln x; \quad x > 0.$$

Question 4 [5] Solve the following linear system of differential equations.

$$\begin{cases} x' = -x + 3y + e^t \\ y' = -2x + 4y. \end{cases}$$